Solutions Manual to Accompany

# MECHANICAL ENGINEERING DESIGN

## FIFTH EDITION

JOSEPH EDWARD SHIGLEY
Professor Emeritus
The University of Michigan

CHARLES R. MISCHKE
Professor of Mechanical Engineering
Iowa State University

McGraw-Hill Book Company
New York St. Louis San Francisco Auckland Bogotá Caracas
Colorado Springs Hamburg Lisbon London Madrid Mexico
Milan Montreal New Delhi Oklahoma City Panama Paris
San Juan São Paulo Singapore Sydney Tokyo Toronto

# FORWARD

This manual has been designed to assist you in teaching from MECHANICAL ENGINEERING DESIGN, 5th edition.

From personal interviews and correspondence we have learned that it is nearly impossible to prevent students from obtaining copies of instructors manuals such as this one. And we have been urged to write this manual so that it will be less useful to the student. This is not so easy to do since the result may turn out to be less useful to you too. So the approach here is varied, but in general takes advantage of your superior knowledge to fill important gaps in the various presentations.

Computer programs are presented in outline form so as to be useful for translation into any computer language, including programmable calculators. The solutions of some of the exercises at the chapter ends have been omitted so that you can be assured that these exercises are uncompromised. And many of the solutions contain clues to inform you of any unauthorized use of the manual. Occasionally supplementary materials are presented, including problems suitable for exams, quizzes, and projects. There are times when make-up exams are needed on short notice and you need a quick source.

We urge you to guard this manual with great care and keep it under lock and key if at all possible. If left on your desktop it can easily be 'borrowed' by a light-fingered student for photocopying. Student access to this manual is counter-productive to education in the long term; and students are not renowned for their long-term concerns. It is possible that, for their educational good, you should not even acknowledge the existence of this manual.

We invite you to write us about any problems you may encounter in using this material. Since you may be pressed for time, short notes or memos from you are just as useful as formal letters. We promise to respond promptly and to keep you informed of various items of interest that have been brought to our attention.

Joseph E. Shigley
125 Timber Trail
Pinecone Beach
Roscommon, MI 48653

Charles R. Mischke
Department of Mechanical Eng.
Iowa State University
Ames, IA 50011

CHAPTER 1

## ATAN 2 Function

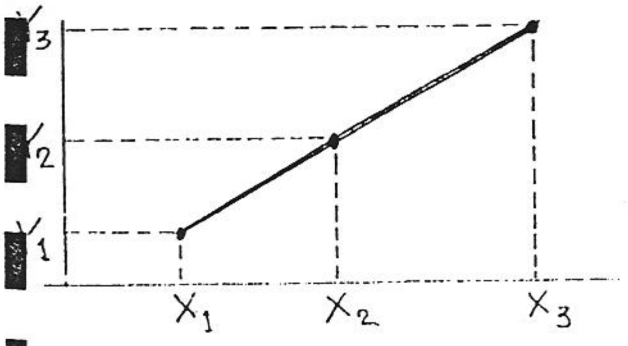
Many computers have only the $\tan^{-1}A$ function. This program computes $\theta = \tan^{-1}(Y/X)$ and yields a positive value of θ for all values of X and Y.

1. Enter X and Y
2. Next if Y = 0; else 7
3. Next if X = 0; else 5
4. θ = 0
5. Next if X < 0; else 4
6. θ = π
7. Next if X = 0; else 11
8. Next if Y > 0; else 10
9. θ = π/2
10. θ = 3π/2
11. Next if X > 0; else 15
12. Next if Y > 0; else 14
13. $\theta = \tan^{-1}(Y/X)$
14. $\theta = 2\pi - \tan^{-1}(\text{abs } Y/X)$
15. Next if Y > 0; else 17
16. $\theta = \pi - \tan^{-1}(Y/\text{abs } X)$
17. $\theta = \pi + \tan^{-1}(\text{abs } Y/\text{abs } X)$

## LINEAR INTERPOLATION

Find $Y_2$ when $X_1$, $Y_1$, $X_2$, $X_3$, and $Y_3$ are given:

$$Y_2 = Y_1 + \frac{(X_2 - X_1)(Y_3 - Y_1)}{X_3 - X_1}$$




1-3. Use Table A-2; results are rounded.

(a) 138 MPa; (b) 1.56 kN;

(c) 136 N · m; (d) 1550 $mm^2$;

(e) 724 $cm^4$; (f) 9.33 $km^2$; (g) 145 GPa;

(h) 72.5 km/h; (i) 0.983 liters

---

1-5. Results are rounded.

(a) σ = 13.1 MPa; (b) σ = 70 MPa;

(c) y = 2.42 μm; (d) θ = 5.18°

---

1-7. Results are rounded.

(a) τ = 381 MPa; (b) σ = 199 MPa;

(c) Z = 3330 $mm^3$; (d) k = 287 N/m

---

1-8. Problem is to show discreteness in size is also imposed on designer. For a given stress level σ in solid and hollow shafts

$$\sigma = \frac{MD/2}{\pi D^4/64} = \frac{Md/2}{\pi(d^4 - d_i^4)/64}$$

from which (problem gives $D/D^4$ as 0.5)

$$\frac{D}{D^4} = 0.5 = \frac{d}{d^4 - d_i'^4} + e$$

where e is error due to rounddown of $d_i$. Hence the equation

$$d_i' = [d^4 - (2d + e)]^{1/4}$$

| d | $d_i$ | $d_i'$ | e |
|---|---|---|---|
| 2.000 | 1.861 | 1.750 | 0.198 |
| 1.875 | 1.713 | 1.625 | 0.152 |
| 1.750 | 1.557 | 1.500 | 0.095 |
| 1.625 | 1.389 | 1.375 | 0.022 |
| 1.500 | 1.198 | 1.125 | 0.067 |
| 1.375 | 0.957 | 0.875 | 0.040 |

1-9.

| x | $x^5$ | | |
|---|---|---|---|
| 0.00 | 0.000 000 000 | 1 | 0.000 000 000 |
| 0.25 | 0.000 976 563 | 4 | 0.003 906 250 |
| 0.50 | 0.031 250 000 | 2 | 0.062 500 000 |
| 0.75 | 0.237 304 688 | 4 | 0.949 218 750 |
| 1.00 | 1.000 000 000 | 1 | 1.000 000 000 |
| | | | 2.015 625 000 |

$$I_4 = \frac{0.25}{3}(2.015\ 625) = 0.167\ 968\ 750$$

| x | $x^5$ | | |
|---|---|---|---|
| 0.00 | 0.000 000 000 | 1 | 0.000 000 000 |
| 0.50 | 0.031 250 000 | 4 | 0.125 000 000 |
| 1.00 | 1.000 000 000 | 1 | 1.000 000 000 |
| | | | 1.125 000 000 |

$$I_2 = \frac{0.50}{3}(1.125\ 000\ 000) = 0.187\ 500$$

$$E = \frac{0.167\ 968\ 750 - 0.187\ 500\ 000}{15}$$

$$= -0.001\ 302\ 083$$

$$I = I_4 + E$$

$$= 0.167\ 968\ 750 - 0.001\ 302\ 083$$

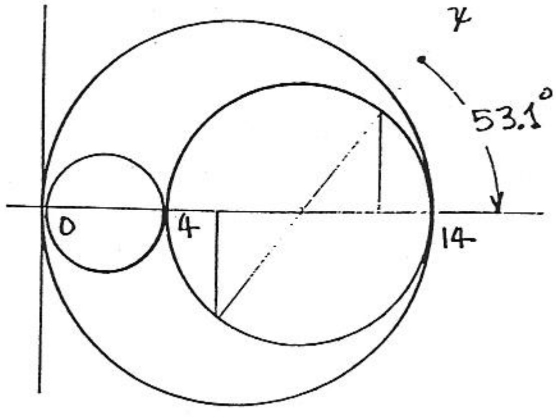
$$= 0.166\ 666\ 667$$

$$\int_0^1 x^5\,dx = \left.\frac{x^6}{6}\right|_0^1 = \frac{1}{6}$$

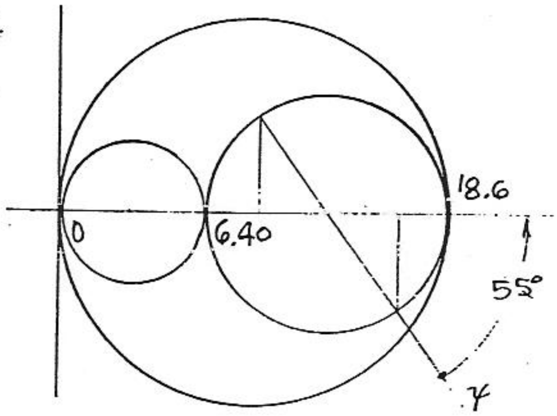
1-10. One can design a program to accept either a column vector of ordinates or to generate them by calling a subprogram created for the purpose. Defining N as number of panels results in N + 1 ordinates. If N is an even number divisible by 4, the error term can be created from ordinates already known.

1. Enter N, a, b
2. $h = (b - a)/N$
3. Initialize sum = 0
4. Stuff column vector with ordinates
5. sum = sum + $y_0$ + $y_n$
6. sum = sum + $4\sum y_i$ (odd interior)
7. sum = sum + $2\sum y_i$ (even interior)
8. I = h(sum)/3

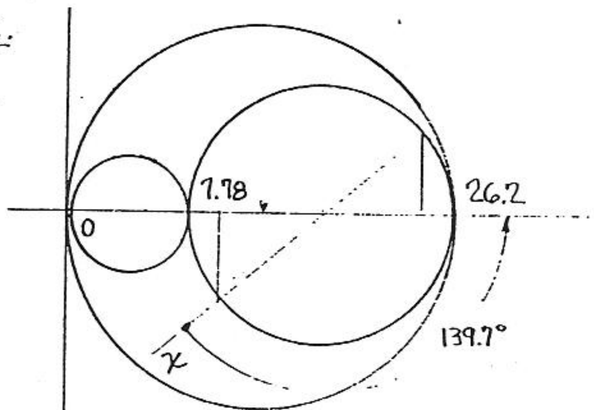
2-1a.



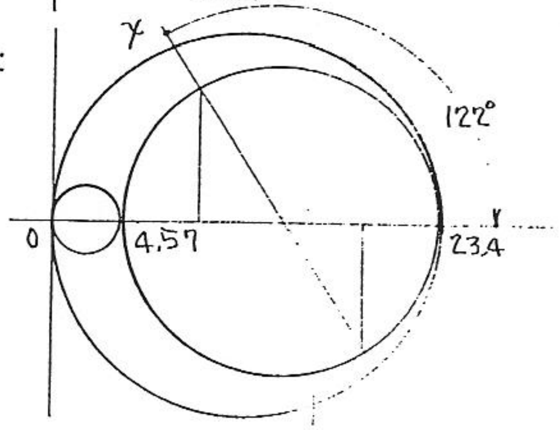

b



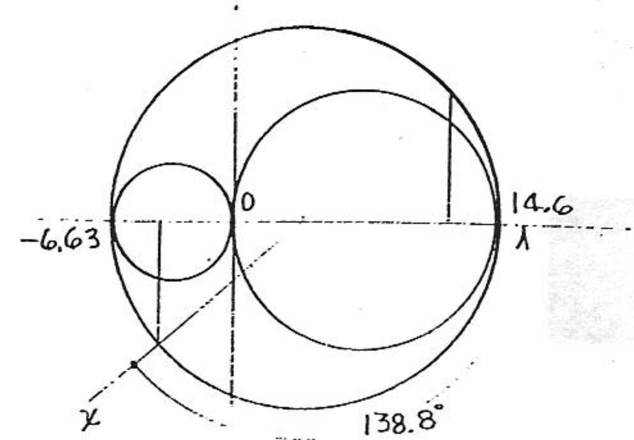

c.



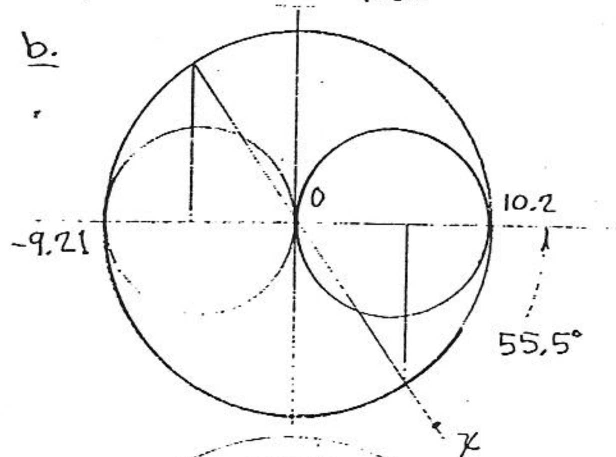

d.



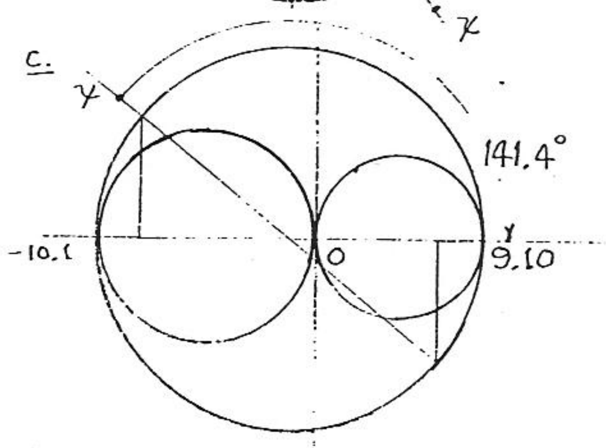

2-2a.



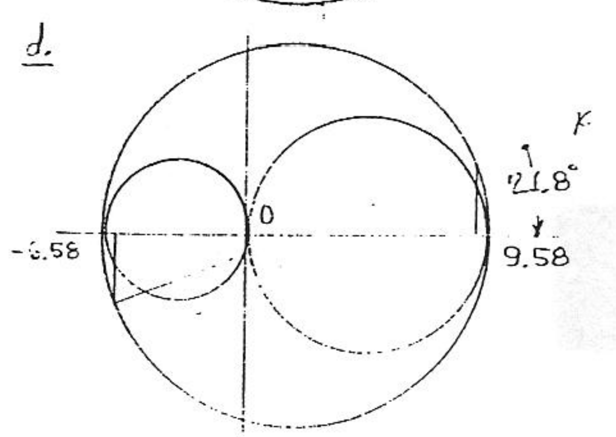

b.



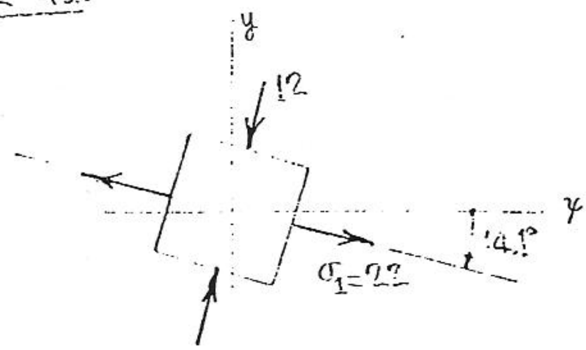

c.



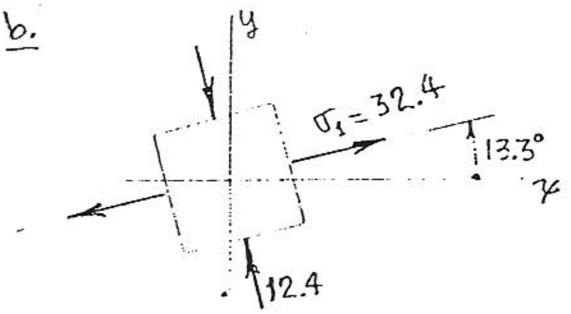

d.



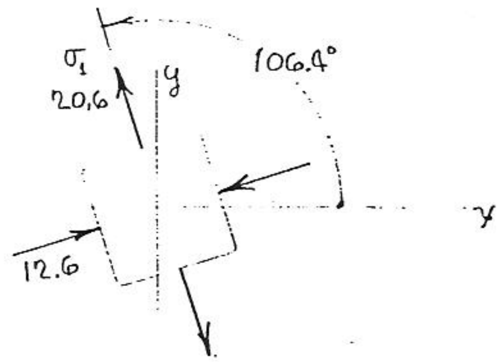
2-4a.



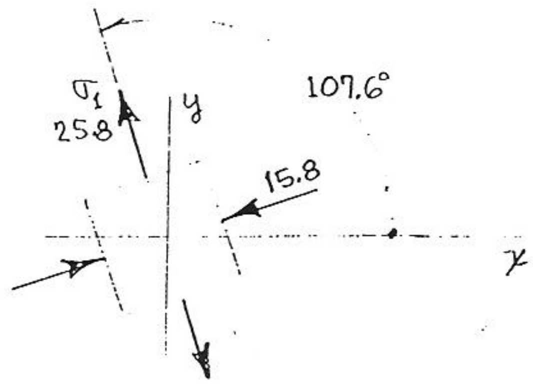

b.




c.




d.




2-5a. $\sigma_1 = 10$, $\sigma_2 = 0$, $\sigma_3 = -4$;

$\tau_{1/2} = 5$, $\tau_{2/3} = 2$, $\tau_{1/3} = 7$;

$\tau_{oct} = 5.89$

b. $\sigma_1 = 11.4$, $\sigma_2 = 0$, $\sigma_3 = -1.4$;

$\tau_{1/2} = 5.7$, $\tau_{2/3} = 0.7$, $\tau_{1/3} = 6.4$;

$\tau_{oct} = 5.73$

c. $\sigma_1 = 0$, $\sigma_2 = 0$, $\sigma_3 = -10$;

$\tau_{1/2} = 0$, $\tau_{2/3} = 5$, $\tau_{1/3} = 5$

$\tau_{oct} = 4.71$

d. $\sigma_1 = 12.4$, $\sigma_2 = 0$, $\sigma_3 = -32.4$;

$\tau_{1/2} = 6.18$, $\tau_{2/3} = 16.2$, $\tau_{1/3} = 22.4$;

$\tau_{oct} = 18.9$

---

2-7a. $\sigma_1 = 0$, $\sigma_2 = -23.0$, $\sigma_3 = -87.0$;

$\tau_{1/2} = 12.0$, $\tau_{2/3} = 32.0$, $\tau_{1/3} = 43.5$;

$\tau_{oct} = 36.9$

b. $\sigma_1 = 39.1$, $\sigma_2 = 0$, $\sigma_3 = -69.1$;

$\tau_{1/2} = 19.5$, $\tau_{2/3} = 34.5$, $\tau_{1/3} = 54.1$;

$\tau_{oct} = 44.7$

c. $\sigma_1 = 48.3$, $\sigma_2 = -8.28$, $\sigma_3 = -30$;

$\tau_{1/2} = 28.3$, $\tau_{2/3} = 10.9$, $\tau_{1/3} = 39.1$;

$\tau_{oct} = 33.0$

d. $\sigma_1 = 64.05$, $\sigma_2 = -14.05$, $\sigma_3 = -20$;

$\tau_{1/2} = 39.05$, $\tau_{2/3} = 2.97$, $\tau_{1/3} = 40.0$;

$\tau_{oct} = 37.3$

---

2-9. $A = 0.196\ in^2$; $\sigma = 10.2$ kpsi Ans

$\delta = 0.024$ in Ans; $\varepsilon_\ell = 0.000\ 340$ in/in;

$\nu = 0.292$; $\varepsilon_d = -0.000\ 099$ in/in Ans;

Solutions Manual to Accompany
MECHANICAL ENGINEERING DESIGN
Fifth Edition


0-07-056900-2

2 3 4 5 6 7 8 9 0 WHT WHT 8 9 3 2 1 0

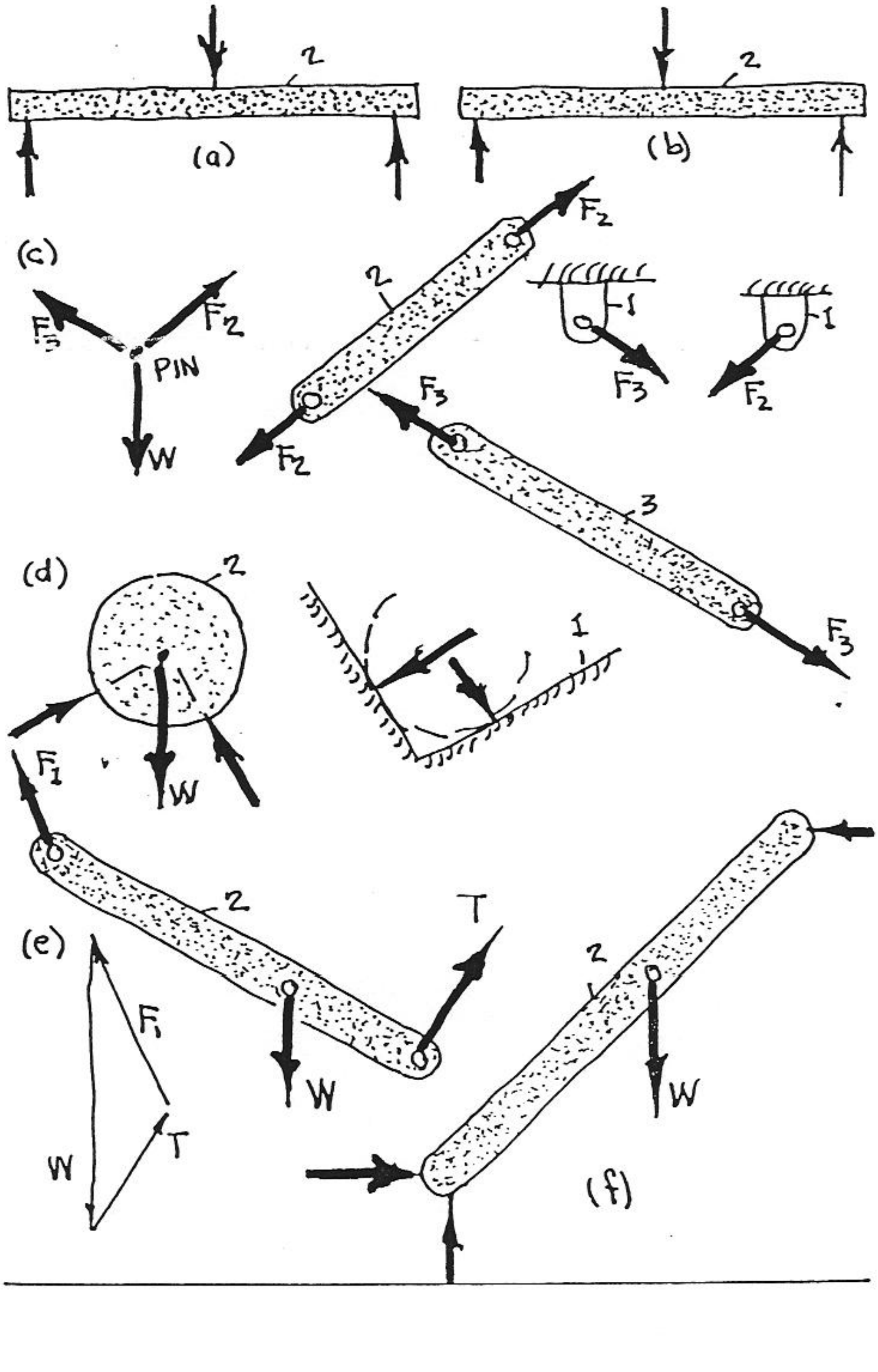
2-9. (Cont.) $\Delta d = -0.000\ 049\ 6$ in Ans

2-11. $\epsilon = 1.90(10^{-3})$;

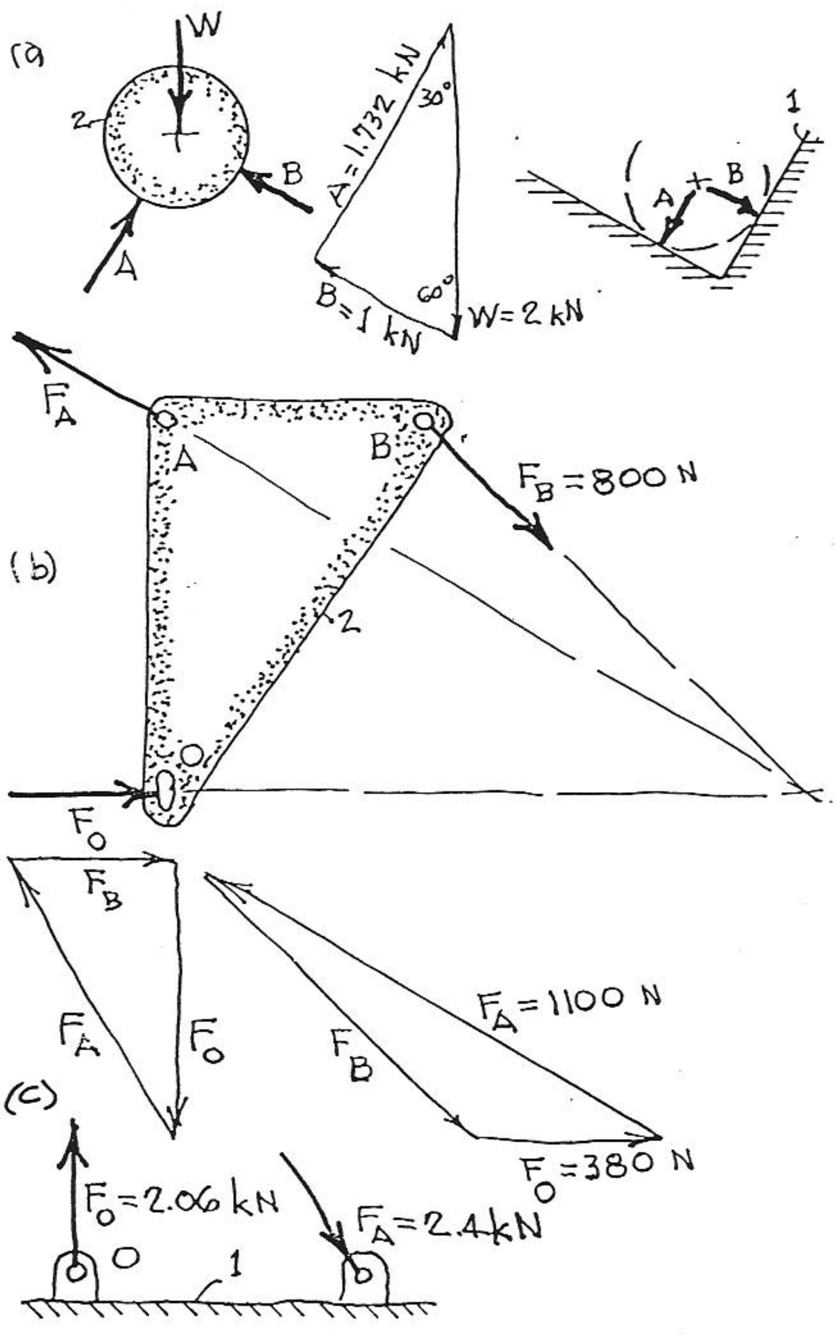
$\delta = 5.7(10^{-3})$ m, or 5.7 mm Ans

QUIZ Suppose counterclockwise shear stresses were plotted up, instead of down, on the Mohr's circle diagram. How would this change affect the analysis?

2-14.

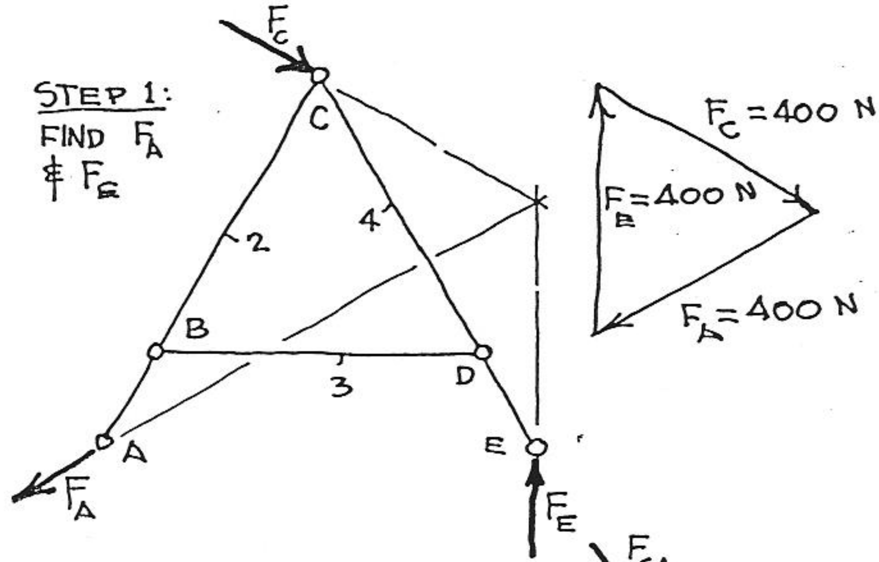


2-15.

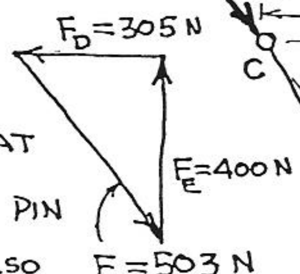

(a)
W
2
B
A
A=1.732 kN
30°
60°
B=1 kN
W=2 kN
1
A
B
$F_A$
A
B
$F_B$=800 N
(b)
2
O
$F_O$
$F_B$
$F_A$
$F_O$
$F_A$=1100 N
$F_B$
(c)
$F_O$=380 N
$F_O$=2.06 kN
$F_A$=2.4 kN
O
1

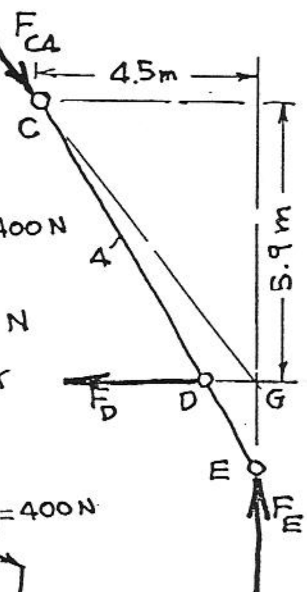
(d)

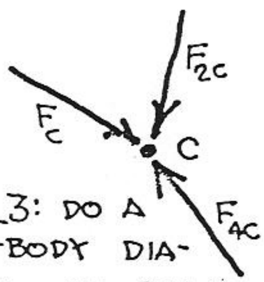
STEP 1:
FIND $F_A$ & $F_E$

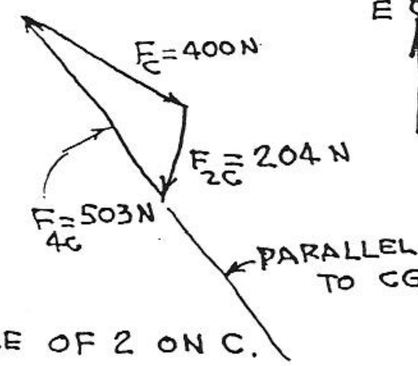



STEP 2: DO A FREE-BODY DIAGRAM OF LINK 2. NOTE THAT $F_{C4}$ IS FORCE OF PIN C ON LINK 4. ALSO $F_D$ CAN ALSO BE OBTAINED BY TAKING MOMENTS ABOUT C.



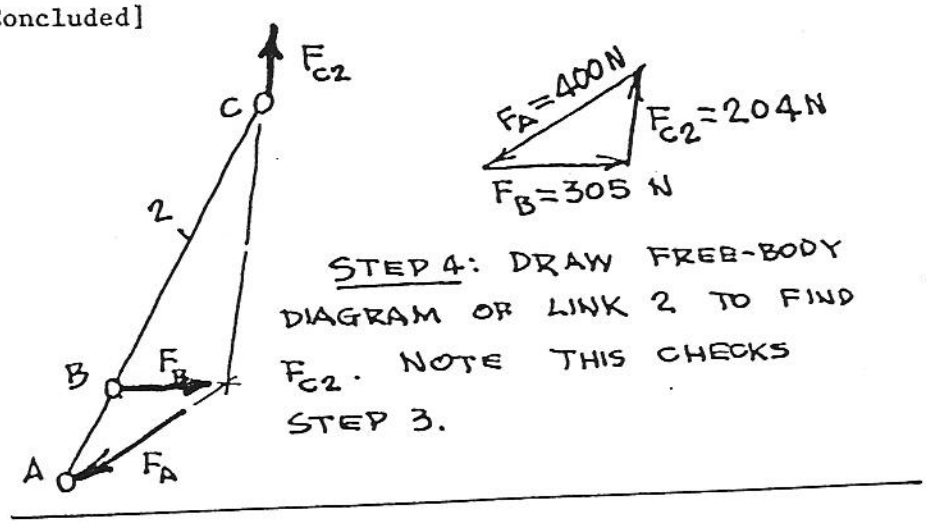



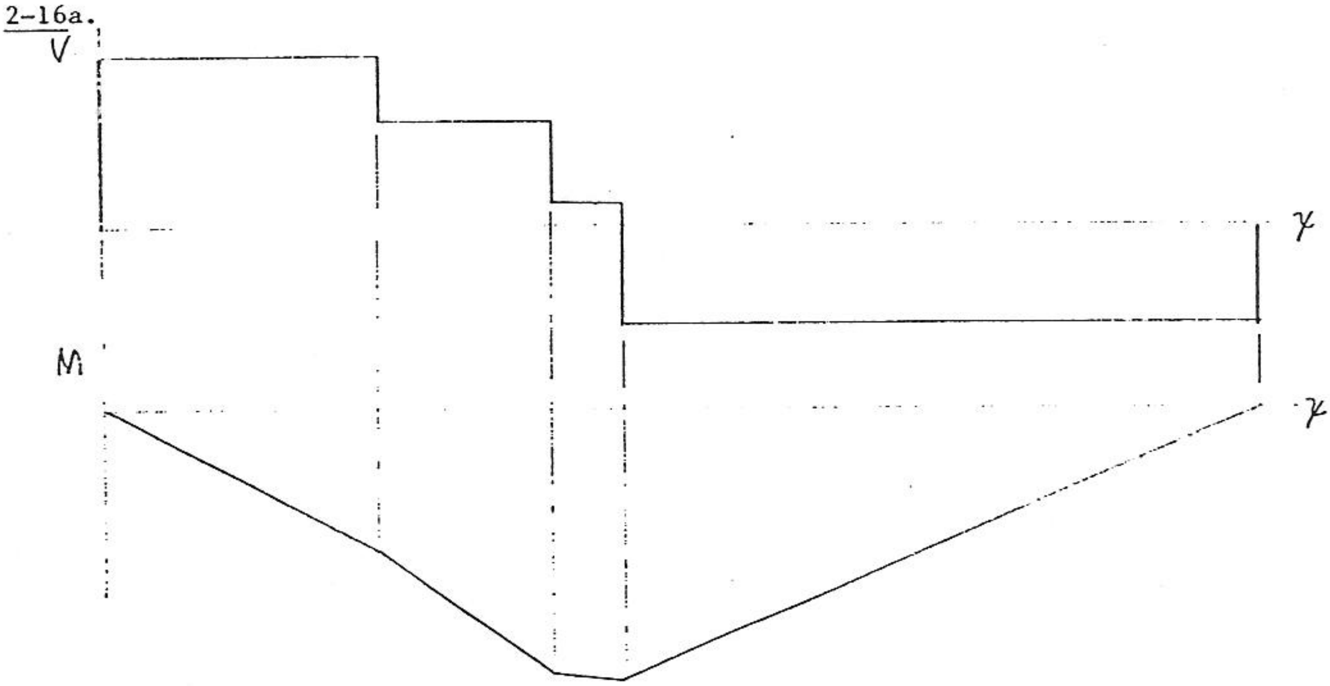




STEP 3: DO A FREE-BODY DIAGRAM OF PIN C.
THIS GIVES $F_{2C}$, FORCE OF 2 ON C.

2-15 (d) [Concluded]




2-16a.




$F_1 = R_1 = 86.94$, $F_2 = -35.00$, $F_3 = -40.00$, $F_4 = -60.00$, $F_5 = R_2 = 48.06$

$x_2 = 3.00$, $x_3 = 7.00$, $x_4 = 8.00$, $x_5 = 18.00$

$V_1 = 86.94$, $V_2 = 51.94$, $V_3 = 11.94$, $V_4 = -48.06$, $V_5 = 0$

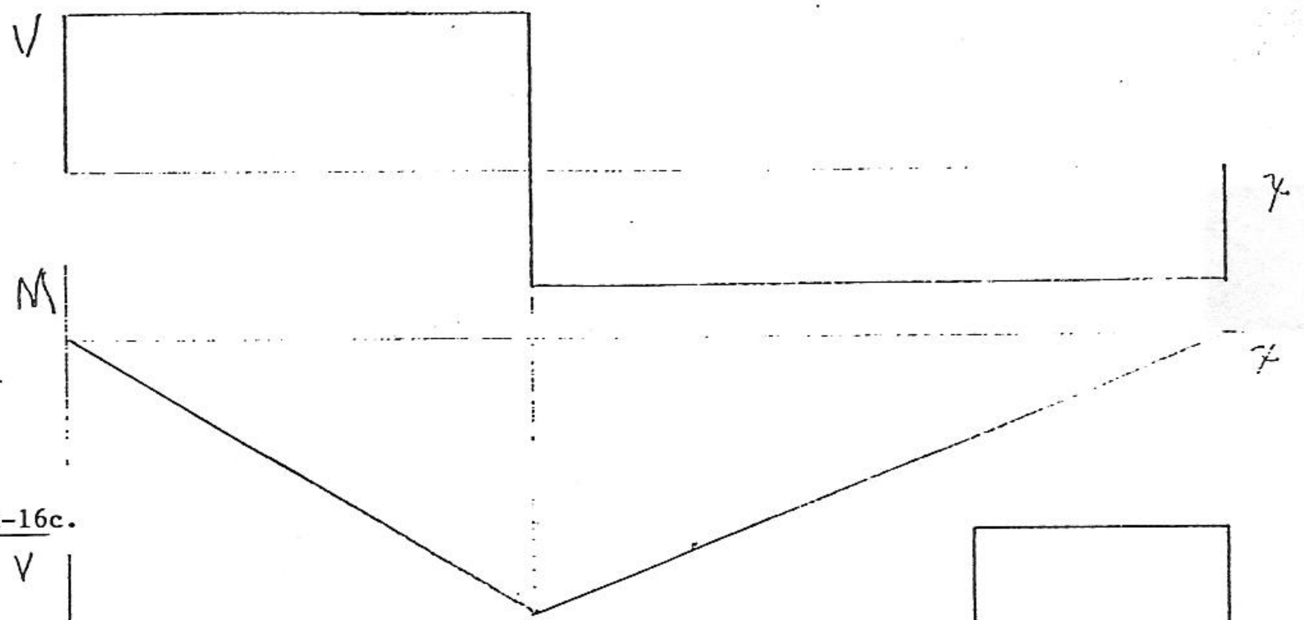
$M_1 = 0$, $M_2 = -260.83$, $M_3 = -468.61$, $M_4 = -480.56$, $M_5 = 0$

2-16b. Using a = 0.4; $F_1 = R_1 = 0.6$, $F_2 = -1.00$, $F_3 = R_2 = 0.40$

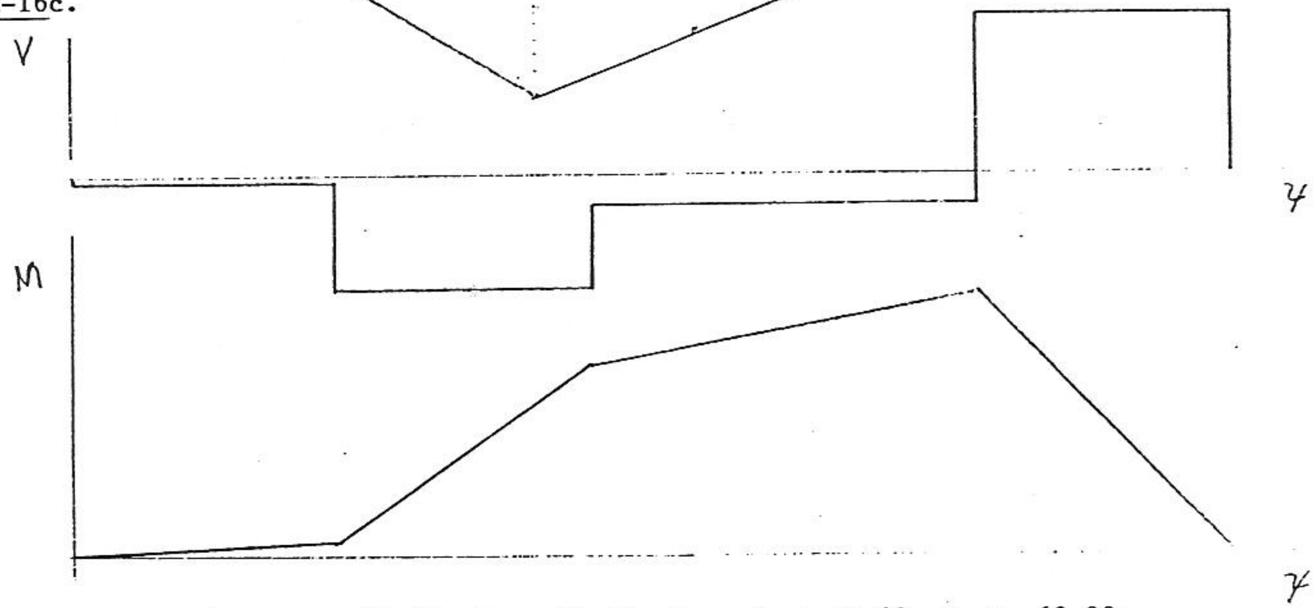
$x_2 = 0.40$, $x_3 = 1.00$; $V_1 = 0.60$, $V_2 = -0.40$, $V_3 = 0.00$;

$M_1 = 0.00$, $M_2 = -0.24$, $V_3 = 0$

2-16b. (Cont.)



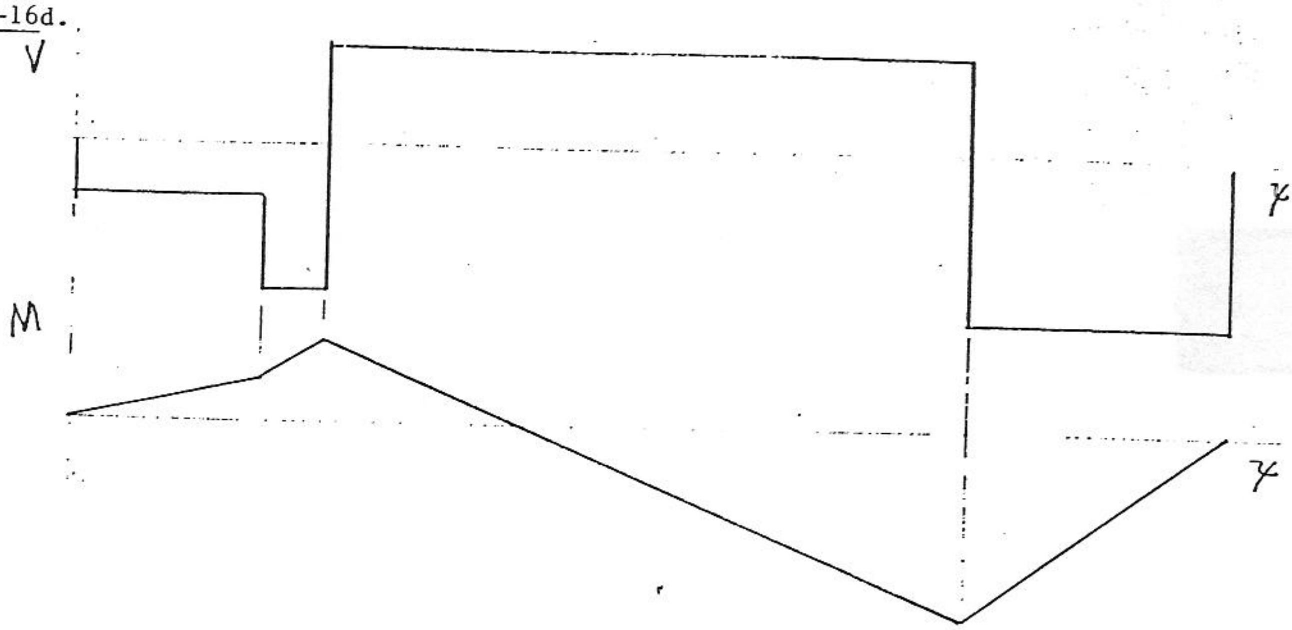

2-16c.




$F_1 = R_1 = -1.43$, $F_2 = -40.00$, $F_3 = 30.00$, $F_4 = R_2 = 71.43$, $F_5 = -60.00$; $x_2 = 4.00$, $x_3 = 8.00$, $x_4 = 14.00$, $x_5 = 18.00$; $V_1 = -1.43$, $V_2 = -41.43$, $V_3 = -11.43$, $V_4 = 60.00$, $V_5 = 0.00$; $M_1 = 0.00$, $M_2 = 5.71$, $M_3 = 171.43$, $M_4 = 240.00$, $M_5 = 0.00$

2-16d. $F_1 = -60.00$, $F_2 = -100.00$, $F_3 = R_1 = 270.00$, $F_4 = -300.00$, $F_5 = R_2 = 190.00$; $x_2 = 3.00$, $x_3 = 4.00$, $x_4 = 14.00$, $x_5 = 18.00$; $V_1 = -60.00$, $V_2 = -160.00$, $V_3 = 110.00$, $V_4 = -190.00$, $V_5 = 0.00$; $M_1 = 0.00$, $M_2 = 180.00$, $M_3 = 340.00$, $M_4 = -760.00$, $M_5 = 0.00$.

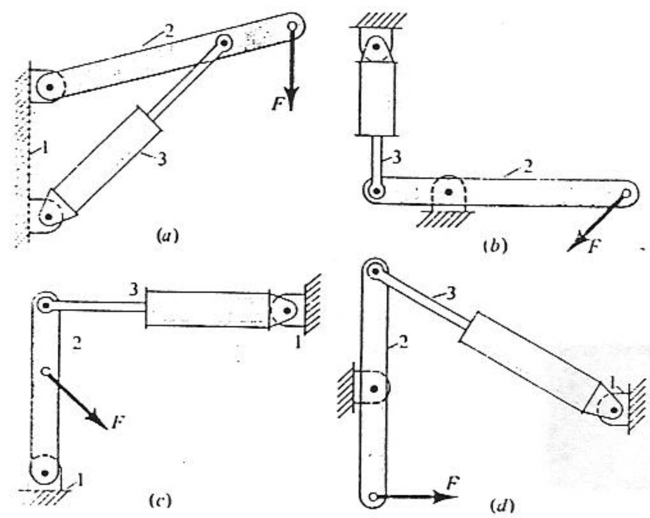
2-16d.




PROGRAM Mohr's circle analysis of plane stress.

1. Enter $\sigma_x$, $\sigma_y$, $\tau_{xy}$; $\tau_{xy}$ is positive when clockwise.
2. $x = (\sigma_x - \sigma_y)/2$; $R = (x^2 + \tau_{xy}{}^2)^{\frac{1}{2}}$
3. If x = 0 next, else 7.
4. If $\tau_{xy} > 0$ next, else 6
5. $\theta = 90°$, go to step 11
6. $\theta = 270°$, go to step 11
7. $AA = -57.296 \tan^{-1}(\tau_{xy}/x)$
8. If x > 0 step 10, else next
9. $\theta = 180 + AA$; go to step 11
10. $\theta = AA$
11. $B = (\sigma_x + \sigma_y)/2$
12. $\sigma_A = B + R$; $\sigma_B = B - R$; $\phi = \theta/2$
13. If $\sigma_A > 0$ next, else step 17
14. If $\sigma_B > 0$ next, else step 18
15. $\sigma_1 = \sigma_A$, $\sigma_2 = \sigma_B$, $\sigma_3 = 0$
16. $\tau_{max} = |\sigma_1/2|$; go to step 20
17. $\sigma_1 = 0$, $\sigma_2 = \sigma_A$, $\sigma_3 = \sigma_B$; go to step 20
18. $\sigma_1 = \sigma_A$, $\sigma_2 = 0$, $\sigma_3 = \sigma_B$
19. $\tau_{max} = |\sigma_1 - \sigma_3|/2$
20. $\phi = \phi$ from x to $\sigma_1$
21. Print $\phi$ ; go to step 24
22. $\tau_{max} = |\sigma_3/2|$
23. $\phi = \phi$ from x to $\sigma_2$; print $\phi$
24. Print $\sigma_1$, $\sigma_2$, $\sigma_3$, $\tau_{max}$

QUIZ Sketch the free-body diagrams for lever 2 in each of the following:



2-18. Moments + or − cause bending stress, so use absolute values.

(a)
$$M = \left| \frac{w}{2} [\ell x - (a + x)^2] \right|$$

Observe moment at center, $x_c = (\ell/2 - a)$

$$M_c = \left| \frac{w\ell}{2} \left(\frac{\ell}{4} - a\right) \right|$$

Observe moment at left reaction, $x = a$

$$M_r = \left| \frac{wa^2}{2} \right|$$

| a | $M_c$ | $M_r$ | M |
|---|---|---|---|
| 0 | 1250 | 0 | 1250 |
| 1 | 750 | 50 | 750 |
| 2 | 250 | 200 | 250 |
| 2.25 | 125 | 253 | 253 |
| 2.50 | 0 | 313 | 313 |
| 3 | 250 | 450 | 450 |
| 4 | 750 | 800 | 800 |
| 5 | 0 | 1250 | 1250 |

(b) Analytically optimum location of support where $|M_r| = |M_c|$

$$\frac{wa^2}{2} = \frac{w\ell}{2} \left(\frac{\ell}{2} - a\right)$$

from which

$$(a/\ell)^2 + (a/\ell) - 0.25 = 0$$

$$(a/\ell) = (\sqrt{2} - 1)/2 = 0.207$$

M in table above has minimum value of

$$M_{min} = wa^2/2 = 0.207^2 w\ell^2/2 = 0.0214 w\ell^2$$

$$= 0.0214(100)(10^2) = 214 \text{ in·lb}$$

---

2-20.

(a)

1. Input R, $\theta$
2. $X = R\cos\theta$
3. $Y = R\sin\theta$
4. Output X, Y

(b)

1. Input x, y
2. $r = [x^2 + y^2]^{1/2}$
3. If $x \geq 0$ and $y = 0$, set $\theta = 0$, go to 12
4. If $x < 0$ and $y = 0$, set $\theta = \pi$, go to 12
5. If $x = 0$ and $y > 0$, set $\theta = \pi/2$, go to 12
6. If $x = 0$ and $y < 0$, set $\theta = 3\pi/2$, go to 12
7. $\phi = \tan^{-1} |y/x|$
8. If $x \geq 0$ and $y > 0$, set $\theta = \phi$
9. If $x < 0$ and $y > 0$, set $\theta = \pi - \phi$
10. If $x < 0$ and $y < 0$, set $\theta = \pi + \phi$
11. If $x > 0$ and $y < 0$, set $\theta = 2\pi - \phi$
12. Output r, $\theta$

(c)

1. Input $\theta$
2. $\bar{x} = \cos\theta$
3. $\bar{y} = \cos(\pi/2 - \theta)$
4. Output $\bar{x}$, $\bar{y}$

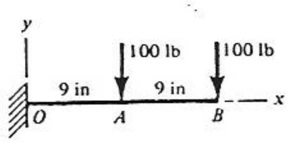
(d)

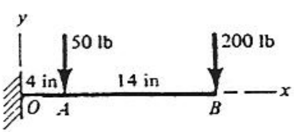
1. sumx = 0, sumy = 0, sumz = 0
2. Input number of forces, n
3. Do through step 7 for i = 1, n
4. Input components $F_i^x$, $F_i^y$, $F_i^z$
5. sumx = sumx + $F_i^x$
6. sumy = sumy + $F_i^y$
7. sumz = sumz + $F_i^z$
8. Output sumx, sumy, sumz

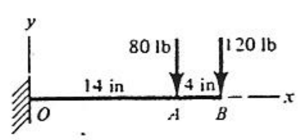
(e)

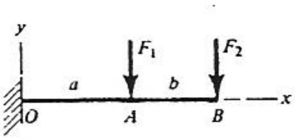
1. Input $C_x$, $C_y$, $C_z$, $C_x{}'$, $C_y{}'$, $C_z{}'$
2. $x = C_yC_z{}' - C_y{}'C_z$
3. $y = C_x{}'C_z - C_xC_z{}'$
4. $z = C_xC_y{}' - C_x{}'C_y$
5. Output x, y, z

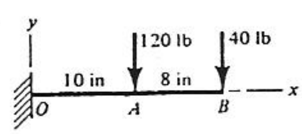
QUIZ PROBLEMS

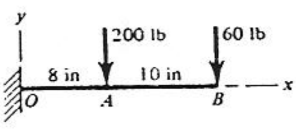


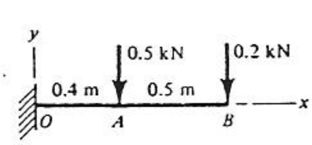



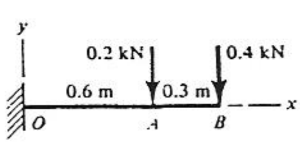



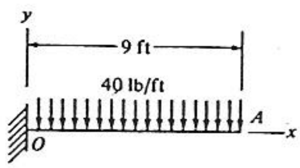



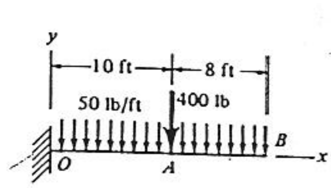



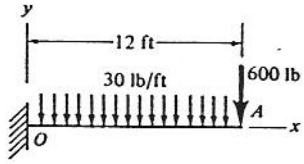



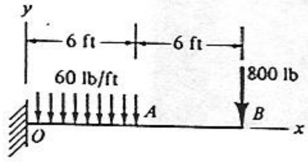



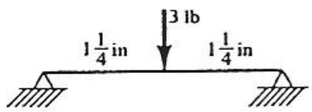



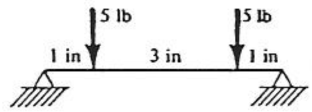



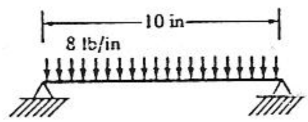



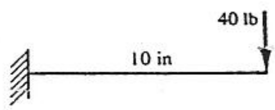



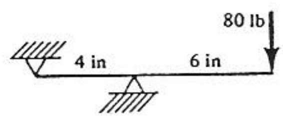



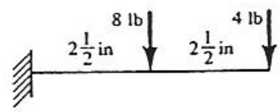



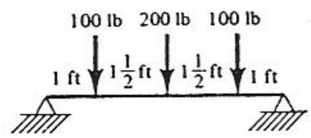



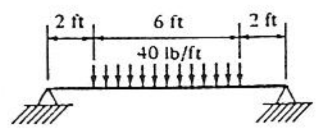



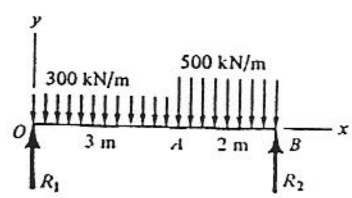



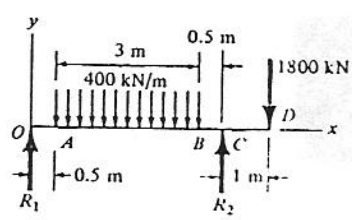



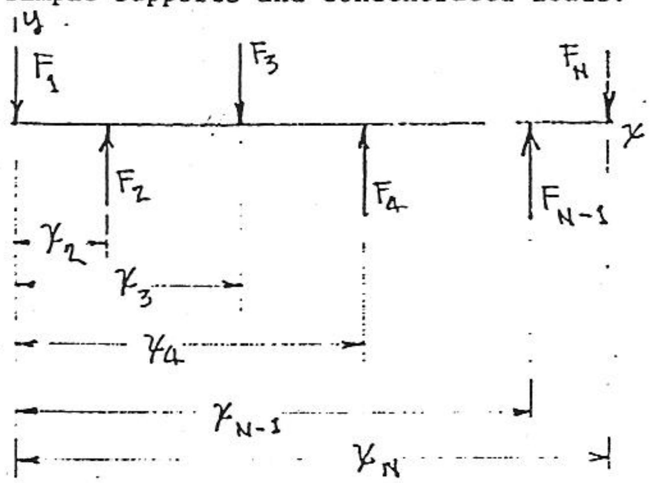



PROGRAM Force analysis of beams with simple supports and concentrated loads.




Forces are positive if upward acting

1. Enter and display names of program and programmer.
2. Enter and display units used.
3. Enter the number of forces plus the number of reactions as N. (A zero-value force can be used for special points of interest.)
4. For i = 1 to N enter $x_i$.
5. For i = 1 to N, next if $F_i$ is a reaction, else 9.
6. If flag is set, 8, else next.
7. $R_A = F_i$, $x_A = x_i$, set flag, go to 10.
8. $R_B = F_i$, $x_B = x_i$, go to 10.
9. Enter $F_i$.
10. Next i.
11. If i = N, next, else 5.
12. A = 0.
13. For i = 1 to N, $A = A + F_i(x_i - x_A)$.
14. $R_B = A/(x_A - x_B)$.
15. B = 0; for i = 1 to N, $B = B + F_i$.
16. $R_A = -B + R_B$.
17. For i = 1 to N, $V_i = V_{i-1} + F_i$, $M_{i+1} = M_i - V_i(x_{i+1} - x_i)$.

---

PROJECT Find or write a program to find the largest number from a group stored in memory.

---

PROGRAM Draw the shear-force diagram. This program can be used with the various beam programs. And similar programs can be written for moment, slope, and deflection of beams.

1. Clear screen.
2. Find $V_{max}$ and $V_{min}$.
3. Length scale; $S_x$ = desired graph length/$x_N$.
4. Height scale: $S_V$ = desired graph height/$(V_{max} - V_{min})$.
5. R = coordinate of right side of screen.
6. $S_0 = \Delta + S_V V_{max}$ where $\Delta$ is a small screen coordinate.
7. For i = 1 to N, $S_i = S_{i-1} - S_V F_i$.
8. Draw x axis as a line from $(R,S_0)$ to $(0,S_0)$.
9. For i = 1 to N draw shear-force diagram as line from $(S_x x_i, S_{i-1})$ to $(S_x x_i, S_i)$ to $(S_x x_{i+1}, S_i)$.

---

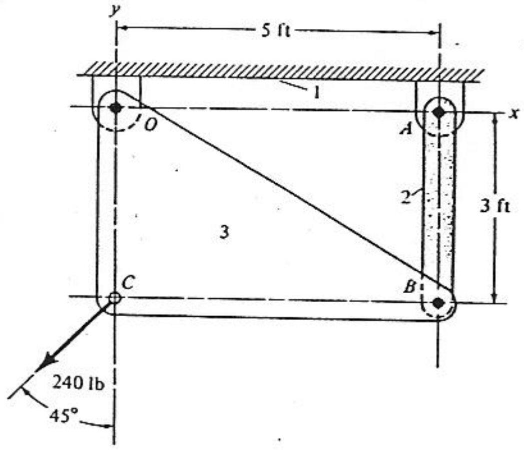
PROGRAM Force analysis of a cantilever with concentrated loads.

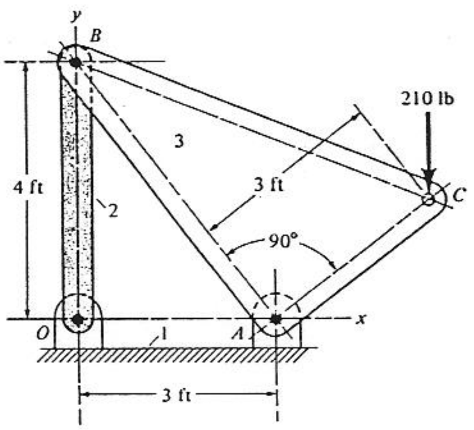
Forces are positive when they act upward.

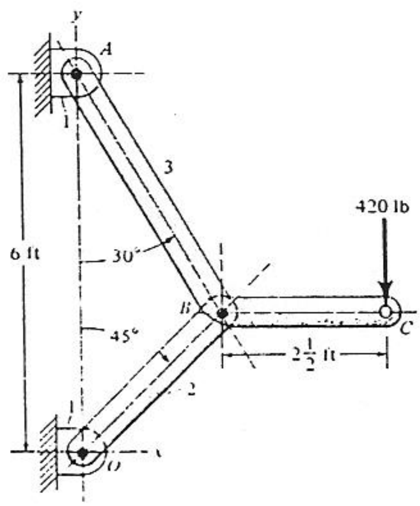
1. Enter and display the units used.
2. Enter number of external forces N.
3. For i = 2 to N + 1, enter location from support and magnitude of each force.
4. A = 0; for i = 2 to N + 1, $A = A + x_i F_i$.
5. $M_1 = A$.
6. B = 0; for i = 2 to N + 1, $B = B - F_i$.

7. For i = 1 to N + 1, $V_i = V_{i-1} + F_i$,
$M_{i+1} = M_i + V_i(x_{i+1} - x_i)$.

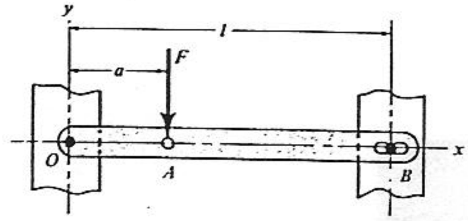
QUIZ  Find the forces acting on part 3.

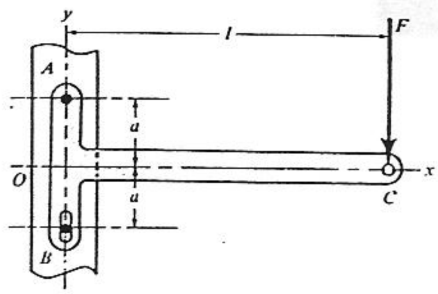


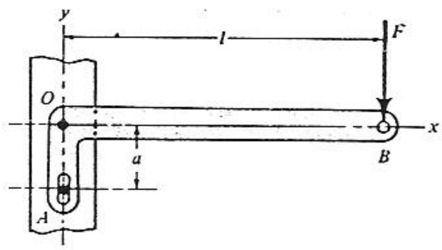



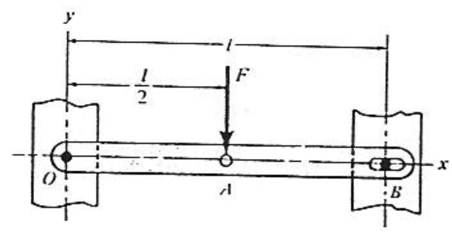



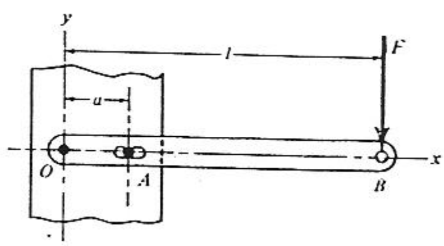

QUIZ  Derive formulas for the reactions.

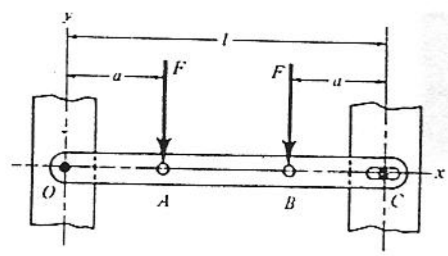


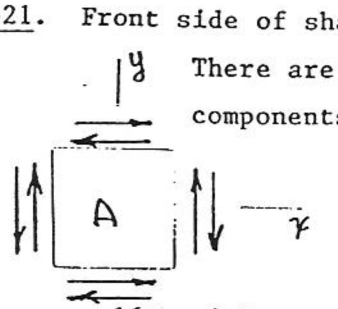



2-21. Front side of shaft:



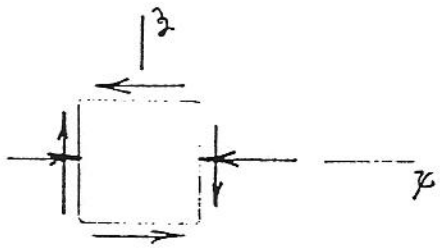

There are two shear-stress components. These sum to

$$\tau_{xy} = -\frac{16T}{\pi d^3} + \frac{4}{3}\frac{V}{A}$$

$$= -\frac{16(1200)}{\pi(0.75)^3} + \frac{4(300)}{3\pi(0.75)^2/4}$$

$$= -14\,500 + 905 = -13\,600 \text{ psi} \quad \underline{\text{Ans.}}$$

Bottom of shaft (looking up):



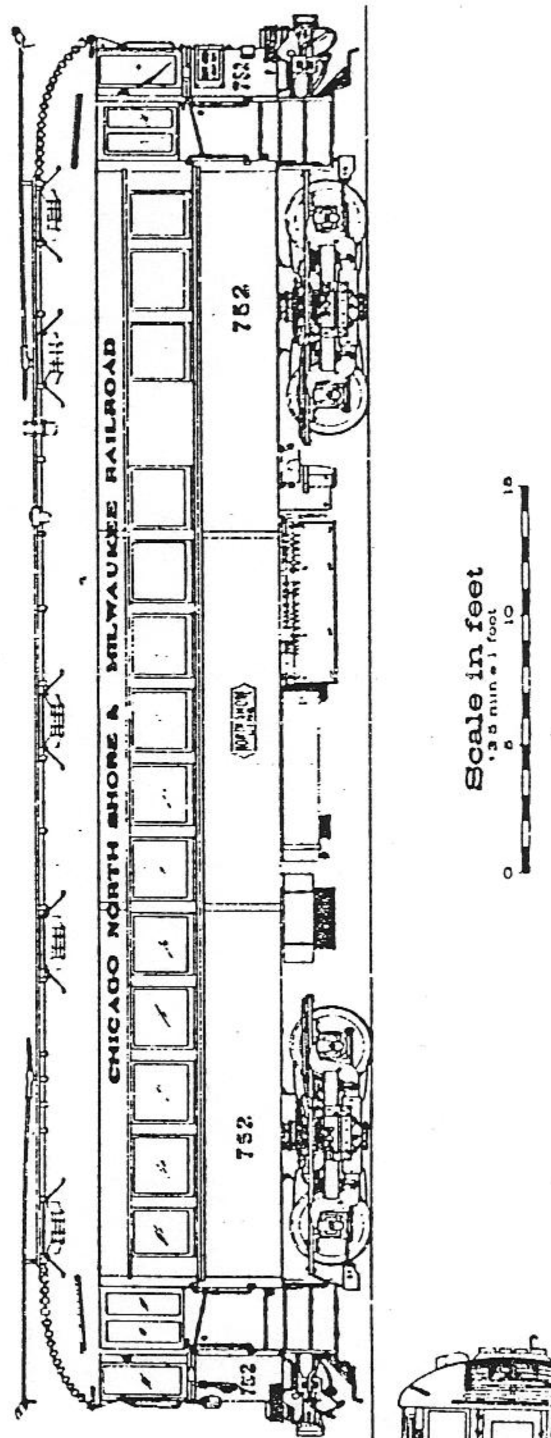

$\tau_{xz} = 14\,500$ psi

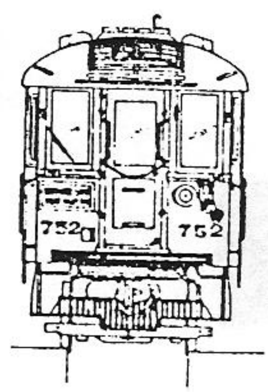
$\sigma_x = -43\,400$ psi

---

PROJECT The Chicago, North Shore and Milwaukee Railroad was an electric railway running between the cities in its corporate title. It had passenger cars as shown in the figure which weighed 104.4 kip, had 32'-8" truck centers, 7' wheelbase trucks and a coupled length of 55'-3¼". When a single car was on a 100-ft long, simply supported deck plate girder bridge

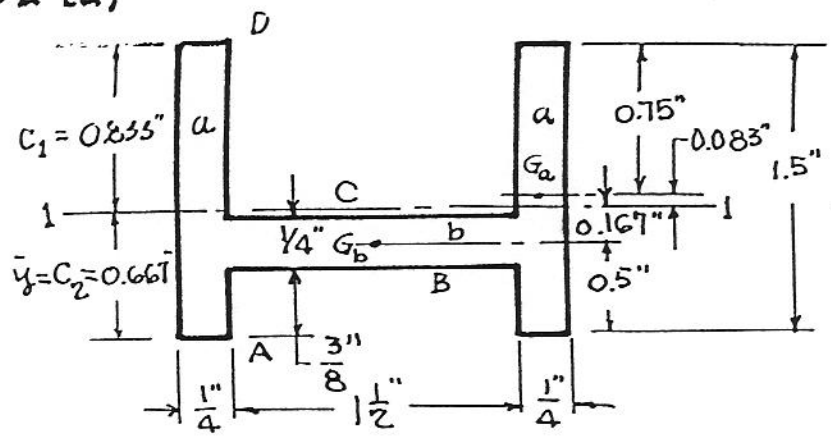
(a) What was the largest bending moment in the bridge?
(b) Where was it located on the bridge?
(c) What was the position of the car on the bridge?
(d) Under which wheel is the maximum bending moment?




(The figure shows the coaches, series 752-776 as originally built. (Copyright 1963 by Central Electric Railfans' Assoc. Bull. 107, p. 145. Reproduced by permission.)

2-22 (a)




$A_a = A_b = 0.25(1.5) = 0.375 \text{ in}^2,$

$A = 3(0.375) = 1.125 \text{ in}^2$

$$\bar{y} = \frac{2(0.375)(0.75) + 0.375(0.5)}{1.125} = 0.667 \text{ in}$$

$$I_a = \frac{0.25(1.5)^3}{12} = 0.070 \text{ in}^4$$

$$I_b = \frac{1.5(0.25)^3}{12} = 0.001\ 95 \text{ in}^4$$

$$I_1 = 2\left[0.070 + 0.376(0.083)^2\right] + \left[0.001\ 95 + 0.375(0.167)^2\right] = 0.158 \text{ in}^4 \quad \underline{\text{Ans}}$$

$$\sigma_A = \frac{10\ 000(0.667)}{0.158} = 42(10)^3 \text{ psi} \quad \underline{\text{Ans.}}$$

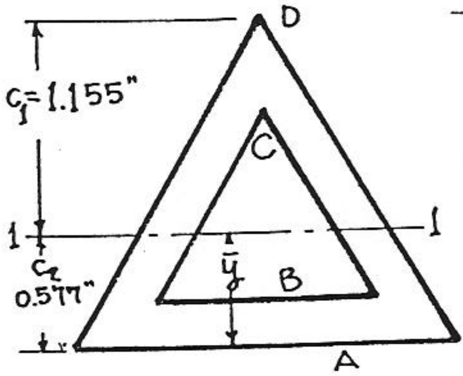
$$\sigma_B = \frac{10\ 000(0.667 - 0.375)}{0.158} = 18.5(10)^3 \text{ psi} \quad \underline{\text{Ans.}}$$

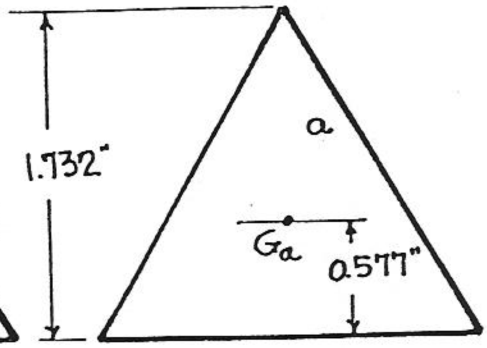
$$\sigma_C = \frac{10\ 000(0.167 - 0.125)}{0.158} = 2.7(10)^3 \text{ psi} \quad \underline{\text{Ans.}}$$

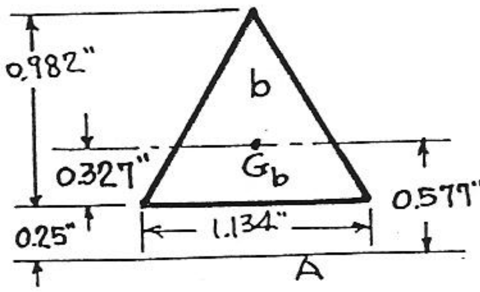
(b)

$$\sigma_D = -\frac{10\ 000(0.833)}{0.158} = 52.7(10)^3 \text{ psi} \quad \underline{\text{Ans.}}$$

2-22
(Cont.)




(b) Here we treat the hole as a negative area.

$$A_a = 1.732 \text{ in}^2$$

$$A_b = 1.134\left(\frac{0.982}{2}\right) = 0.557 \text{ in}^2$$

$$A = 1.732 - 0.557 = 1.175 \text{ in}^2$$

$$\bar{y} = \frac{1.732(0.577) - 0.557(0.577)}{1.175} = 0.577 \text{ in}$$

$$I_a = \frac{bh^3}{36} = \frac{2(1.732)^3}{36} = 0.289 \text{ in}^4$$

$$I_b = \frac{1.134(0.982)^3}{36} = 0.0298 \text{ in}^4$$

$$I_1 = I_a - I_b = 0.289 - 0.0298 = 0.259 \text{ in}^4$$

because the c.g.'s are coincident.

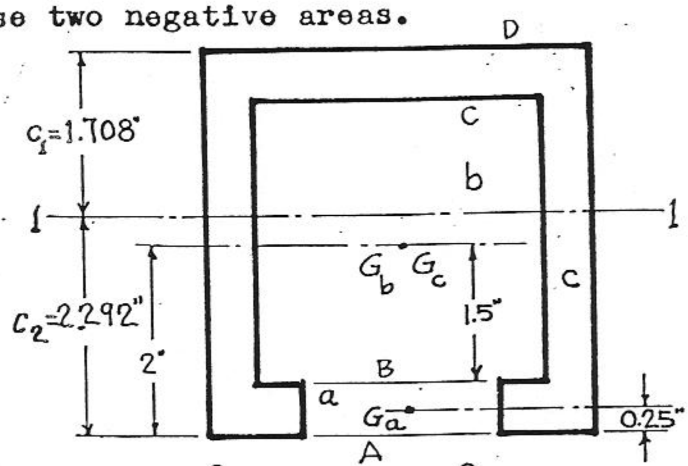
$$\sigma_A = \frac{10\ 000(0.577)}{0.259} = 22.3(10)^3 \text{ psi}$$ Ans.

$$\sigma_B = \frac{10\ 000(0.327)}{0.259} = 12.6(10)^3 \text{ psi}$$ Ans.

$$\sigma_C = -\frac{10\ 000(0.982 - 0.327)}{0.259} = -25.3(10)^3 \text{ psi}$$ Ans.

$$\sigma_D = -\frac{10\ 000(1.155)}{0.259} = -44.6(10)^3 \text{ psi}$$ Ans.

2-23. (a) Use two negative areas.




$A_a = 1\ in^2,\ A_b = 9\ in^2,\ A_c = 16\ in^2,$

$A = 16 - 9 - 1 = 6\ in^2\ ;\ \bar{y}_a = 0.25\ in,$

$\bar{y}_b = 1.5\ in,\ \bar{y}_c = 2\ in$

$$\bar{y} = \frac{16(2) - 9(2) - 1(0.25)}{6} = 2.292\ in$$

$c_1 = 4 - 2.292 = 1.708\ in$

$$I_a = \frac{2(0.5)^3}{12} = 0.020\ 83\ in^4$$

$$I_b = \frac{3(3)^3}{12} = 6.75\ in^4$$

$$I_c = \frac{4(4)^3}{12} = 21.333\ in^4$$

$$I_1 = \left[21.333 + 16(0.292)^2\right] - \left[6.75 + 9(0.292)^2\right] - \left[0.020\ 83 + 1(2.292 - 0.25)^2\right] = 10.99\ in^4$$

$$\sigma_A = \frac{10\ 000(2.292)}{10.99} = 2080\ psi \quad \text{Ans.}$$

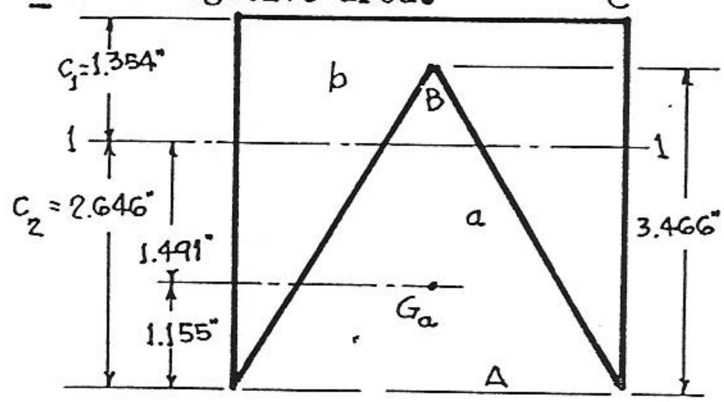
$$\sigma_B = \frac{10\ 000(2.292 - 0.5)}{10.99} = 1630\ psi \quad \text{Ans.}$$

$$\sigma_C = -\frac{10\ 000(1.708 - 0.5)}{10.99} = -1100\ psi \quad \text{Ans.}$$

2-23. (Cont.)

$$\sigma_D = -\frac{10\,000(1.708)}{10.99} = -1550 \text{ psi} \quad \underline{\text{Ans.}}$$

(b) Use <u>a</u> as a negative area.




(b) $A_a = 6.932 \text{ in}^2$, $A_b = 16 \text{ in}_2$,

$A = 9.068 \text{ in}^2$; $\bar{y}_a = 1.155$ in, $\bar{y}_b = 2$ in

$$\bar{y} = \frac{2(16) - 1.155(6.932)}{9.068} = 2.646 \text{ in}$$

$$c_1 = 4 - 2.646 = 1.354 \text{ in}$$

$$I_a = \frac{bh^3}{36} = \frac{4(3.466)^3}{36} = 4.626 \text{ in}^4$$

$$I_b = \frac{4(4)^3}{12} = 21.33 \text{ in}^4$$

$$I_1 = [21.33 + 16(0.646)^2] - [4.626 + 6.932(1.491)^2]$$
$$= 7.97 \text{ in}^4$$

$$\sigma_A = \frac{10\,000(2.646)}{7.97} = 3320 \text{ psi} \quad \underline{\text{Ans.}}$$

$$\sigma_B = -\frac{10\,000(3.466 - 2.646)}{7.97} = -1030 \text{ psi} \quad \underline{\text{Ans.}}$$

$$\sigma_C = -\frac{10\,000(1.354)}{7.97} = -1700 \text{ psi} \quad \underline{\text{Ans.}}$$

2-25a. The moment is maximum and constant from A to B. $M_{max} = -50(20) = -1000$ lb·in

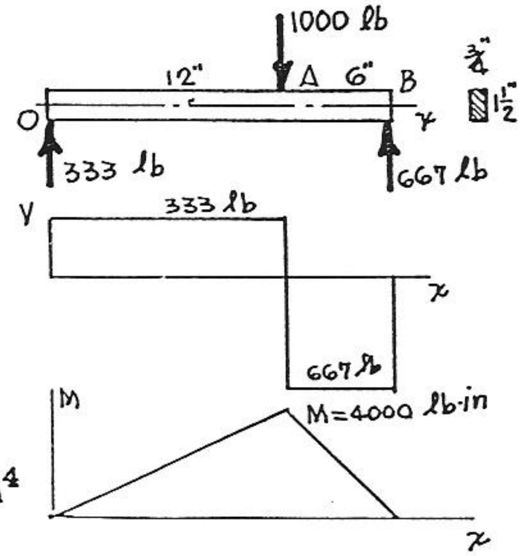
$\rho = EI/M = 1.6(10^6)(1/3)/(-1000) = -533$ in; so center of curvature at x = 30 in, and y = -533 in Ans.

(b) Moment is maximum and constant from A to B. $M_{max} = 250$ lb·in. Then

$\rho = EI/M = 1.6(10^6)(1/3)/250 = 2130$ in. So center of curvature is at x = 20 in y = 2130 in. Ans.

---

2-26a.




$$\tau = \frac{3(667)}{2(1.125)}$$

= 890 psi on neutral axis from A to B.

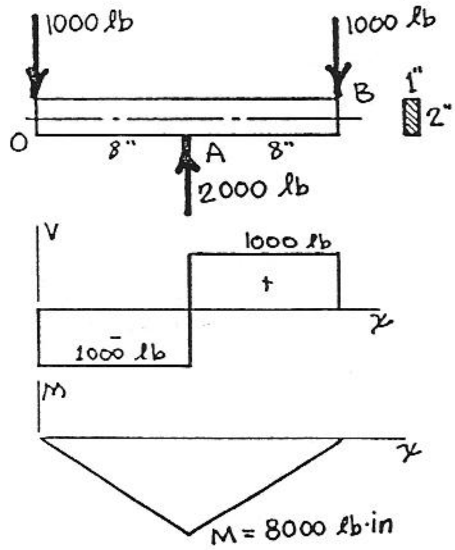
(b) c = 1 in

$A = 2\ in^2$

$$I = \frac{2(1)}{3} = 0.667\ in^4$$

$$\sigma = \frac{8000(1)}{0.667}$$

= 12 000 psi at A on the top surface.




$$\tau = \frac{3(1000)}{2(2)} = 750\ psi$$

on neutral surface from O to B.

(c) $A = 1.5\ in^2$

c = 1 in

$$I = \frac{1.5}{3} = 0.5\ in^4$$

$M_{max} = -600(5/2) + 900(7.5/2) = 1875$ lb·in

$$\sigma = \frac{1875(1)}{0.5} = 3750\ psi$$ on bottom at midbeam

2-26 (Cont.)

$\tau = \frac{3(900)}{2(1.5)}$

$= 900$ psi

on neutral axis

at A & B.

(d) $A = 2$ in$^2$

$c = 1$ in, $I = 0.667$ in$^4$

$\sigma = \frac{1800(1)}{0.667}$

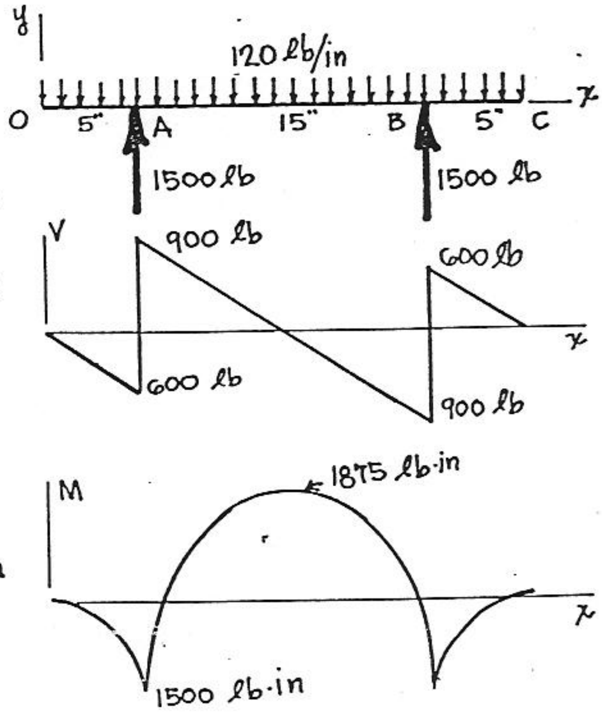
$= 2700$ psi on

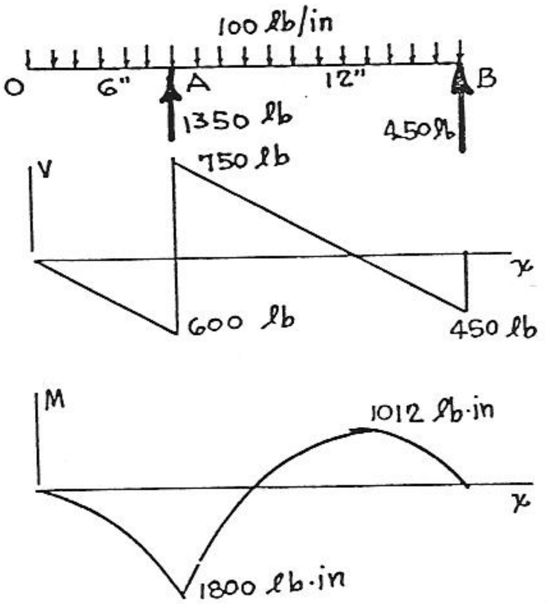
top at A

$\tau = \frac{3(750)}{2(2)}$

$= 562$ psi on

neutral axis at A.




---

2-27 $M_{max} = w(\ell/2)/2[\ell/2] = w\ell^2/8$; $\sigma = Mc/I = w\ell^2c/8I$

(a) $I = bh^3/12 = 1.5(9.5)^3/12 = 107.17$ in$^4$; $w = 8(107.17)(1200)/[(144)^2(9.5/2)]$,

or $w = 10.4$ lb/in (125 lb/ft) Ans.

2-27 (Cont.)

(b) $I = \frac{\pi}{64}[2^4 - (1.25)^4] = 0.666 \text{ in}^4$

$$w = \frac{8I\sigma}{\ell^2 c} = \frac{8(0.666)(12)(10^3)}{(48)^2(1)} = 27.7 \text{ lb/in}$$ Ans.

(c) Assume square corners.

$$I = \frac{2(3)^3}{12} - \frac{1.625(2.625)^3}{12} = 2.05 \text{ in}^4$$

$$w = \frac{8(2.05)(12)(10^3)}{(48)^2(1.5)} = 56.9 \text{ lb/in}$$ Ans.

(d) $I = 2(1.18) = 2.36 \text{ in}^4$

$$w = \frac{8(2.36)(12)(10^3)}{(72)^2(0.82)} = 53.3 \text{ lb/in}$$

based on compression. For tension we have

$w = 53.3[0.82/(3 - 0.82)] = 20.0$ lb/in Ans.

(e) $I = 3.83 \text{ in}^4$

$$w = \frac{8(3.83)(12)(10^3)}{(72)^2(2)} = 35.5 \text{ lb/in}$$ Ans.

(f) $I = 1(4)^3/12 = 5.33 \text{ in}^4$

$$w = \frac{8(5.33)(12)(10^3)}{(72)^2(2)} = 49.4 \text{ lb/in}$$ Ans.

---

2-28 Use $d = [16T/(\pi\sigma)]^{1/3}$ and

$T = 63\ 000H/n$

(a) $T = 63\ 000(50)/2000 = 1580$ lb·in,

$d = [16(1580)/(8000\pi)]^{1/3} = 1.00$ in Ans.

(b) $T = 63\ 000(50)/200 = 15\ 800$ lb·in,

$d = [16(15\ 800)/(8000\pi)]^{1/3}$

$= 2.16$ in Ans.

---

2-29 $J = \pi d^4/32 = \pi(1.5)^4/32$
$= 0.497 \text{ cm}^4$

$\theta = \frac{30\pi}{180} = \frac{\pi}{6}$ rad; also $\theta = T\ell/GJ$, so

$\ell = GJ\theta/T$; since $\tau = Tr/J$, $T = J\tau/r$

So $\ell = \frac{GJ\theta}{T} = \frac{GJ\theta}{J\tau}\cdot r = \frac{Gr\theta}{\tau}$

Here $\ell$ is in cm if G is in MPa, r in cm, $\theta$ in rad and $\tau$ in MPa. So

$$\ell = \frac{79.3(10^3)(0.75)(\ /6)}{110} = 283 \text{ cm}$$

or 2.83 m Ans.

---

2-30 Since $\tau = Tc/J$, let $T = KJ$

$J_{solid} = \pi d^4/32 = \pi(7^4)/32 = 236 \text{ cm}^4$

$J_{hollow} = (\pi/32)[7^4 - (5.8)^4] = 125 \text{ cm}^4$

$$\text{Percent reduction} = \frac{236 - 125}{236}(100)$$

$= 47.0\%$ Ans.

(b) $W_{solid} = KA = K\pi d^2/4 = C_1 d^2 = 49C_1$

$W_{hollow} = (K\pi/4)(d^2 - d_i^2)$

$= C_1[49 - (5.8)^2] = 15.4C_1$

$$\text{Percent reduction} = \frac{49 - 15.4}{49}(100)$$

$= 68.6\%$ Ans.

---

QUIZ A cold-drawn square steel tube has a wall thickness t and an outside dimension w. Develop an approximate expression for the maximum shear stress due to pure torsion. (Open books)

33. If support $R_2$ is between $F_1$ and $F_2$, the moment about $R_1$ is zero,

00(3) + 7.75(1100) − $R_2 \ell$ = 0,

or $R_2 = 14\ 525/\ell$ . Now

$= 3100 - R_2 = (3100\ell - 14\ 525)/\ell$

$= 3R_1$, $M_{x=\ell} = -(7.75 - \ell)(1100)$.

e moment diagram is first triangular, en quadralateral, then crosses axis. Controlling moment at $x = 3$ or $x = \ell$.

| | $R_2$ | $R_1$ | $M_3$ | $M_\ell$ |
|---|---|---|---|---|
| | 4842 | −1742 | −5225 | −5225 |
| 4 | 3631 | −531 | −1594 | −4125 |
| | 2905 | 195 | 585 | −3025 |
| | 2420 | 680 | 2038 | −1925 |
| 7 | 2075 | 1025 | 3075 | −825 |

e minimum largest absolute moment at $\ell \doteq 6$ in. Analytically, minimum occurs en moment $|M_3|$ has same magnitude as $|M_\ell|$ . $M_3 = 3R_1 = (9300\ell - 43\ 575)/\ell$

$M_\ell = -(7.75 - \ell)1100 = 8525 - 1100\ell$

rom $|M_3| = |M_\ell|$, $\ell = 5.95$ in.

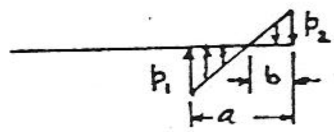
If second bearing is near B of figure, hen $R_2 = 14\ 525/\ell$ , $R_1 = 3100 - R_2$. Equating $|R_1| = |R_2|$ gives R = 1550 lb nd $\ell = 14\ 525/1550 = 9.37$ in.

---

2-34. Reaction consists of the superosition of a uniform and a linear distributed loading. The uniform part is $q_2 = F/a$ and the lincar part is

$q_1 = 6F(\ell + a/2)/a^2$

The intensity at the wall is $p_1 = q_1 + q_2$ and at the pin end $p_2 = q_1 - q_2$.




Now $b = ap_2/(p_1 + p_2)$. Moment at distance x to right of load is

$$M = \frac{p_1}{2}\,\frac{\ell - x + 2a - 2b}{a - b}\,\frac{(x-\ell)^2}{3} \cdot \frac{\ell - x + 3a - 3b}{\ell - x + 2a - 2b} - Fx$$

Maximum moment at $x_{max} = \ell - a - 2b$ and

$$M_{max} = -F\left[\frac{(a - 2b)(a + b)}{3a} + \ell\right]$$

Conventional moment is $-F\ell$ so fractional increase $\Delta$ is

$$\Delta = \frac{(a - 2b)(a + b)}{3a\ell} = \left(1 - \frac{2b}{a}\right)\left(1 + \frac{b}{a}\right)\frac{a}{3\ell}$$

The permissible values of b/a range $0.333 \le b/a \le 0.5$, so largest possible discrepancy is $0.148a/\ell$ . For F = 1500 lb, $\ell = 1.5$ in, a = 1.2 in,

$q_2 = 1500/2 = 750$ lb/in

$q_1 = 6(1500)(1.5 + 1.2/2)/1.2^2 = 13\ 125$

$p_1 = 750 + 13\ 125 = 13\ 875$ lb/in

$p_2 = 13\ 125 - 750 = 12\ 375$ lb/in

$b = 1.2(12\ 375)/(13\ 875 + 12\ 375) = 0.566$

$\Delta = (1.2-2(0.566))(1.2+0.566)/[3(1.2)1.5]$

$= 0.022$; the increase is about 2%.

For more, see Steve Smith, "Evaluating Socket Joint Stress," MACHINE DESIGN, Jan. 10, 1985, p. 128-9.

**2-36** $\omega = 2\pi n/60 = 2\pi(8)/60 = 0.838$ rad/s

$T = H/\omega = 1000/0.838 = 1190$ N · m

$d = (16T/\tau\pi)^{1/3} = [16(1190)/(75\pi)]^{1/3}$

$= 4.32$ cm

Using preferred sizes, select 45 mm Ans.

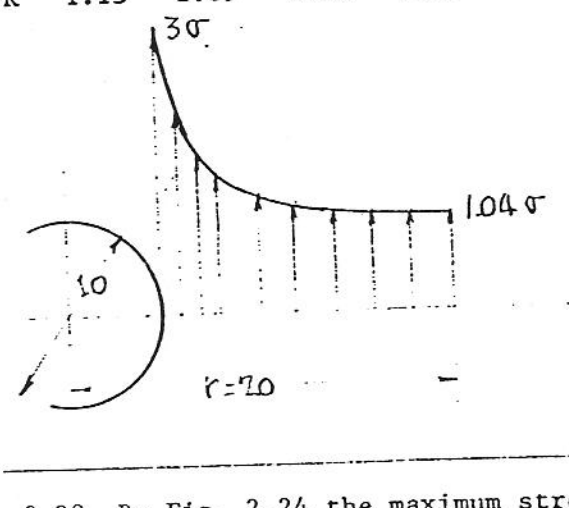
---

**2-37** (a) At $\theta = 90°$, $\sigma_r = 0$, $\sigma_\theta = -\sigma$, $\sigma_r = 0$. At $\theta = 0°$, $\sigma_r = 0$, $\sigma_\theta = 3\sigma$, and $\tau_{r\theta} = 0$

(b)

| r | 5 | 6 | 7 | 8 | 10 |
|---|---|---|---|---|---|
| $K = \sigma_\theta/\sigma$ | 3 | 2.07 | 1.65 | 1.42 | 1.22 |

| r | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|
| K | 1.13 | 1.09 | 1.06 | 1.05 | 1.04 |




---

**2-38** By Fig. 2-24 the maximum stresses occur at the inside fiber where $r = r_i$. Therefore, from Eq. (2-51)

$$\sigma_{t,max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right)$$

$$= p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) \quad \text{Ans.}$$

$$\sigma_{r,max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right) = -p_i \quad \text{Ans.}$$

---

**2-39** If $p_i = 0$, Eq. (2-50) becomes

$$\sigma_t = \frac{-p_o r_o^2 - r_i^2 r_o^2 p_o/r^2}{r_o^2 - r_i^2}$$

$$= -\frac{p_o r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

The maximum tangential stress occurs at $r = r_i$. So

$$\sigma_{t,max} = -\frac{2p_o r_o^2}{r_o^2 - r_i^2} \quad \text{Ans.}$$

For $\sigma_r$, we have

$$\sigma_r = \frac{-p_o r_o^2 + r_i^2 r_o^2 p_o/r^2}{r_o^2 - r_i^2}$$

$$= \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2} - 1\right)$$

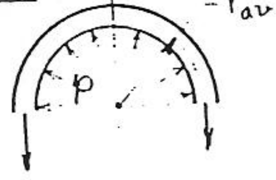
So $\sigma_r = 0$ at $r = r_i$. Thus at $r = r_o$

$$\sigma_{r,max} = \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2 - r_o^2}{r_o^2}\right) = -p_o$$

Ans.

---

**2-40**




$F = pA = \pi r_{av}^2 p$

$$\sigma_1 = \sigma_2 = \frac{F}{A}$$

$$= \frac{\pi r_{av}^2 p}{2\pi r_{av} t}$$

$$= \frac{p r_{av}}{2t} \quad \text{Ans.}$$

---

2-41 $\tau_{max} = \dfrac{\sigma_t - \sigma_r}{2}$ at $r = r_i$ where is

termediate in value. From Prob. 2-38

$$\tau_{max} = \frac{1}{2}(\sigma_{t,max} - \sigma_{r,max})$$

$$\tau_{max} = \frac{p_i}{2}\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} + 1\right)$$

w solve for $p_i$ using $r_o = 3$ in, $r_i = 2.75$ in, and $\tau_{max} = 4000$ psi.

his gives $p_i = 639$ psi <u>Ans.</u>

---

2-42 $r_o = 120$ mm, $r_i = 110$ mm

See Prob. 2-41

$$\tau_{max} = \frac{2.4}{2}\left[\frac{(120)^2 + (110)^2}{(120)^2 - (110)^2} + 1\right]$$

$= 15.0$ MPa <u>Ans.</u>

---

2-43 From Table A-20 $S_y = 57$ kpsi;

also, $r_o = 0.875$ in and $r_i = 0.625$ in.

From Prob. 2-39

$$\sigma_{t,max} = -\frac{2p_o r_o^2}{r_o^2 - r_i^2} \quad \text{and so}$$

$$p_o = \frac{(r_o^2 - r_i^2)(0.8S_y)}{2r_o^2}$$

Solving, gives $p_o = 11\ 200$ psi <u>Ans.</u>

---

2-44 From Table A-20 $S_y = 57$ kpsi;

also $r_o = 1.1875$ in, $r_i = 0.875$ in.

From Prob. 2-38

$$\sigma_{t,max} = p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right); \text{ therefore}$$

$$p_i = 0.8S_y\left(\frac{r_o^2 - r_i^2}{r_o^2 + r_i^2}\right); \text{solving, gives}$$

$p_i = 13\ 500$ psi <u>Ans.</u>

---

2-45 $r_o = 5$ in, $r_i = 0.375$ in,

$\omega = 2\pi n/60 = 2\pi(7200)/60 = 754$ rad/s

$\rho = w/g = 0.0.282/386 = 0.000\ 730\ 6$

$\nu = 0.292$

| $r$ | $\sigma_t$ | $\sigma_r$ | |
|---|---|---|---|
| 0.375 | 8556 | 0 | |
| 0.5 | 6676 | 1851 | |
| 0.75 | 5310 | 3133 | |
| 1.0 | 4800 | 3525 | |
| 1.3693 | 4435 | 3656 | $(r_o r_i)^{\frac{1}{2}}$ |
| 1.5 | 4345 | 3645 | |
| 2 | 4058 | 3463 | |
| 2.5 | 3784 | 3133 | |
| 3 | 3487 | 2692 | |
| 3.5 | 3153 | 2154 | |
| 4 | 2776 | 1525 | |
| 4.5 | 2354 | 806 | |
| 5 | 1886 | 0 | |

At $r = 0.375$ $\tau_{max} = 8556/2 = 4280$ psi <u>Ans.</u>

$\sigma_{r,max} = 3656$ psi <u>Ans.</u>

---

2-46 $\omega = 2\pi(2069)/60 = 216.7$ rad/s,

$\rho = 3320$ kg/m³, $\nu = 0.24$, $r_i = 0.0125$ m,

$r_o = 0.15$ m; use Eq. (2-56)

$$\sigma_t = 3320(216.7)^2\left(\frac{3 + 0.24}{8}\right) \times \left[(0.0125)^2 + (0.15)^2 + (0.15)^2 - \frac{1 + 3(0.24)}{3 + 0.24}(0.0125)^2\right](10)^{-6}$$

$= 2.85$ MPa <u>Ans.</u>

2-48 to 2-53 $\nu = 0.292$, $E = 30$ Mpsi, $E = 207$ GPa), $r_o = 1.5$ in (40 mm), $r_i = 0$, $R = 0.75$ in (20 mm).

Since $p = K \frac{E\delta}{R} = K'E$, p is in same units as E.

2-48

$$\delta_{max} = \frac{1}{2}(40.025 - 40.010) = 0.0075 \text{ mm}$$

$$\delta_{min} = \frac{1}{2}(40.000 - 40.026) = -0.013 \text{ mm}$$

$P = 0$, $P = 50.5$ MPa

2-49

$$\delta_{max} = \frac{1}{2}(1.5010 - 1.5004) = 0.0003 \text{ in}$$

$$\delta_{min} = \frac{1}{2}(1.5000 - 1.5010) = -0.0010 \text{ in}$$

$P = 0$, $P = 15\ 000$ psi

---

PROGRAM Analysis of press and shrink fits. This program computes the interface pressure p and the stangential stresses $\sigma_{it}$ and $\sigma_{ot}$ at the interface radius. It is easy to extend it to solve for stresses in either member at other radii.

1. Enter $E_o$, $\nu_o$, $E_i$, $\nu_i$, and $\delta$.
2. Enter outside radius $r_o$ of outer member.
3. Enter inside radius $r_i$ of inner member.
4. Enter interface radius R.
5. Solve Eq. (2-59) for p.
6. Solve Eqs. (2-57) and (2-58) for $\sigma_{it}$ and $\sigma_{ot}$.

---

PROGRAM This is a program to compute the tangential and radial stresses in rotating rings at any specified radius r using equation pair (2-56).

1. Enter $\rho$, $\omega$, $\nu$, $r_o$, and $r_i$.
2. Enter the radius r at which the stresses are desired.
3. $AA = \rho\omega^2$
4. $BB = (3 + \nu)/8$
5. $CC = (1 + 3\nu)/(3 + \nu)$
6. $\sigma_t = (AA)(BB)\{r_i^2 + r_o^2 + [(r_i^2 r_o^2)/r^2] - (CC)(r^2)\}$
7. $\sigma_r = (AA)(BB)\{r_i^2 + r_o^2 - [(r_i^2 r_o^2)/r^2] - r^2\}$
8. Go to step 2.

---

CURVED BEAM PROGRAM See pp. 65-69.
A positive moment produces tension on the inside fiber; a positive force produces tension in the cross section. Program written for rectangular, trapezoidal, circular, and tee sections.

1. Enter $r_i$ and $r_o$.
2. $h = r_o - r_i$
3. If section is rectangular, step 7, else next.
4. If section is trapezoidal, step 12, else next.
5. If section is circular, step 17, else next.
6. Section must be a tee; go to step 21.
7. Rectangular; enter width b.
8. $A = bh$
9. Use Eq. (2-67) to solve for R.

(Continued on page 28)

2-58. $D = 3$ in, $d = 1$ in, $r_i = 1.5$ in, $r_o = 2.5$ in, Eq.(2-73) $R = (3 + 1)/2 = 2$ in, Eq.(2-74)

$$r_n = \frac{1^2}{4(2[2] - \sqrt{4[2^2] - 1^2})} = 1.968\ 246$$

$$e = R - r_n = 2 - 1.968\ 246 = 0.031\ 754$$

$$c_i = r_n - r_i = 1.968\ 246 - 1.5 = 0.468\ 246 \text{ in}$$

$$c_o = r_o - r_n = 2.5 - 1.968\ 246 = 0.531\ 754 \text{ in}$$

$$A = \pi d^2/4 = \pi(1)^2/4 = 0.7854 \text{ in}^2$$

$$M = FR = 1000(2) = 2000 \text{ in lb}$$

Using Eqs.(2-66)

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{1000}{0.7854} + \frac{2000(0.468\ 246)}{0.7854(0.031\ 754)1.5} = 26\ 300 \text{ psi}$$

$$\sigma_o = \frac{1000}{0.7854} - \frac{2000(0.531\ 754)}{0.7854(0.031\ 754)2.5} = -15\ 800 \text{ psi}$$

---

2-59. Section AA

$D = 0.75$ in, $A = \pi(0.25)^2/4 = 0.0491$ in$^2$, $r_i = 0.75/2 = 0.375$ in, $r_o = 0.75/2 + 0.25 = 0.625$ in, $R = (0.75 + 0.25)/2 = 0.500$ in, from Eq.(2-74)

$$r_n = \frac{0.25^2}{4[2(0.5) - \sqrt{4(0.5)^2 - 0.25^2}]} = 0.492\ 061 \text{ in.}$$

$$e = R - r_n = 0.5 - 0.492\ 061 = 0.007\ 939 \text{ in}$$

$$c_o = r_o - r_n = 0.625 - 0.492\ 061 = 0.132\ 939 \text{ in}$$

$$c_i = r_n - r_i = 0.492\ 061 - 0.375 = 0.117\ 061 \text{ in}$$

$$M = FR = 100(0.5) = 50 \text{ in lb}$$

$$\sigma_i = \frac{100}{0.0491} + \frac{50(0.117\ 061)}{0.0491(0.007\ 939)(0.375)} = 42\ 100 \text{ ksi}$$

$$\sigma_o = \frac{100}{0.0491} - \frac{50(0.132\ 939)}{0.0491(0.007\ 939)(0.625)} = -25\ 200 \text{ psi}$$

Section BB: Abscissa angle $\theta$ of line of radius centers is

$$\theta = \cos^{-1}\left(\frac{r_2 + d/2}{r_2 + d + D/2}\right) = \cos^{-1}\left(\frac{0.375 + 0.25/2}{0.375 + 0.25 + 0.75/2}\right) = 60°$$

$$M = F\frac{D + d}{2}\cos\theta = 100(0.5)\cos 60° = 25 \text{ in lb}, \quad r_i = r_2 = 0.375 \text{ in},$$

$$r_o = r_2 + d = 0.375 + 0.25 = 0.625 \text{ in}$$

$e = 0.007\ 939$ in as before

$$\sigma_i = \frac{F\cos\theta}{A} - \frac{Mc_i}{Aer_i} = \frac{100\cos 60°}{0.0491} - \frac{25(0.117\ 061)}{0.0491(0.007\ 939)0.375}$$

$= -19\ 000$ psi

$$\sigma_o = \frac{100\cos 60°}{0.0491} + \frac{25(0.132\ 939)}{0.0491(0.007\ 939)0.625}$$

$= 14\ 700$ psi

On section BB shear stress due to shear force is zero at surface.

---

2-60. $r_i = 0.125$ in, $r_o = 0.125 + 0.1094 = 0.2344$ in, Eq.(2-67)

$R = 0.125 + 0.1094/2 = 0.1797$ in,

Eq.(2-68) $r_n = 0.1094/\ln(0.2394/0.125)$

$= 0.174\ 006$ in,

$e = R - r_n = 0.1797 - 0.174\ 006$

$= 0.005\ 694$ in

$c_i = r_n - r_i = 0.174\ 006 - 0.125$

$= 0.049\ 006$ in

$c_o = r_o - r_n = 0.2344 - 0.174\ 006$

$= 0.060\ 394$ in

$A = 0.75(0.1094) = 0.082\ 050\ in^2$

$M = F(4 + h/2) = 3(4 + 0.1094/2)$

$= 12.16$ in lb

$$\sigma_i = -\frac{3}{0.082} - \frac{12.16(0.049)}{0.082(0.005\ 694)0.125}$$

$= -10\ 200$ psi

$$\sigma_o = -\frac{3}{0.082} + \frac{12.16(0.060)}{0.082(0.005\ 694)0.2344}$$

$= 6630$ psi

---

CURVED BEAM PROGRAM (Continued from p. 26).

10. Use Eq. (2-68) to solve for $r_n$.
11. Go to step 26.
12. Trapezoidal; enter $b_i$ and $b_o$.
13. $A = h(b_i + b_o)/2$
14. Use Eq. (2-69) to solve for R.
15. Use Eq. (2-70) to solve for $r_n$.
16. Go to Step 26.
17. Circular; $A = \pi h^2/4$
18. Use Eq. (2-73) to solve for R.
19. Use Eq. (2-74) to solve for $r_n$.
20. Go to step 26.
21. Tee; enter $c_1$ and $c_2$ from Fig. 2-29c.
22. Enter $b_i$ and $b_o$.
23. $A = b_i c_1 + b_o c_2$
24. Use Eq. (2-71) to solve for R.
25. Use Eq. (2-72) to solve for $r_n$.
26. $e = R - r_n$
27. $c_i = r_n - r_i$
28. $c_o = r_o - r_n$
29. Next if stress is produced by a moment, else step 31.
30. Enter M; go to step 33.
31. Enter F.
32. Enter the moment about the centroid produced by F.
33. $\sigma_i = (F/A) + Mc_i/(Aer_i)$
34. $\sigma_o = (F/A) - Mc_o/(Aer_i)$

---

2-62. Evaluate Eqs.(2-84), (2-85), and (2-86) using Simpson's rule.

| r | b | | |
|---|---|---|---|
| 2 | 0.75 | 1 | 0.75 |
| 3 | 0.75 | 4 | 3.00 |
| 4 | 0.75 | 2 | 1.50 |
| 5 | 0.75 | 4 | 3.00 |
| 6 | 0.75 | 1 | 0.75 |
| | | | 9.00 |

Eq.(2-84): $A = 9/3 = 3\ \text{in}^2$

| r | br | | |
|---|---|---|---|
| 2 | 1.50 | 1 | 1.50 |
| 3 | 2.75 | 4 | 9.00 |
| 4 | 3.00 | 2 | 6.00 |
| 5 | 3.75 | 4 | 15.00 |
| 6 | 4.50 | 1 | 4.50 |
| | | | 36.00 |

Eq.(2-85): $R = \frac{36}{3(3)} = 4$ in

| r | b/r | | |
|---|---|---|---|
| 2 | 0.3750 | 1 | 0.375 |
| 3 | 0.2500 | 4 | 1.000 |
| 4 | 0.1875 | 2 | 0.375 |
| 5 | 0.1500 | 4 | 0.600 |
| 6 | 0.1250 | 1 | 0.125 |
| | | | 2.475 |

Eq.(2-86): $r_n = \frac{3}{2.475/3}$

$= 3.\underline{63}$ in

Underlined digits are repeating digits as $3.\underline{63} = 3.636363\ldots$

$e = R - r_n = 4 - 3.\underline{63} = 0.\underline{36}$ in

$c_o = r_o - r_n = 6 - 3.\underline{63} = 2.\underline{36}$ in

$c_i = r_n - r_i = 3.\underline{63} - 2 = 1.\underline{63}$

$$\sigma_i = \frac{5000}{3} + \frac{5000(4)1.\underline{63}}{3(0.\underline{36})2} = 16\ 667 \text{ psi}$$

$$\sigma_o = \frac{5000}{3} + \frac{5000(4)2.\underline{36}}{3(0.\underline{36})6} = -5556 \text{ psi}$$

Exact: $R = 2 + 4/2 = 4$ in

$r_n = 4/\ln(6/2) = 3.640\ 957$ in

$e = 4 - 3.640\ 957 = 0.359\ 043$ in

$$\text{error in } e = \frac{0.\underline{36} - 0.359\ 042}{0.359\ 042} = 0.0128$$

or about 1 percent.

$c_o = 6 - 3.640\ 957 = 2.359\ 043$ in

$c_i = 3.640\ 957 - 2 = 1.640\ 957$ in

$$\sigma_i = \frac{5000}{3} + \frac{5000(4)(1.640\ 957)}{3(0.359\ 043)2}$$

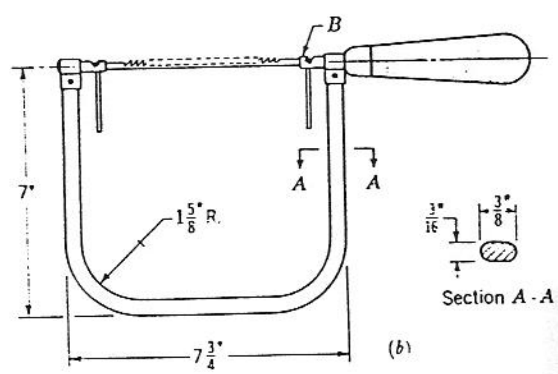
$= 16\ 901$ psi

$$\sigma_o = \frac{5000}{3} - \frac{5000(4)2.359\ 043}{3(0.359\ 043)6}$$

$\doteq -5634$ psi

Numerical results can be improved by applying Richardson's correction to Eq.(2-86) changing $3.\underline{63}$ to 3.639 795, or simply increasing the number of ordinates used. The error in the eccentricity e parallels errors in bending stress, so make efforts to determine e accurately.

Make your own curved beam problem:



2-63. Eq.(2-85):

$$A = 0.75[(3.5 - 2) + (6 - 4.5)]$$

$= 2.25$ in$^2$, Eq.(2-86):

$$R = \frac{0.75}{2.25}\left[\frac{3.5^2 - 2^2}{2} + \frac{6^2 - 4.5^2}{2}\right]$$

$= 4$ in, Eq.(2-87):

$$r_n = \frac{2.25/0/75}{\ln(3.5/2) + \ln(6/4.5)}$$

$= 3.540\ 668$ in

$$e = R - r_n = 4 - 3.540\ 668 = 0.459\ 332$$

$$c_i = 3.540\ 668 - 2 = 1.540\ 668 \text{ in}$$

$$c_o = 6 - 3.540\ 668 = 2.459\ 332 \text{ in}$$

$$\sigma_i = \frac{5000}{2.25} + \frac{5000(4)(1.540\ 668)}{2.25(0.459\ 332)2} = 17.1 \text{ kpsi}$$

$$\sigma_o = \frac{5000}{2.25} - \frac{5000(4)(2.459\ 332)}{2.25(0.459\ 332)6} = -5.7 \text{ kpsi}$$

---

2-65. Simpson's rule evaluation of Eqs. (2-85), (2-86) and (2-87).

| r | b | br | b/r | |
|---|---|---|---|---|
| 10.0 | 1.000 000 | 0.000 000 | 0.000 000 | 1 |
| 10.4 | 1.200 000 | 12.480 000 | 0.115 385 | 4 |
| 10.8 | 1.600 000 | 17.280 000 | 0.148 148 | 2 |
| 11.2 | 1.833 030 | 20.529 935 | 0.163 663 | 4 |
| 11.6 | 1.959 592 | 22.731 267 | 0.168 930 | 2 |
| 12.0 | 2.000 000 | 24.000 000 | 0.166 667 | 4 |
| 12.4 | 1.959 592 | 24.298 941 | 0.158 032 | 2 |
| 12.8 | 1.833 030 | 23.462 784 | 0.143 205 | 4 |
| 13.2 | 1.600 000 | 21.120 000 | 0.121 212 | 2 |
| 13.6 | 1.200 000 | 16.320 000 | 0.088 235 | 4 |
| 14.0 | 0.000 000 | 0.000 000 | 0.000 000 | 1 |

$$A = \frac{0.4}{3}(46.502\ 608) = 6.200\ 348 \text{ in}^2$$

$$R = \frac{0.4(558.031\ 292)}{3(6.200\ 348)} = 11.999\ 999 \text{ in}$$

$$r_n = \frac{6.200\ 348}{0.4(3.901\ 264)/3} = 11.919\ 883 \text{ in}$$

$$e = 11.999\ 999 - 11.919\ 883 = 0.080\ 116$$

(For comparison, using 21 ordinates; $A = 6.254\ 061$ in$^2$, $R = 12$ in, $r_n = 11.917\ 330$, $e = 0.082\ 672$ in)

Continuing with the 11-ordinate analysis

$$c_i = 11.919\ 883 - 10 = 1.919\ 883 \text{ in}$$

$$c_o = 14 - 11.919\ 883 = 2.080\ 117 \text{ in}$$

$$M = F(2 + 2) = 20\ 000(4) = 80\ 000 \text{ in lb}$$

$$\sigma_i = \frac{20\ 000}{6.200\ 348} + \frac{80\ 000(1.919\ 883)}{6.200\ 348(0.080\ 118)10}$$

$= 34\ 145$ psi

$$\sigma_o = \frac{20\ 000}{6.200\ 348} - \frac{80\ 000(2.080\ 117)}{6.200\ 348(0.080\ 116)14}$$

$= -20\ 703$ psi

Note that A, R and $r_n$ were evaluated by numerical integration even though it is possible to write $A = \pi ab$ and $R = 12$ by inspection. This is because $R = I_2/I_1$ where integral errors are correlated. Similarly $r_n = I_1/I_3$. This approach leads to more accurate R and $r_n$ and also improves the estimate of e, something to keep in mind when writing computer programs.

---

2-66 The area A can be obtained by subtracting the area of a circle of 0.4 in radius from the rectangle. Thus

$$A = (1)(1.6) - \pi(0.4)^2 = 1.097\ 345\ \text{in}^2$$

The radius to the centroid $\bar{r}$ can be obtained by inspection of the symmetrical section. Thus $\bar{r} = 1 + (1.6/2) = 1.8$ in. The distance to the neutral surface $r_n$ involves three integrals.

$$r_n = \frac{A}{\int_1^{1.4} \frac{(1)dr}{r} + \int_{1.4}^{2.2} \frac{b(r)dr}{r} + \int_{2.2}^{2.6} \frac{(1)dr}{r}}$$

$$= \frac{A}{\ln\frac{1.4}{1} + \int_{1.4}^{2.2} \frac{b(r)dr}{r} + \ln\frac{2.6}{2.2}} = \frac{1.097\ 345}{0.503\ 526 + \int_{1.4}^{2.2} \frac{b(r)dr}{r}}$$

The integral in the denominator of the last expression can be evaluated numerically. From the geometry of a circle, the width b(r) at each station in the interval $1.4 \leq r \leq 2.2$ can be found. Choosing nine ordinates for Simpson's rule in the interval

| r | b | b/r | × | |
|---|---|---|---|---|
| 1.4 | 1.000 000 | 0.714 286 | 1 | 0.714 286 |
| 1.5 | 0.470 850 | 0.313 900 | 4 | 1.255 600 |
| 1.6 | 0.307 180 | 0.191 987 | 2 | 0.383 975 |
| 1.7 | 0.225 403 | 0.132 590 | 4 | 0.530 360 |
| 1.8 | 0.200 000 | 0.111 111 | 2 | 0.222 222 |
| 1.9 | 0.225 403 | 0.118 633 | 4 | 0.474 533 |
| 2.0 | 0.307 180 | 0.153 590 | 2 | 0.307 180 |
| 2.1 | 0.470 850 | 0.224 214 | 4 | 0.896 857 |
| 2.2 | 1.000 000 | 0.454 545 | 1 | 0.454 545 |
| | | | | $\Sigma = 5.239\ 558$ |

The integral is $(\Delta r/3)\Sigma = (0.1/3)(5.239\ 558) = 0.174\ 652$ making the radius to the neutral axis equal to

2-66 (Continued) $r_n = 1.097\ 345/(0.503\ 526 + 0.174\ 652) = 1.618\ 078$ in; the eccentricity e is

$$e = \bar{r} - r_n = 1.8 - 1.618\ 078 = 0.181\ 922 \text{ in}$$

An estimate of the error in the integral can be had by performing a second integration using every other ordinate. The difference between the two estimates divided by 15 is a useful error estimate (See B. Carnahan, H. A. Luther, and J. O. Wilkes, APPLIED NUMBERICAL METHODS, John Wiley, p. 79, 1969) with the sign significant. The second integration produces $\Sigma = 2.773\ 361$ and the second integral estimate is $(0.2/3)(2.773\ 361) = 0.184\ 891$. The error estimate is

$$(0.174\ 652 - 0.184\ 891)/15 = -0.000\ 6826$$

So the estimate of the integral is high. The result is that we have about two correct digits on the estimate of e. This could be improved by doubling the number of ordinates in the interval. Proceeding,

$$c_i = \bar{r} - r_i - e = 1.8 - 1 - 0.181\ 922 = 0.618\ 078 = 0.618 \text{ in}$$

$$c_o = r_o - \bar{r} + e = 2.8 - 1.8 + 0.181\ 922 = 0.981\ 622 = 0.982 \text{ in}$$

The stress at the inner surface is

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{R}{A} + \frac{F(a + r)c_i}{Aer_i} = \frac{3000}{1.097} + \frac{3000(4 + 1.8)(0.618)}{1.097(0.18)(1)} = 57.2 \text{ kpsi}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{F}{A} - \frac{F(a + r)c_o}{Aer_o} = \frac{3000}{1.097} - \frac{3000(4 + 1.8)(0.982)}{1.097(0.18)(2.6)} = -30.5 \text{ kpsi}$$

---

2-67

$$a = K F^{1/3} = F^{1/3}\left[\frac{3}{8}\ \frac{2\left(\frac{1 - \nu^2}{E}\right)}{2(1/d)}\right]$$

Use $\nu = 0.292$, F in newtons, E in N/mm² and d in mm. Then

$$K = \frac{3}{8}\left\{\frac{\left[\frac{1 - (0.292)^2}{207{,}000}\right]}{1/25}\right\} = 0.0346$$

$$p_{max} = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi(KF^{1/3})^2} = \frac{3F^{1/3}}{2\pi K^2}$$

$$= \frac{3F^{1/3}}{2\ (0.0346)^2} = 399F^{1/3} \text{ MPa}$$

---

2-68

$$K = \frac{3}{8}\ \frac{2\ \frac{1 - (0.202)^2}{207\ 000}}{(1/25) + 0} = 0.0436$$

$$p_{max} = \frac{3F^{1/3}}{2\pi K^2} = \frac{3F^{1/3}}{2\pi(0.0436)^2} = 251F^{1/3}$$

And so $\sigma_z = -251F^{1/3}$ MPa Ans.

From Fig. 2-36,

$$\tau_{oct}\ (\max) = 0.294(251F^{1/3}$$
$$= 73.8F^{1/3} \text{ MPa}$$

Then, from Fig. 2-33 we have

$$\tau_{max}(\max) = 0.312(251F^{1/3}) = 78.3F^{1/3}$$

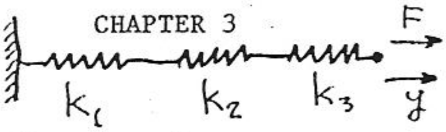
MPa Ans.

$$z = 0.49a = 0.49(0.0436)(18)^{1/3}$$
$$= 0.056 \text{ mm}$$ Ans.

---

# CHAPTER 3

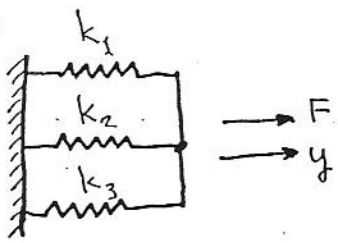
3-1 (a)




$k = \frac{F}{y}$ ; $y = \frac{F}{k_1} + \frac{F}{k_2} + \frac{F}{k_3}$

so $k = \dfrac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}}$ Ans.

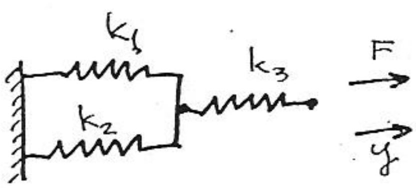
(b)




$F = k_1 y + k_2 y + k_3 y$

$k = F/y = k_1 + k_2 + k_3$ Ans.

(c)




$k = \dfrac{1}{\frac{1}{k_1 + k_2} + \frac{1}{k_3}}$ Ans.

---

3-2 Torsion bar, $k_T = \frac{T}{\theta} = \frac{F\ell}{\theta}$, and so $\theta = \frac{F\ell}{k_T}$; cantilever, $k_C = \frac{F}{\delta}$, $\delta = \frac{F}{k_C}$

Assembly, $k = \frac{F}{y}$, $y = \frac{F}{k} = \ell\theta + \delta$

So $y = \frac{F}{k} = \frac{F\ell^2}{k_T} + \frac{F}{k_C}$

Or $k = \dfrac{1}{\frac{\ell^2}{k_T} + \frac{1}{k_C}}$ Ans.

---

3-3 For a torsion bar $k = \frac{T}{\theta} = \frac{GJ}{\ell}$, where $J = \frac{\pi d^4}{32}$, so $k = \frac{\pi d^4 G}{32\ell} = K\frac{d^4}{\ell}$

The springs, 1 and 2, are in parallel

so $k = k_1 + k_2 = K\frac{d^4}{\ell_1} + K\frac{d^4}{\ell_2}$

$= Kd^4\left(\frac{1}{x} + \frac{1}{\ell - x}\right)$

So $\theta = \frac{T}{k} = \dfrac{T}{Kd^4\left(\frac{1}{x} + \frac{1}{\ell - x}\right)}$

Then $T = k\theta = \frac{Kd^4}{x}\theta + \frac{Kd^4\theta}{\ell - x}$

Thus $T_1 = \frac{Kd^4}{x}\theta$ ; $T_2 = \frac{Kd^4\theta}{\ell - x}$

If $x = \ell/2$, then $T_1 = T_2$

If $x < \ell/2$, then $T_1 > T_2$

Using $\tau = 16T/\pi d^3$ and $\theta = 32T\ell/(G\pi d^4)$

gives $T = \frac{\pi d^3 \tau}{16}$

and so

$$\theta_{all} = \frac{32\ell}{G\pi d^4} \cdot \frac{\pi d^3 \tau}{16} = \frac{2\ell\tau_{all}}{Gd}$$

Thus, if $x < \ell/2$, the allowable twist is

$$\theta_{all} = \frac{2x\tau_{all}}{Gd} \quad \text{Ans.}$$

Since $k = Kd^4\left(\frac{1}{x} + \frac{1}{\ell - x}\right)$

$$= \frac{\pi G d^4}{32}\left(\frac{1}{x} + \frac{1}{\ell - x}\right) \quad \text{Ans.}$$

Then the maximum torque is found to be

$$T_{max} = \frac{\pi d^3 x \tau_{all}}{16}\left(\frac{1}{x} + \frac{1}{\ell - x}\right) \quad \text{Ans.}$$

---

3-4 Both legs have the same twist angle. From Prob. 3-3, $d_1 = 0.2d_2$ Ans.

$k = \frac{\pi G}{32\ell}(1.258 d_2^4)$ Ans.

$\theta_{all} = \frac{2(0.8\ell)\tau_{all}}{Gd_2}$ Ans.

$T_{max} = 0.198 d_2^3 \tau_{all}$ Ans.

---

PROGRAM This routine is used to compute the shear force, bending moment, slope, and deflections for a simply supported beam with any number of cross section changes. See pages 101-105 for details. The units used are: F, lb(kN); X, in (mm); E, Mpsi (GPa); I, $in^4$ ($cm^4$); M, lb · in (N ·m); θ, rad and deg; Y, in (mm).

If U. S. customary units are used, K = 1E-06; if SI units are used, K = 1E-05.

1. Enter E.
2. Enter number of stations, NO.
3. N = 4(NO - 1) + 1
4. X(0) = X(1) = 0
5. Beginning with station # 2 enter the successive station locations.
6. For I = 5 to N in steps of 4 enter X(I).
7. X(I - 1) = X(I)
   X(I - 3) = X(I - 4) + X(I)/2
   X(I - 2) = X(I - 3); next I
8. Enter number of cross section changes, NI.
9. For I = 1 to NI + 1, enter second moment of area beginning at X = 0.
10. Enter location of each change in cross section. For I = 1 to NI, enter XX(I).
11. I = 2; J = 1
12. IN(I) = II(J); IN(I - 1) = II(J)
13. If X(I) = XX(J), go to 15, else next.
14. I = I + 2; go to 12.
15. If J = NI go to 17, else next.
16. J = J + 1; go to 14.
17. I = I + 1; IN(I) = II(NI + 1)
18. Next if I = N, else 17.
19. For I = 1 to N in steps of 4
20. If station number (I + 3)/4 is a reaction go to 23; if a force go to 21; if neither, go to 22.
21. Enter F[(I + 3)/4]
22. Next I; go to 26
23. If flag 1 go to 25, else next.
24. $R_A$ = F(I); $X_A$ = X(I); set flag 1; go to 22.
25. $R_B$ = F(I); $X_B$ = X(I); go to 22.
26. A = 0; B = 0; for I = 1 to N in steps of 4
27. A = A + F(I)[X(I) - $X_A$]
28. B = B + F(I)
29. Next I
30. $R_B = A/(X_A - X_B)$
    $R_A = -(B + R_B)$
31. Print the reactions $R_A$ and $R_B$.
32. For I = 1 to N in steps of 2
33. If flag 2 then 37, else next.
34. If X(I) = $X_A$ next, else 36.
35. F(I) = $R_A$; set flag 2.
36. Next I.
37. If X(I) = $X_B$ then next, else 36
38. F(I) = $R_B$
39. For I = 1 to N
40. D(I) = D(I - 1) + F(I)
41. M(I + 1) = M(I) + D(I)[X(I+1)-X(I)]
42. Next I
43. For I = 1 to N in steps of 4 print the shear forces D(I).
44. For I = 1 to N in steps of 4 print the bending moments M(I).
45. For I = 1 to (N - 2) in steps of 2
46. $AM = \frac{M(I - 1)}{E[IN(I + 1)]} + \frac{M(I)}{E[IN(I)]}$

47. AX = X(I + 2) - X(I)

48. Solve Eq. (3-19) for $\phi(I + 2)$

49. Next I

50. For I = 1 to N - 4 in steps of 4

51. Solve Eq. (3-20) for $\psi(I + 4)$

52. Next I

53. For I = 1 to N in steps of 4

54. If flag 3 go to 58, else next

55. If $X(I) = X_A$ then next, else 57

56. $\psi_A = \psi(I)$; set flag 3

57. Next I

58. If $X(I) = X_B$ then next, else 57

59. $\psi_B = \psi(I)$

60. Solve Eqs. (3-18) for $C_1$ and $C_2$.

61. For I = 1 to N in steps of 4

62. $Y(I) = K[\psi(I) + C_1X(I) + C_2]$

63. $\theta(I) = K[\phi(I) + C_1]$

64. Next I

65. For I = 1 to N in steps of 4

66. If flag 4 then 70, else next

67. If $X(I) = X_A$ then next, else 69

68. $\theta_A = 57.3K[\phi(I) + C_1]$

69. Next I

70. If $X(I) = X_B$ then next, else 69

71. $\theta_B = 57.3K[\phi(I) + C_1]$

72. Print $\theta_A$ and $\theta_B$ in degrees.

73. For I = 1 to N in steps of 4

74. Print Y(I); next I

---

3-5 $I = 2(5.56) = 11.12 \text{ in}^4$

$$y = -\frac{w\ell^4}{8EI} + \frac{Fa^2}{6EI}(a - 3\ell)$$

Here $w = 50/12 = 4.167$ lb/in, and $a = 7(12) = 84$ in, and $\ell = 10(12) = 120$ in.

$$y_1 = -\frac{4.167(120)^4}{8(30)(10^6)(11.12)} = -0.324 \text{ in}$$

$$y_2 = -\frac{600(84)^2[3(120) - 84]}{6(30)(10^6)(11.12)} = -0.584 \text{ in}$$

So $y = -0.324 - 0.584 = -0.908$ in Ans.

$$M_1 = -Fa - (w\ell^2/2)$$

$$= -600(84) - [4.167(120)^2/2]$$

$$= -80\,000 \text{ lb}\cdot\text{in}$$

$c = 4 - 1.17 = 2.83$ in

$$\sigma = \frac{Mc}{I} = \frac{80\,000(2.83)}{11.12}(10^{-3}) = 20.4 \text{ kpsi}$$

Ans.

---

3-6 Use 2 6-in by 8.2-lb channels.

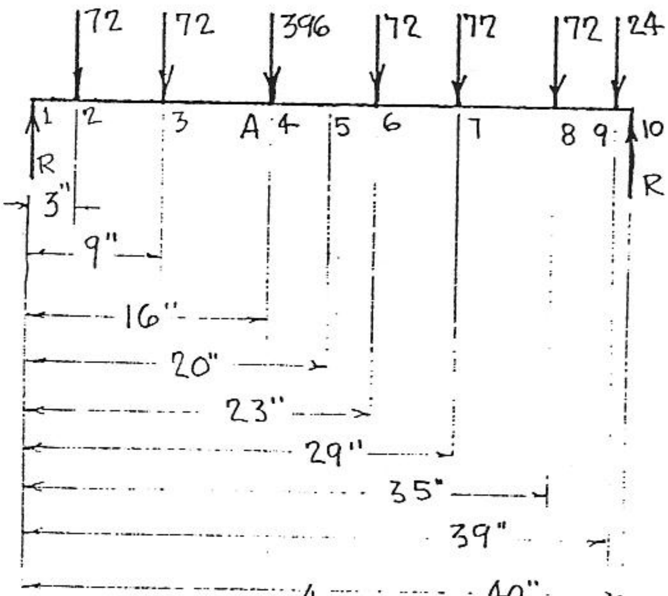
$I = 26.2 \text{ in}^4$; $y = -0.054$ in,

$M_{max} = 32\,400$ lb · in,

$\sigma = 3700$ psi

---

3-7 Computer solution; 10 stations




$I = 0.248 \text{ in}^4$

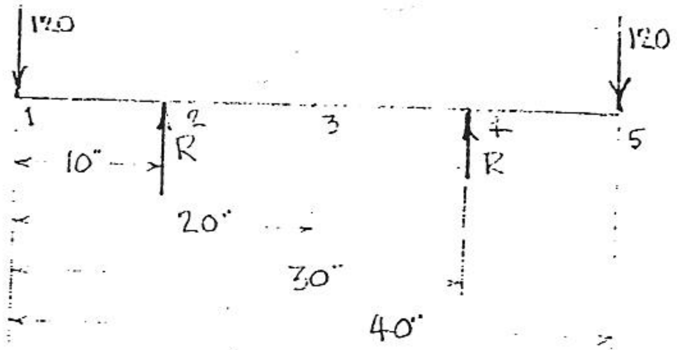
$y_A = -0.101$ in; midspan $y_5 = -0.105$ in

$$\%\text{ diff} = \frac{0.105 - 0.101}{0.105}(100) = 3.81\%$$ Ans.

$R_1 = 420$ lb; $R_{10} = 360$ lb

---

3-8 Computer solution; 5 stations




$I = 0.105 \text{ in}^4$

$R_2 = R_4 = 120$ lb

$y_1 = -0.0508$ in; $y_2 = 0$; $y_3 = 0.01905$ in;

$y_4 = 0$; $y_5 = -0.0508$ in

---

3-9 $a = 36$ in, $\ell = 72$ in, $I = 13 \text{ in}^4$, $E = 30$ Mpsi

$$y = \frac{F_1 a^2}{6EI}(a - 3\ell) - \frac{F_2 \ell^3}{3EI}$$

$$= \frac{400(36)^2(36 - 216)}{6(30)(10^6)(13)}$$

$$- \frac{400(72)^3}{3(30)(10^6)(13)} = -0.167 \text{ in} \quad \text{Ans.}$$

---

3-10 $I = 3.85 \text{ in}^4$; $E = 30$ Mpsi

$$y = -\frac{F\ell^3}{3EI} - \frac{w\ell^4}{8EI} = -\frac{\ell^3}{EI}\left(\frac{F}{3} + \frac{w\ell}{8}\right)$$

$$= -\frac{(48)^3}{30(10^6)(3.85)}\left[\frac{220}{3} + \frac{10(48)}{8}\right]$$

$$= -0.128 \text{ in} \quad \text{Ans.}$$

---

3-11 $I = \pi d^4/64 = \pi(2)^4/64 = 0.785 \text{ in}^4$

$$y = -\frac{F_2 \ell^3}{48EI} + \frac{F_1 a}{24EI}(4a^2 - 3\ell^2)$$

$$= -\frac{120(40)^3}{48(30)(10^6)(0.785}$$

$$+ \frac{80(10)(400 - 4800)}{24(30)(10^6)(0.785)} = -0.0130 \text{ in} \quad \text{ANS}$$

---

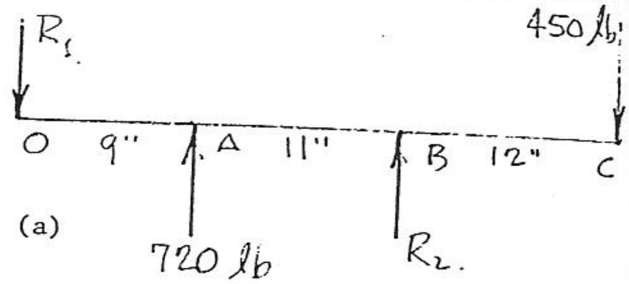
3-12 OMITTED

---

3-13 SEE NEXT PAGE

---

3-14 $T = 2800$ lb · in; $T_1 = 80$ lb, $T_2 = 640$ lb. Computer solution:




(a)

$R_1 = -666$ lb; $R_2 = 396$ lb

$I = 0.120 \text{ in}^4$; $\theta_O = 0.58°$;

$\theta_B = -0.847°$; $y_A = 0.0686$ in;

$y_C = -0.2494$ in

(b) Let $K = I\theta = 0.120(0.847) = 0.1016$;

Then $I' = K/\theta = 0.1016/0.06 = 1.694 \text{ in}^4$;

So $d = (64I'/\pi)^{1/4} = 2.42$ in; use $d = 2.5$ in to nearest 1/8 in. Then $I = \pi(2.5)^4/64 = 1.917 \text{ in}^4$.

For this I we get $\theta_O = 0.036°$ and $\theta_B = -0.053°$; $y_C = -0.01561$ in Ans.

---

3-15 Let P designate the increment of load in the wire rope. Then

$A_m = 0.4(0.625)^2 = 0.156 \text{ in}^2$

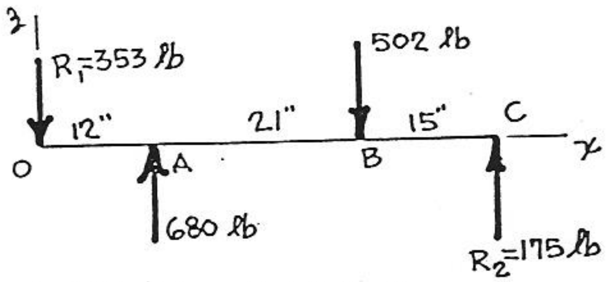
$$\delta = \frac{P\ell}{AE} = \frac{2000(12)(100)}{0.156(12)(10^6)} = 1.28 \text{ in} \quad \text{Ans.}$$

---

3-16 OMITTED

---

3-13




$$\text{Torque} = (600 - 80)(9/2) = 2340 \text{ lb}\cdot\text{in}$$

$$(T_2 - T_1)\frac{12}{2} = T_2(1 - 0.125)(6) = 2340$$

$$T_2 = \frac{2340}{6(0.875)} = 446 \text{ lb}, \quad T_1 = 0.125(446) = 56 \text{ lb}$$

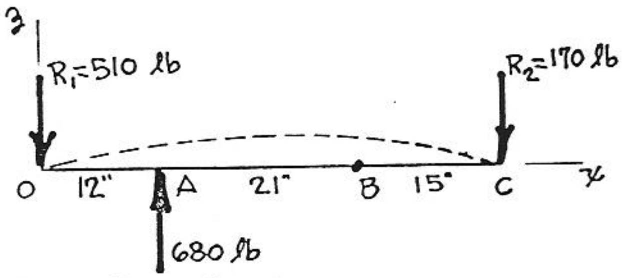
$$\sum M_0 = 12(680) - 33(502) + 48R_2 = 0$$

$$R_2 = \frac{33(502) - 12(680)}{48} = 175 \text{ lb}$$

$$R_1 = 680 - 502 + 175 = 353 \text{ lb}$$

We will treat this as two separate problems and then sum the results.

First, consider the 680 lb load as acting alone.




$$z_{OA} = -\frac{Fbx}{6EI\ell}(x^2 + b^2 - \ell^2); \text{ here } b = 36",$$

$x = 12"$, $\ell = 48"$, $F = 680$ lb. Also

$$I = \frac{\pi d^4}{64} = \frac{\pi(1.5)^4}{64} = 0.249 \text{ in}^4$$

$$z_A = -\frac{680(36)(12)(144 + 1296 - 2304)}{6(30)(10)^6(0.249)(48)}$$

$$= +\,0.11798 \text{ in}$$

3-13 (Concluded)

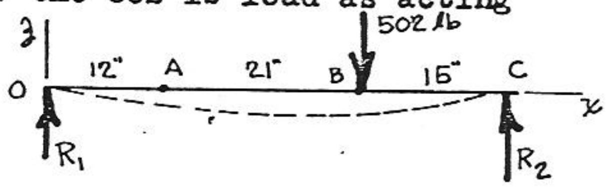
$$z_{AC} = -\frac{Fa(\ell - x)}{6EI\ell}(x^2 + a^2 - 2\ell x)$$

Here a = 12", x = 21 + 12 = 33";

$$z_B = -\frac{680(12)(15)(1089 + 144 - 3168)}{6(30)(10)^6(0.249)(48)}$$

$= + 0.110\ 09$ in

Next, consider the 502 lb load as acting alone.




$$z_{OB} = \frac{Fbx}{6EI\ell}(x^2 + b^2 - \ell^2),$$ where b = 15",

x = 12", $\ell$ = 48", I = 0.249 in$^4$. Then

$$z_A = \frac{502(15)(12)(144 + 225 - 2304)}{6(30)(10)^6(0.249)(48)} = -0.081\ 27\ \text{in}$$

For $z_B$ use x = 33";

$$z_B = \frac{502(15)(33)(1089 + 225 - 2304)}{6(30)(10)^6(0.249)(48)}$$

$= - 0.114\ 35$ in

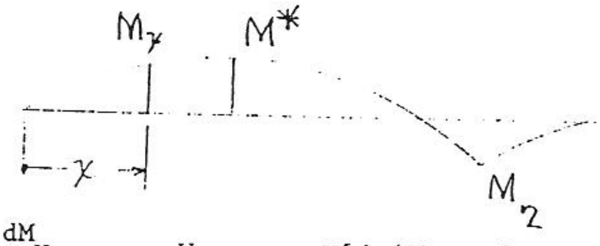
Therefore, by superposition

$z_A = + 0.117\ 98 - 0.081\ 27 = + 0.03671$ in **Ans.**

$z_B = + 0.110\ 09 - 0.114\ 35 = - 0.004\ 26$ in **Ans.**

---

3-17




Total load = W = $w\ell$

$$M_2 = \frac{Wa^2}{2\ell} \quad R_1 = \frac{W[(\ell/2) - a]}{\ell - a}$$

$$M_x = R_1 x - \frac{wx^2}{2} = R_1 x - \frac{Wx^2}{2\ell}$$

$$\frac{dM_x}{dx} = R_1 - \frac{Wx}{\ell} = 0 = \frac{W[(\ell/2) - a]}{\ell - a} - \frac{Wx}{\ell}$$ from which $$x^* = \frac{(\ell/2) - a}{\ell - a}$$

3-17 (Continued)

$$M_x^* = R_1 x^* - \frac{W(x^*)^2}{2}$$

$$= \frac{W[(\ell/2) - a]}{\ell - a} \frac{[(\ell/2) - a]}{\ell - a}$$

$$- \frac{W[(\ell/2) - a)^2 \ell^2}{2(\ell - a)^2}$$

$$= \frac{[(\ell/2) - a]^2 W \ell}{2(\ell - a)^2}$$

Now set $M_x^* = M_2$; this gives

$$\frac{[(\ell/2) - a]^2 W\ell}{2(\ell - a)^2} = \frac{Wa^2}{\ell} \text{ from which}$$

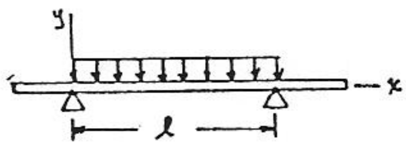
$$(a/\ell)^4 - 2(a/\ell)^3 + (a/\ell) - \tfrac{1}{4} = 0$$

The roots are 0.293, 1.707, 0.707, -0.707, of which 0.293 is significant.

Thus $(a/\ell) = 0.293$ Ans.

---

3-18 (a) = 36(12) = 432 in




$$y_{max} = -\frac{5w\ell^4}{384EI} = -\frac{5(5000/12)(432)^4}{384(30)(10^6)(5450)}$$

$$= -1.16 \text{ in}$$

The frame is bowed up 1.16 in with respect to the bolsters. It is fabricated upside down and then inverted.

(b) The equation in xy coordinates is for center sill neutral surface

$$y = \frac{wx}{24EI}(2\ell x^2 - x^3 - \ell^3)$$

Differentiating this equation and solving for the slope at the left bolster gives

$$\frac{dy}{dx} = \frac{w}{24EI}(6\ell x^2 - 4x^3 - \ell^3); \text{ thus}$$

$$\left.\frac{dy}{dx}\right|_{x=0} = -\frac{w\ell^3}{24EI} = -\frac{(5000/12)(432)^3}{24(30)(10^6)(5450)}$$

$$= -0.00857$$

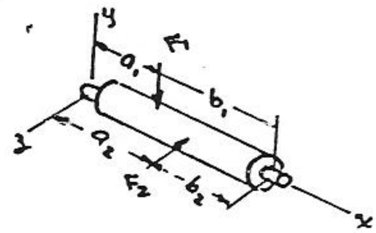
The slope at the right bolster is 0.00857, so equation at left end is $y = -0.00857x$ and at the right end is $y = 0.00857(x - \ell)$.

---

3-19 OMITTED

---

3-20




The slope at x = 0 due to $F_1$ in the xy plane is

$$\theta_{xy} = \frac{F_1 b_1 (b_1^2 - \ell^2)}{6EI\ell}$$

and in the xz plane due to $F_2$ is

$$\theta_{xz} = \frac{F_2 b_2 (b_2^2 - \ell^2)}{6EI\ell}$$

For small angles the slopes add as vectors. Thus

$$\theta_L = (\theta_{xy}^2 + \theta_{xz}^2)^{\frac{1}{2}}$$

$$= \left[\left(\frac{F_1 b_1 (b_1^2 - \ell^2)}{6EI\ell}\right)^2 + \left(\frac{F_2 b_2 (b_2^2 - \ell^2)}{6EI\ell}\right)^2\right]^{\frac{1}{2}}$$

3-20 (Continued) Designating the slope constraint as $\xi$ we then have

$$\xi = \left|\theta_L\right| = \frac{1}{6EI}\left\{\Sigma[F_i b_i(b_i^2 - \ell^2)]^2\right\}^{\frac{1}{2}}$$

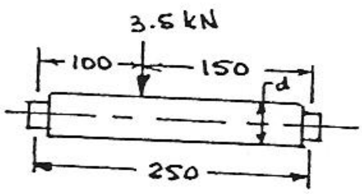
Setting $I = \pi d^4/64$ and solving for d

$$d = \left|\frac{32}{3\pi E\ell\xi}\left\{\Sigma[F_i b_i(b_i^2 - \ell^2)]^2\right\}^{\frac{1}{2}}\right|^{\frac{1}{4}}$$

For the LH brg $E = 30$ Mpsi, $\xi = 0.001$, $b_1 = 12$, $b_2 = 6$, and $\ell = 16$. The result is $d_L = 1.31$ in. Using a similar procedure we get $d_R = 1.35$ in for the RH brg. So use $d = 1\ 3/8$ in Ans.

---

3-21




$$d = \left|\frac{32n}{3\pi E\ell\xi} Fb(b^2 - \ell^2)\right|^{\frac{1}{4}}$$

$$= \left|(\text{mm } 10^{-3})\ \frac{\text{kN mm}^3}{\text{GPa mm}}\ \frac{10^3 10^{-9}}{10^9 10^{-3}}\right|^{\frac{1}{4}}$$

$$d = \sqrt[4]{\frac{32(1.28)(3/5)(150)|(150^2-250^2)|}{3\pi(207)(250)(0.001)}10^{-12}}$$

$= 36.4$ mm Ans.

---

3-22 OMITTED

---

3-23

$$d_L = \left|\frac{32n}{3\pi E\ell\xi}\left(\sum[F_i b_i(b_i^2 - \ ^2)]^2\right)^{1/2}\right|^{\frac{1}{4}}$$

$$= \left|\frac{32(1.5)}{3\pi(29.8)(10^6)(10)(0.001)}\left([800(6)(6^2 - 10^2)]^2 + [600(3)(3^2 - 10^2)]^2\right)^{\frac{1}{2}}\right|^{\frac{1}{4}}$$

$= 1.56$ in

$$d_R = \left|\frac{32(1.5)}{3\pi(29.8)(10^6)(10)(0.001)}\left([800(4)(10^2 - 4^2)]^2 + [600(7)(10^2 - 7^2)]^2\right)^{\frac{1}{2}}\right|^{\frac{1}{4}}$$

Solving gives $d_R = 1.57$ in. So choose $d \geq 1.57$ in to meet the requirements.

---

3-24 From Table A-9-8 we have

$$y_L = \frac{M_B x}{6EI\ell}(x^2 + 3a^2 - 6a\ell + 2\ell^2)$$

$$\frac{dy_L}{dx} = \frac{M_B}{6EI}(3x^2 + 3a^2 - 6a\ell + 2\ell^2)$$

AT $x = 0$, the LH slope is

$$\theta_L = \frac{dy_L}{dx} = \frac{M_B}{6EI\ell}(3a^2 - 6a\ell + 2\ell^2)$$

from which

$$\xi = \left|\theta_L\right| = \frac{M_B}{6EI\ell}(\ell^2 - 3b^2)$$

Setting $I = \pi d^4/64$ and solving for d

$$d = \left|\frac{32M_B(\ell^2 - 3b^2)}{3\pi E\ell\xi}\right|^{\frac{1}{4}}$$

For a multiplicity of moments the slopes add vectorially and

$$d_L = \left|\frac{32}{3\pi E\ell\xi}\ \Sigma[M_i(\ell^2 - 3b_i^2)]^{\frac{1}{2}}\right|^{\frac{1}{4}}$$

$$d_R = \left|\frac{32}{3\pi E\ell\xi}\ \Sigma[M_i(3a_i^2 - \ell^2]^2\ ^{\frac{1}{2}}\right|^{\frac{1}{4}}$$

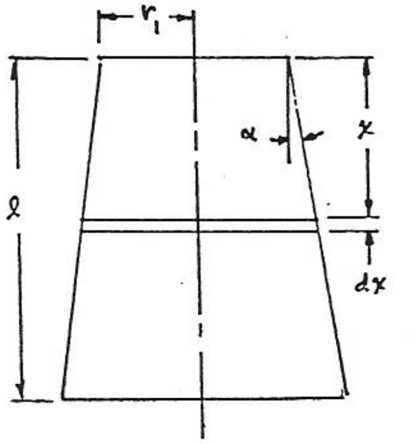
Greatest slope is at LH brg. So

$$d = \left|\frac{32(1200)[81 - 3(4^2)]}{3\pi(30)(10^6)(9)(0.002)}\right|^{\frac{1}{4}} = 0.706 \text{ in}$$

So use $d = 3/4$ in Ans.

3-25 OMITTED

3-26




Use Eq. (a) of Sec. 2-5 to get

$$d\delta = \frac{F\,dx}{AE} = \frac{F\,dx}{E\pi(r_1 + x\tan\alpha)^2}$$

$$\delta = \frac{F}{\pi E}\int_0^\ell \frac{dx}{(r_1 + x\tan\alpha)^2}$$

$$= \frac{F}{\pi E}\left(-\frac{1}{\tan\alpha\,(r_1 + x\tan\alpha)}\right)\Bigg|_0^\ell$$

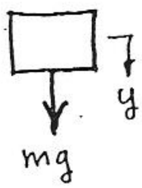
$$\delta = \frac{F}{\pi E}\,\frac{1}{r_1(r_1 + \ell\tan\alpha)}$$

Then

$$k = \frac{F}{\delta} = \frac{\pi E r_1(r_1 + \ell\tan\alpha)}{\ell}$$

$$= \frac{EA_1}{\ell}\left(1 + \frac{2\ell}{d_1}\tan\alpha\right)$$

3-27 For free fall with




$y \leq h$; $\Sigma F_y - m\ddot{y} = 0$

$mg - m\ddot{y} = 0$, so $\ddot{y} = g$

Using $y = a + bt + ct^2$

we have at $t = 0$, $y = 0$,

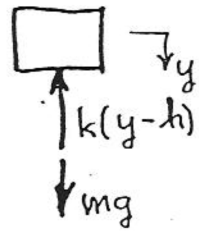
and $\dot{y} = 0$, and so $a = 0$, $b = 0$, and

$c = g/2$. Thus

$y = \frac{1}{2}gt^2$ and $\dot{y} = gt$, for $y \leq h$

At impact, $y = h$, $t = (2h/g)^{\frac{1}{2}}$, and

$v_0 = (2gh)^{\frac{1}{2}}$




After contact the D. E. is

$mg - k(y - h) - m\ddot{y} = 0$

for $y > h$

NOw let $x = y - h$; then

$\dot{x} = \dot{y}$, and $\ddot{x} = \ddot{y}$. So the D. E. is

$\ddot{x} + \frac{k}{m}x = g$ with solution $\omega = (k/m)^{\frac{1}{2}}$

and

$$x = A\cos\omega t' + B\sin\omega t' + \frac{mg}{k}$$

At contact $t' = 0$, $x = 0$, and $\dot{x} = v_0$

Evaluating A and B then yields

$$x = -\frac{mg}{k}\cos\omega t' + \frac{v_0}{\omega}\sin\omega t' + \frac{mg}{k}$$

or

$$y = -\frac{W}{k}\cos\omega t' + \frac{v_0}{\omega}\sin\omega t' + \frac{W}{k} + h$$

and

$$\dot{y} = \frac{W\omega}{k}\sin\omega t' + v_0\cos\omega t'$$

To find $y_{max}$ set $\dot{y} = 0$. Solving gives

$$\tan\omega t' = -\frac{v_0 k}{W\omega} \text{ or}$$

$$(\omega t')^* = \tan^{-1}\left(-\frac{v_0 k}{W\omega}\right)$$

The first value of $(\omega t')^*$ is a minimum and negative. So add $\pi$ radians to it to get the maximum.

Numerical example: $h = 1$ in, $W = 30$ lb, $k = 100$ lb/in. Then

$\omega = (k/m)^{\frac{1}{2}} = [100(386)/30]^{\frac{1}{2}} = 35.87$ rad/s

$W/k = 30/100 = 0.3$

$v_0 = (2gh)^{\frac{1}{2}} = [2(386)(1)]^{\frac{1}{2}} = 27.78$ in/s

Then

$$y = -0.3\cos 35.87t' + \frac{27.78}{35.87}\sin 35.87t' + 0.3 + 1$$

For $y_{max}$

$$\tan\omega t' = -\frac{v_0 k}{W\omega} = -\frac{27.78(100)}{30(35.87)} = -2.58$$

$(\omega t')^* = -1.20$ rad minimum

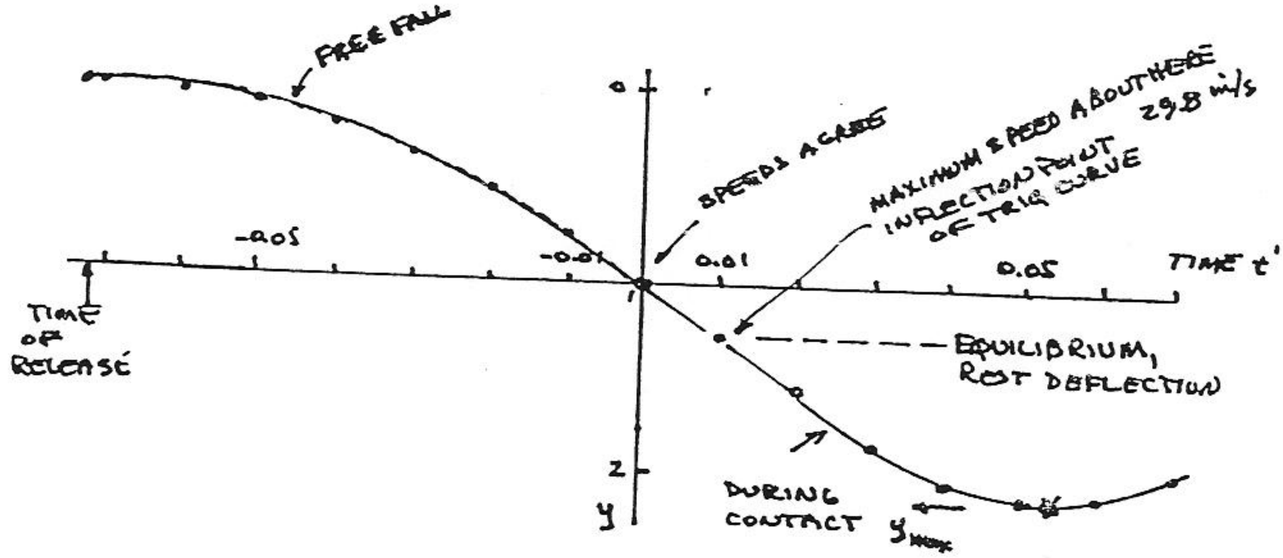
$(\omega t')^* = -1.20 + \pi = 1.940$ maximum

3-27 (Concluded) Then $t'^* = 1.940/35.87 = 0.0541$ s. This means that the spring bottoms out at $t'^*$ seconds. Then $(\omega t')^* = 35.87(0.0541) = 1.94$ rad

So $y_{max} = -0.3 \cos 1.94 + \dfrac{27.78}{35.87} \sin 1.94 + 0.3 + 1 = 2.130$ in  Ans.

The maximum spring force is $F_{max} = k(y_{max} - h) = 100(2.130 - 1) = 113$ lb  Ans.

The action is illustrated by the graph below. Applications: Impact, such as a dropped package. Also, a pogo stick with a passive rider. The idea has been used for a one-legged robotic walking machine.



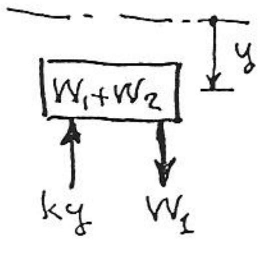

---

3-28 Choose $t' = 0$ at the instant of impact. At this instant, $v_1 = (2gh)^{\frac{1}{2}}$. Using momentum, $m_1 v_1 = m_2 v_2$. Thus

$$\frac{W_1}{g}(2gh)^{\frac{1}{2}} = \frac{W_1 + W_2}{g} v_2$$

So $$v_2 = \frac{W_1 (2gh)^{\frac{1}{2}}}{W_1 + W_2}$$

So, at $t' = 0$, $y = 0$, and $\dot{y} = v_2$




Let $W = W_1 + W_2$

The D. E. is

$$\frac{W}{g}\ddot{y} = -ky + W_1$$

because the spring force at $y = 0$ includes a reaction to $W_2$.

With $\omega = (kg/W)^{\frac{1}{2}}$ the solution is

$$y = A \cos \omega t' + B \sin \omega t' + W_1/k$$

$$\dot{y} = -A\omega \sin \omega t' + B\omega \cos \omega t'$$

At $t' = 0$, $y = 0$, so $A = -W_1/k$

At $t' = 0$, $\dot{y} = v_2$, so $v_2 = B\omega$

Then

$$B = \frac{v_2}{\omega} = \frac{W_1 (2gh)^{\frac{1}{2}}}{(W_1 + W_2)[kg/(W_1 + W_2)]^{\frac{1}{2}}}$$

3-28 (Concluded) We now have

$$y = -\frac{W_1}{k}\cos\omega t' + W_1\left[\frac{2h}{k(W_1 + W_2)}\right]^{\frac{1}{2}}\sin\omega t' + \frac{W_1}{k}$$

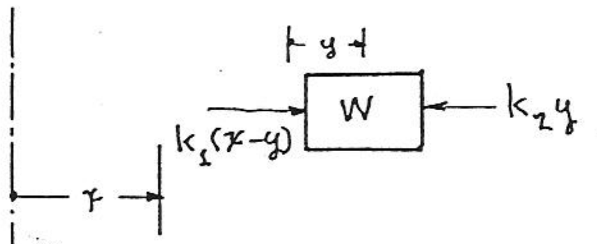
Transforming gives

$$y = \frac{W_1}{k}\left(\frac{2hk}{W_1 + W_2} + 1\right)^{\frac{1}{2}}\cos(\omega t' - \phi) + \frac{W_1}{k}$$ where $\phi$ is a phase angle. We now have

$$y_{max} = \frac{W_1}{k}\left(\frac{2hk}{W_1 + W_2} + 1\right)^{\frac{1}{2}} + \frac{W_1}{k}$$ Ans.

---

3-29 Assume x > y to get a free-body diagram.




Then

$$\frac{W}{g}\ddot{y} = k_1(x - y) - k_2y$$

A particular solution is

$$y = \frac{k_1a}{k_1 + k_2}$$

Then the complementary plus the particular solution is

$$y = A\cos\omega t + B\sin\omega t + \frac{k_1a}{k_1 + k_2}$$

where $\omega = \left[\frac{(k_1 + k_2)g}{W}\right]^{\frac{1}{2}}$

At t = 0, y = 0, and $\dot{y}$ = 0; so B = 0

and $A = -\dfrac{k_1a}{k_1 + k_2}$

Then

$$y = \frac{k_1a}{k_1 + k_2}(1 - \cos\omega t)$$

y is maximum when the cosine is -1. So

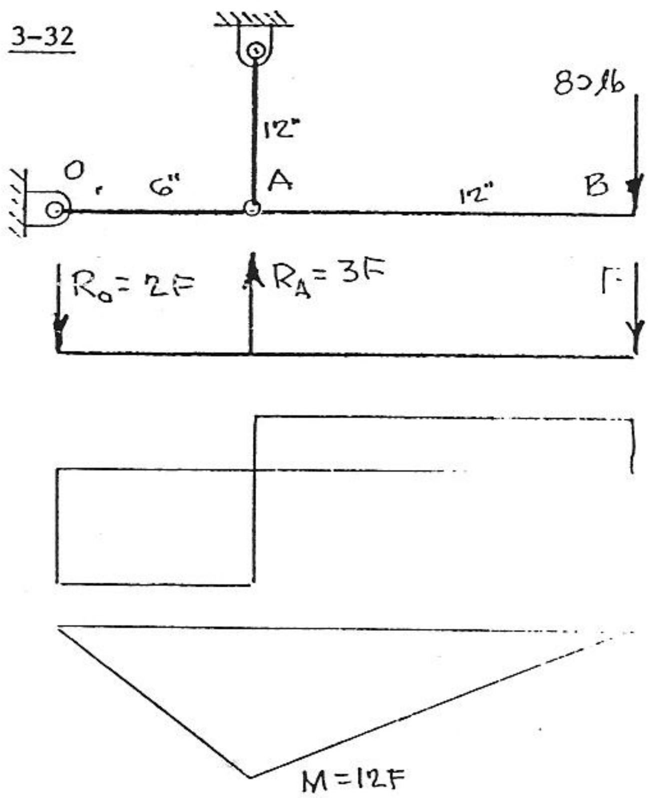
$$y_{max} = \frac{2k_1a}{k_1 + k_2}$$ Ans.

---

3-30 and 3-31 OMITTED

---

3-32




$$M_{OA} = -2Fx;\ M_{AB} = -12F + F(x - 6)$$

$$U = \int_0^6 \frac{M_{OA}^2}{2EI}dx + \int_6^{18}\frac{M_{AB}^2}{2EI}dx + \frac{F^2\ell}{2AE}$$

$$y_B = \frac{\partial U}{\partial F} = \frac{1}{EI}\int_0^6 M_{OA}\left(\frac{\partial M_{OA}}{\partial F}\right)dx + \frac{1}{EI}\int_6^{18} M_{AB}\left(\frac{\partial M_{AB}}{\partial F}\right)dx + \frac{F\ell}{AE}$$

3-32 (Continued)

$$\frac{\partial M_{OA}}{\partial F} = -2x \qquad \frac{\partial M_{AB}}{\partial F} = (x - 6) - 12$$
$$= x - 18$$

$$y_B = \frac{1}{EI}\int_0^6 (-2Fx)(-2x)\,dx$$
$$+\frac{F}{EI}\int_6^{18} (x - 6 - 12)(x - 18)\,dx$$
$$+\frac{12F}{AE} = \frac{864F}{EI} + \frac{12F}{AE}$$

$$I = \frac{bh^3}{12} = \frac{0.25(2)^3}{12} = 0.167 \text{ in}^4$$

$$A = \frac{\pi d^2}{4} = \frac{\pi(0.5)^2}{4} = 0.196 \text{ in}^2$$

$$y_B = \frac{864(80)}{10(10^6)(0.167)} + \frac{12(80)}{10(10^6)(0.196)}$$
$$= 0.0414 + 0.0005 = 0.0419 \text{ in } \underline{\text{Ans.}}$$

---

3-33 As a check on Castigliano's method, here is an independent one.

Let $L_1 = 0.1$ m and $L_2 = 1.50$ m.

Then $\theta = \frac{TL}{GJ} = \frac{FL_1L_2}{GJ}$

The cantilever deflection is

$y = \frac{FL_1^3}{3EI}$ So the total deflection is

$$y_T = y + \theta L_1 = \frac{FL_1^3}{3EI} + \frac{FL_1^2L_2}{GJ}$$

With E = 207 GPa, G = 79.3 GPa, $I = 0.102 \text{ cm}^4$, and $J = 0.204 \text{ cm}^4$, the result is

$y = F/y = 1/0.0943 = 10.60$ kN/m $\underline{\text{Ans.}}$

---

3-34 Here $\Sigma F = 250 - R_c - R_s = 0$ where $R_c$ is the reaction of the 3 copper wires and $R_s$ is that of the steel wire. Let $R_c$ be the redundant reaction. Then

$$R_s = 250 - R_c$$

$$U = \frac{R_s^2L}{2A_sE_s} + \frac{R_c^2L}{2A_cE_c}$$
$$= \frac{(250 - R_c)^2L}{2A_sE_s} + \frac{2R_cL}{2A_cE_c}$$

Now take the derivative and set it equal to zero. Thus

$$\frac{\partial U}{\partial R_c} = \frac{2(250 - R_c)(-1)L}{2A_sE_s} + \frac{2R_cL}{2A_cE_c} = 0$$

Solving gives

$$R_c = 250K, \text{ where } K = \frac{A_cE_c}{A_sE_s + A_cE_c}$$

The areas are for each wire

$$A_c = \pi(0.801)^2/4 = 5.03(10^{-3}) \text{ in}^2$$
$$A_s = \pi(0.0625)^2/4 = 3.07(10^{-3}) \text{ in}^2$$

Also, $E_c = 17.2$ Mpsi and $E_s = 30$ Mpsi

Thus
$$K = \frac{3(5.03)(17.2)}{3.07(30) + 3(5.03)(17.2)}$$
$$= 0.738$$

Then $R_c = 250(0.738) = 184.5$ lb

and $R_s = 250 - 184.5 = 65.5$ lb

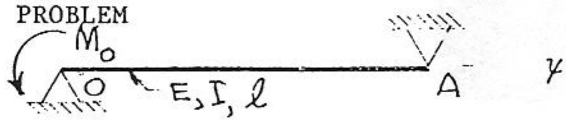
The stresses are

$$\sigma_c = \frac{R_c}{3A_c} = \frac{184.5}{3(5.03)} = 12.2 \text{ kpsi} \quad \underline{\text{Ans.}}$$

$$\sigma_s = \frac{R_s}{A_s} = \frac{64.5}{3.07} = 21 \text{ kpsi} \quad \underline{\text{Ans.}}$$

---

PROBLEM




Find maximum linear deflection due to $M_0$

---

3-35 (a) The bolt tensile stress is

$\sigma_b = 0.9S_p = 0.9(85) = 76.5$ kpsi  Ans.

Also

$A_b = \pi(0.375)^2/4 = 0.110\ \text{in}^2$

The preload is

$F_i = 6\sigma_b A_b = 6(76.5)(0.110) = 50.5$ kip

$$A_c = \frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}[(4.5)^2 - (4)^2]$$

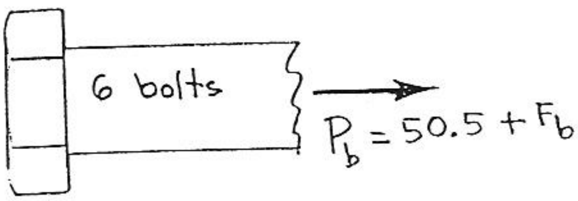
$$= 3.34\ \text{in}^2$$

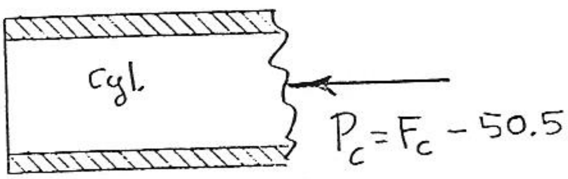
Then

$$\sigma_c = \frac{F_i}{A_c} = \frac{-50.5}{3.34} = -15.1 \text{ kpsi} \quad \text{Ans.}$$

(b) The force due to the internal pressure is

$P = \pi D^2 p/4 = \pi(4)^2(600)/4 = 7540$ lb




From diagrams $F_b + F_c = P = 7.54$  (1)

The loss of compression in the cylinder is

$$\Delta\delta_c = \frac{F_c L_c}{A_c E} = \frac{F_c(11)}{3.34(30)(10^6)}$$

$$= 0.110(10^{-6})F_c \quad (2)$$

The increase in the bolt elongation is

$$\Delta\delta_b = \frac{F_b L_b}{6A_b E} = \frac{F_b(12)}{6(0.110)(30)(10^6)}$$

$$= 0.606(10^{-6})F_b$$

Or, since $F_b = 7.54 - F_c$

$$\Delta\delta_b = 0.606(10^{-6})(7.54 - F_c) \quad (3)$$

Since Eqs. (2) and (3) must equal each other, we find $F_c = 6.38$ kip; and hence $F_b = 7.54 - 6.38 = 1.16$ kip. Then

$P_b = 50.5 + 1.16 = 51.66$ kip

and

$$\sigma_b = \frac{51.66}{6(0.110)} = 78.2 \text{ kpsi} \quad \text{Ans.}$$

Also $P_c = 6.38 - 50.5 = -44.12$ kip

and

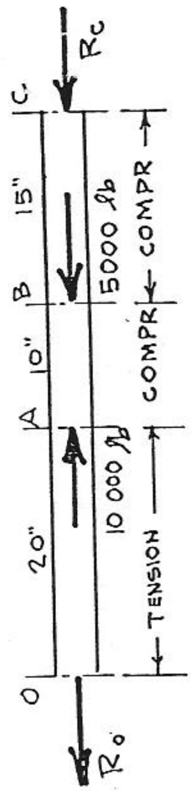
$$\sigma_c = \frac{-44.12}{3.34} = -13.2 \text{ kpsi} \quad \text{Ans.}$$

---

3-36 OMITTED

---

3-37




$$\delta_{OA} = \delta_{AB} + \delta_{BC} \quad (1)$$

$$\Sigma F = -R_O + 10 - 5 - R_C = 0$$

From which we get

$$R_O + R_C = 5 \quad (2)$$

From (1)

$$\frac{20R_O}{AE} = \frac{10(5)}{AE} + \frac{15R_C}{AE} \quad (3)$$

Combining (2) and (3) and solving gives

$$20(5 - R_C) - 15R_C = 50$$

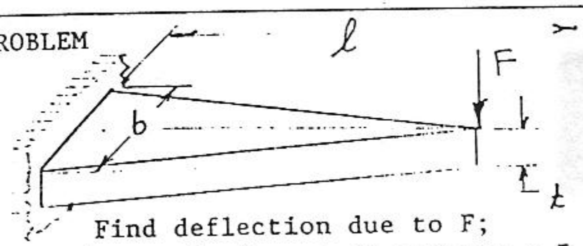
$$R_C = \frac{50}{35} = 1.43 \text{ kip} \quad \text{Ans.}$$

$R_O = 5 - 1.43 = 3.57$ kip  Ans.

Note that $R_O$ is tension and $R_C$ is compression.

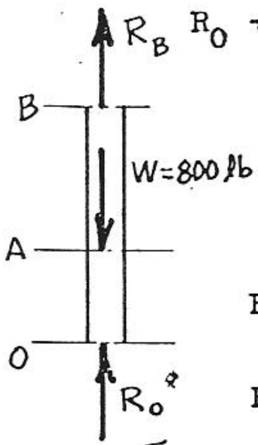
---

PROBLEM




Find deflection due to F; second moment of area at support = I

3-38




$R_O + R_B = W \quad (1) \qquad \delta_{OA} = \delta_{AB} \quad (2)$

$$\frac{20R_O}{AE} = \frac{30R_B}{AE}, \quad R_O = \frac{3}{2} R_B$$

$$800 - R_B = \frac{3}{2} R_B$$

$$R_B = \frac{1600}{5} = 320 \text{ lb} \quad \underline{\text{Ans.}}$$

$$R_O = 800 - 320 = 480 \text{ lb} \quad \underline{\text{Ans.}}$$

$$\sigma_O = - \frac{480}{1} = - 480 \text{ psi} \quad \underline{\text{Ans.}}$$

$$\sigma_B = + \frac{320}{1} = 320 \text{ psi} \quad \underline{\text{Ans.}}$$

---

3-39

$$\theta_{OA} = \theta_{AB}$$

$$\frac{T_{OA}(4)}{GJ} = \frac{T_{AB}(6)}{GJ}; \quad T_{OA} = \frac{3}{2} T_{AB}; \quad T_{OA} + T_{AB} = T$$

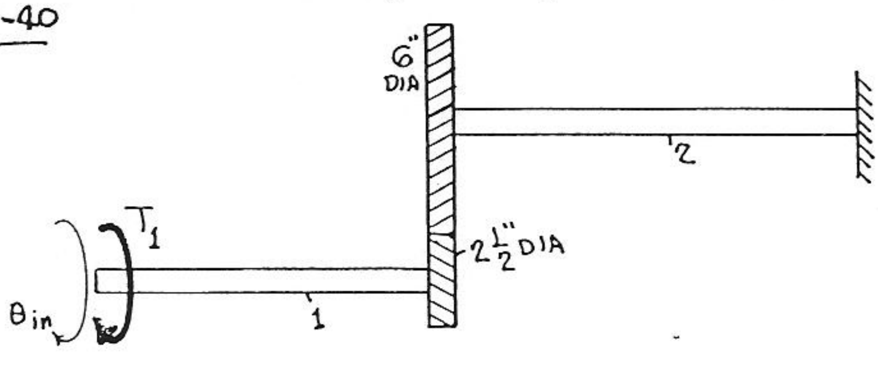
$$T_{AB}(\tfrac{3}{2} + 1) = T; \quad T_{AB} = \frac{T}{2.5} \quad \underline{\text{Ans.}}$$

$$T_{OA} = \frac{3}{2} T_{AB} = \frac{3T}{5} \quad \underline{\text{Ans.}}$$

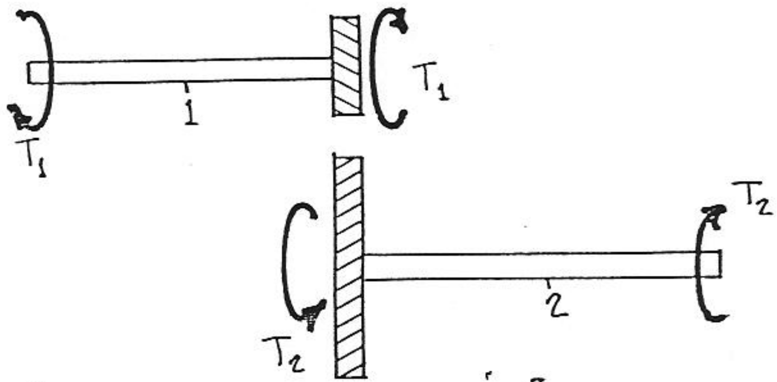
---

The shafts may be diagrammed as follows:

3-40



The free-body diagrams follow:




$$T_2 = \frac{3}{1.25} T_1 \qquad (1) \qquad \theta_{in} = \theta_1 + \frac{3}{1.25} \theta_2$$

$$\theta_1 = \frac{T_1 \ell}{GJ_1} \qquad \theta_2 = \frac{T_2 \ell}{GJ_2}$$

$$G = 11.5 \text{ Mpsi}, \quad J_1 = \frac{\pi(0.875)^4}{32} = 0.0575 \text{ in}^4$$

$$J_2 = \frac{\pi(1.25)^4}{32} = 0.240 \text{ in}^4$$

$$\theta_{in} = \frac{4\pi}{180} \text{ rad}$$

$$\frac{4\pi}{180} = \frac{T_1 \ell}{GJ_1} + \left(\frac{3}{1.25}\right)^2 \frac{T_1 \ell}{GJ_2}$$

$$T_1 \left[\frac{1}{J_1} + \left(\frac{3}{1.25}\right)^2 \frac{1}{J_2}\right] = \frac{4\pi G}{180 \ell}$$

$$T_1 \left[\frac{1}{0.0575} + \left(\frac{3}{1.25}\right)^2 \frac{1}{0.240}\right] = \frac{4\pi(11.5)(10)^6}{180(48)}$$

Solving gives $T_1 = 404$ lb·in

3-40 (Concluded)

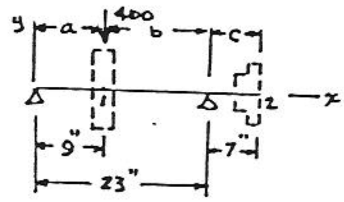
$$\text{Then } T_2 = \frac{3}{1.25}(404) = 970 \text{ lb·in}$$

$$\tau_1 = \frac{16T_1}{\pi d_1^3} = \frac{16(404)}{\pi(0.875)^3} = 3070 \text{ psi} \quad \underline{\text{Ans.}}$$

$$\tau_2 = \frac{16T_2}{\pi d_2^3} = \frac{16(970)}{\pi(1.25)^3} = 2530 \text{ psi} \quad \underline{\text{Ans.}}$$

---

3-41 Define $\delta_{ij}$ as the deflection in the direction of the load at station i due to a unitload at station i. If U is the potential energy of strain for a body obeying Hooke's law, apply $P_1$ first. Then




$I = \frac{\pi d^4}{64}$

$U = \frac{1}{2}P_1(P_1\delta_{11})$

When the second load is added U becomes

$U = \frac{1}{2}P_1(P_1\delta_{11}) + \frac{1}{2}P_2(P_2\delta_{22}) + P_1(P_2\delta_{12})$

For loading in the reverse order

$U' = \frac{1}{2}P_2(P_2\delta_{22}) + \frac{1}{2}P_1(P_1\delta_{11}) + P_2(P_1\delta_{21})$

Since the order of loading is immaterial $U = U'$ and

$P_1P_2\delta_{12} = P_2P_1\delta_{21}$

when $P_1 = P_2$, $\delta_{12} = \delta_{21}$

which states that the deflection at station 1 due to a unit load at station 2 is the same as the deflection at station 2 due to a unit load at 1. $\delta$ is sometimes called an <u>influence coefficient</u>.

---

3-42 (a)

$$y_{AB} = \frac{Fcx(\ell^2 - a^2)}{6EI\ell}$$

$$\delta_{12} = \frac{y}{F}\bigg|_{x=a} = \frac{ca(\ell^2 - a^2)}{6EI}$$

$$y_2 = F\delta_{21} = F\delta_{12}$$

$$= \frac{Fca(\ell^2 - a^2)}{6EI\ell}$$

$$= \frac{400(7)(9)(23^2 - 9^2)(64)}{6(30)(10^6)(\pi)(2)^4(23)}$$

$$= 0.0035 \text{ in}$$

(b) The slope of the shaft at <u>left</u> bearing at $x = 0$ is

$$\theta = \frac{Fb(b^2 - \ell^2)}{6EI\ell}$$

Viewing the illustration in Sec. 6 of Table A-9 from the back of the page is to view this problem. Noting that a is to be interchanged with b and -x with x leads to

$$\theta = \frac{Fa(\ell^2 - a^2)}{6EI\ell} = \frac{Fa(\ell^2 - a^2)(64)}{6E\pi d^4\ell}$$

3-42 (Concluded)

$$\theta = \frac{400(9)(23^2 - 9^2)(64)}{6(30)(10^6)\pi(2)^4(23)} = 0.000\ 496 \text{ in/in}$$

So $y_2 = 7\theta = 7(0.000\ 496) = 0.0035$ in

---

3-43 The cross section at A does not rotate, and so we have for a single quadrant that

$$\frac{\partial U}{\partial M_A} = 0$$

The bending moment at angle θ to the x axis is

$$M = M_A - \frac{F}{2}(R - x) = M_A - \frac{FR}{2}(1 - \cos\theta)$$

because $x = R\cos\theta$. Next

$$U = \int \frac{M^2\,ds}{2EI} = \int_0^{\pi/2} \frac{M^2 R\,d\theta}{2EI}$$

since $ds = R\,d\theta$. Then

$$\frac{\partial U}{\partial M_A} = \frac{R}{EI}\int_0^{\pi/2} M\frac{\partial M}{\partial M_A}\,d\theta = 0$$

But $\partial M/\partial M_A = 1$. Therefore

$$\int_0^{\pi/2} M\,d\theta = \int_0^{\pi/2}\left[M_A - \frac{FR}{2}(1 - \cos\theta)\right]d\theta$$

Since this term is zero, we have

$$M_A = \frac{FR}{2}\left(\frac{1}{2} - \frac{1}{\pi}\right)$$

Then we find that

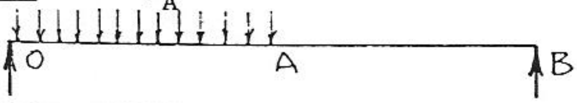
$$M = \frac{FR}{2}\left(\cos\theta - \frac{2}{\pi}\right)$$

The maximum occurs at B where $\theta = \pi/2$. It is

$$M_B = -\frac{FR}{\pi}$$

---

QUIZ Find $y_A$




---

3-44 For one quadrant

$$M = \frac{FR}{2}\left(\cos\theta - \frac{2}{\pi}\right); \quad \frac{\partial M}{\partial F} = \frac{R}{2}\left(\cos\theta - \frac{2}{\pi}\right)$$

$$\delta = \frac{\partial U}{\partial F} = 2\int_0^{\pi/2} \frac{MR}{EI}\frac{\partial M}{\partial F}\,d\theta$$

$$= \frac{FR^3}{2EI}\int_0^{\pi/2}\left(\cos\theta - \frac{2}{\pi}\right)^2 d\theta$$

$$= \frac{FR^3}{2EI}\left(\frac{\pi}{4} - \frac{2}{\pi}\right)$$

For the entire ring

$$\delta_{Total} = 2\delta = \frac{FR^3}{EI}\left(\frac{\pi}{4} - \frac{2}{\pi}\right)$$

---

3-45 Equation (3-40) becomes

$$U = 2\int_0^{\pi} \frac{M^2 R\,d\theta}{2EI} \qquad R/h > 10$$

where $M = FR(1 - \cos\theta)$ and

$$\frac{\partial M}{\partial F} = R(1 - \cos\theta)$$

$$\delta = \frac{\partial U}{\partial F} = \frac{2}{EI}\int_0^{\pi} MR\frac{\partial M}{\partial F}\,d\theta$$

$$= \frac{2}{EI}\int_0^{\pi} FR^3(1 - \cos\theta)^2\,d\theta$$

$$= \frac{3\pi FR^3}{EI}$$

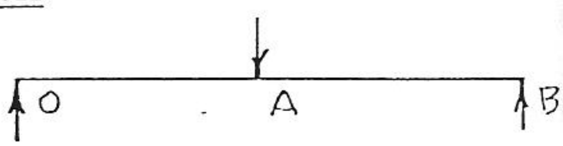
Since $I = bh^3/12 = 4(6)^3/12 = 72$ mm$^4$ and $R = 81/2 = 40.5$ mm, we have

$$\delta = \frac{3\pi(40.5)^3 F}{131(72)} = 66.4F \text{ mm} \quad \underline{\text{Ans.}}$$

where F is in kN.

---

QUIZ




Find slope at O due to shear

---

3-46 and 3-47 OMITTED

COLUMN ANALYSIS PROGRAM

1. Next if metric units, else 9
2. Use MPa for strength, GPa for E, mm for dimensions, and kN for force
3. Enter $S_y$
4. $S_y = 10^6 S_y$
5. Enter E
6. $E = 10^9 E$
7. Enter L
8. $L = 10^{-3} L$; go to 15
9. Use kpsi for strength, Mpsi for E, inches for dimensions.
10. Enter $S_y$
11. $S_y = S_y$
12. Enter E
13. $E = 10^6 E$
14. Enter L
15. Next if end-condition constants are same in both directions, else 18
16. Enter C
17. $C_x = C_y = C$; go to 19
18. Enter $C_x$ and $C_y$
19. If section is a solid round go to 23, else next
20. If section is a hollow round go to 29, else next
21. If section is rectangular, go to 39, else next
22. If section is a rolled shape, go to 48, else stop
23. Next if metric units, else 25
24. Enter D; go to 26
25. Enter D
26. $k_x = k_y = D/4$
27. $A = \pi D^2/4$
28. $I_x = I_y = AD^2/16$; go to 64
29. Next if metric units, else 33
30. Enter outside diameter $D_o$ and wall thickness t
31. $D_o = 10^{-3} D_o$
32. $T = 10^{-3} t$; go to 35
33. Enter outside diameter $D_o$ and wall thickness t
34. $D_o = D_o$; $T = t$
35. $D_i = D_o - 2T$
36. $A = \pi(D_o^2 - D_i^2/4$
37. $I_x = I_y = \pi(D_o^4 - D_i^4)/64$
38. $k_x = [(D_o^2 + D_i^2)/16]^{\frac{1}{2}}$; $k_y = k_x$; go to 64
39. Next if metric, else 43
40. Enter x and y dimensions of rectangle in mm
41. $WX = 10^{-3} x$
42. $WY = 10^{-3} y$; go to 45
43. Enter x and y dimensions of rectangle in inches
44. $WX = x$; $WY = y$
45. $A = (WX)^2$; $k_x = 0.289WY$; $k_y = 0.289WX$
46. $I_x = WX(WY)^3/12$
47. $I_y = WY(WX)^3/12$; go to 64
48. Next if metric, else 51
49. Enter section area in $cm^4$
50. $A = 10^4 A$; go to 53
51. Enter section area in square inches
52. A = A
53. Next if metric, else 57
54. Enter $I_x$ and $I_y$ in $cm^4$
55. $I_x = 10^{-8} I_x$

COLUMN PROGRAM (Continued)

56. $I_y = 10^{-8} I_y$; go to 59

57. Enter $I_x$ and $I_y$ in $mm^4$

58. $I_x = I_x$; $I_y = I_y$

59. Next if metric, else 62

60. Enter $k_x$ and $k_y$ in mm

61. $k_x = 10^{-3} k_x$; $k_y = 10^{-3} k_y$; go to 64

62. Enter $k_x$ and $k_y$ in inches

63. $k_x = k_x$; $k_y = k_y$

64. $B_x = (S_y/2\pi)^2/C_x E$

65. $B_y = B_x C_x / C_y$

66. Euler $P_x = \pi^2 C_x E I_x / L^2$

67. Euler $P_y = \pi^2 C_y E I_y / L^2$

68. Johnson $P_x = A[S_y - B_x(L/k_x)^2]$

69. Johnson $P_y = A[S_y - B_y(L/k_y)^2]$

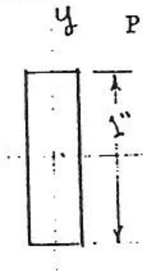
70. Print desired output

---

3-48 See solution below

$P = 298$ lb, $P_{cr} = 5(298)$

$= 1490$ lb



$L = 41.7$ in

(a) Use $t = \frac{1}{2}$ in

(b) $\sigma_b = -298/0.5 \cong -600$ psi

No.

```
A =  .375 SQUARE IN
KX =  .289   IN  KY =   .108375 IN
L/KX =  144.2907   L/KY =   384.7751
EULER PX =   5321.061 LB
EULER PY =   897.9291 LB
JOHNSON PX =   5202.919 LB
JOHNSON PY = -13501.23 LB
```

```
A =   .5 SQUARE IN
KX =  .289   IN  KY =   .1445 IN
L/KX =  144.2907   L/KY =   288.5813
EULER PX =   7094.749 LB
EULER PY =   2128.424 LB
JOHNSON PX =   6937.225 LB
JOHNSON PY = -4875.916 LB
```

```
A =  .4375 SQUARE IN
KX =  .289   IN  KY =   .1264375 IN
L/KX =  144.2907   L/KY =   329.8072
EULER PX =   6207.905 LB
EULER PY =   1425.878 LB
JOHNSON PX =   6070.072 LB
JOHNSON PY = -8786.76 LB
```

---

3-49 OMITTED

---

3-50 $S_y = 68$ kpsi, $E = 30$ Mpsi

$$P_{cr} = np\frac{\pi D^2}{4} = 3(800)\frac{\pi(3)^2}{4} = 17\ 000 \text{ lb}$$

$P = P_{cr}/n = 5650$ lb

See computer output; then

(a) $d = 1.5$ in; (b) $d = 0.875$ in;

(c)
$$n = \frac{20\ 439}{5650} = 3.61$$

$$n = \frac{24\ 994}{5650} = 4.42$$

```
PROB. NO. 3-50A(D = 1.375")
A =  1.484892 SQUARE IN
KX =  .34375   IN  KY =   .34375 IN
L/KX =  174.5455   L/KY =   174.545
EULER PX =   14431.06 LB
EULER PY =   14431.06 LB
JOHNSON PX = -75651.27 LB
JOHNSON PY = -75651.27 LB
```

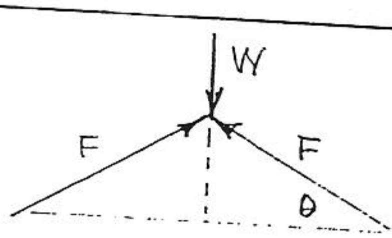
```
PROB. NO. 3-50A (D = 1.5")
A =  1.767145 SQUARE IN
KX =  .375   IN  KY =   .375 IN
L/KX =  160   L/KY =   160
EULER PX =   20438.66 LB
EULER PY =   20438.66 LB
JOHNSON PX = -56458.13 LB
JOHNSON PY = -56458.13 LB
```

3-50 (Concluded)

```
PROB. NO. 3-50B   (D = .75")
A =   .4417861 SQUARE IN
KX =   .1875   IN   KY =   .1875 IN
L/KX =   96    L/KY =   96
EULER PX =   14193.51 LB
EULER PY =   14193.51 LB
JOHNSON PX =   14145.3 LB
JOHNSON PY =   14145.3 LB
```

```
PROB. NO. 3-50B   (D = .875")
A =   .60132 SQUARE IN
KX =   .21875   IN   KY =   .21875 IN
L/KX =   82.28571    L/KY =   82.2857
EULER PX =   26295.24 LB
EULER PY =   26295.24 LB
JOHNSON PX =   24993.6 LB
JOHNSON PY =   24993.6 LB
```

3-51




$2F \sin\theta = 9.8W$, $F = \dfrac{9.8W}{2\sin\theta}$

$F = \dfrac{9.8(400)}{2\sin 15°}(10^{-3}) = 7.57$ kN

Then $P_{cr} = nF = 2.5(7.57) = 18.9$ kN

See computer output below

$S_y = 380$ MPa, $L = 300$ mm, $C_x = 1$,

$C_y = 1.4$, $E = 207$ GPa

Use 2 bars, each 25 x 4.5 mm,

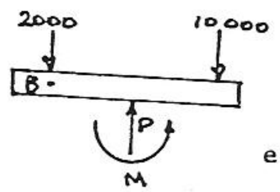
$P_{cr} = 32.6$ kN, $n = 32.6/7.57 = 4.31$

```
PROB. NO. 3-51  (25 X 8)
A =   2 SQUARE CM
KX =   2.312   MM   KY =   7.225001 MM
L/KX =   .1297578    L/KY =   4.152249E-02
EULER PX =   24.2134 KN
EULER PY =   331.0425 KN
JOHNSON PX =   16.49757 KN
JOHNSON PY =   71.64782 KN
```

```
PROB. NO. 3-51  (25 X 9)
A =   2.25 SQUARE CM
KX =   2.601   MM   KY =   7.225001 MM
L/KX =   .1153403    L/KY =   4.152249E-02
EULER PX =   34.47571 KN
EULER PY =   372.4227 KN
JOHNSON PX =   32.60895 KN
JOHNSON PY =   80.6038 KN
```

3-52 $\Sigma F = 0 = 2000 + 10\,000 - P$

$P = 12\,000$ lb; $\Sigma M_B = 12\,000\,\dfrac{5.68}{2}$

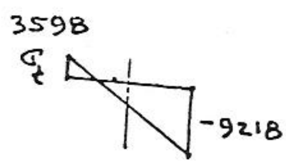



$- 10\,000(5.68) + M = 0$

$M = 22\,720$ in·lb

$e = \dfrac{M}{P} = \dfrac{22\,720}{12\,000} = 1.893"$

From Table A-8




$A = 4.271$ in²

$I = 7.090$ in⁴

$k^2 = \dfrac{I}{A} = \dfrac{7.090}{4/271}$

$= 1.66$ in²

$$\sigma_c = -\frac{12\,000}{4.271}\left[1 + \frac{1.893(2)}{1.66}\right] = -9218 \text{ psi}$$

$$\sigma_b = \frac{Mc}{I} = \frac{22\,720(2)}{7.090} = 6409 \text{ psi}$$

$$y = -k^2/e = -1.66/1.893 = -0.877 \text{ in}$$

$$\sigma_t = -\frac{12\,000}{4.271}\left[1 - \frac{1.893(2)}{1.66}\right] = 3598 \text{ psi}$$

3-53 $$\frac{P}{A} = \frac{S_y}{1 + \left(\frac{ec}{k^2}\right)\sec(x)}$$

$\sec(x) = 1.01$; $\cos x = \dfrac{1}{\sec x} = \dfrac{1}{1.01}$

$$x = \cos^{-1}\left(\frac{1}{1.01}\right) = 0.141 \text{ rad}$$

$$x = \frac{\ell}{2k}\left(\frac{P}{AE}\right)^{\frac{1}{2}} = 0.141$$

$$\frac{\ell}{k} = \frac{2(0.141)}{(P/AE)^{\frac{1}{2}}} = 0.282\,(AE/P)^{\frac{1}{2}}$$

## CHAPTER 4

Statistics is a fundamental, independent discipline which is a pervasive means of gaining knowledge by inferring underlying reality. Engineers have need for data. Variation in process is omnipresent. Data production should be designed with variation in mind. Statistics allows the quantification and explanation of variation. Nature is full of individual variability with long term regularity. Stability is not uniqueness but a pattern of variation, a mix of systematic and random effects. Statistics allows these to be separated, and is about variation and its measurement, the ways models describe data and data criticizes models, and about sensitive use of data to illuminate dark places.

One reference is Kennedy & Neville, Basic Statistical Methods for Engineers and Scientists, 3rd Ed., Harper & Row, 1986. See Lipson & Sheth, Statistical Design and Analysis of Engineering Experiments, McGraw-Hill, 1973, also.

4-1.

| x | n | nx | $nx^2$ |
|---|---|---|---|
| 60 | 2 | 120 | 7 200 |
| 70 | 1 | 70 | 4 900 |
| 80 | 3 | 240 | 19 200 |
| 90 | 5 | 450 | 40 500 |
| 100 | 8 | 800 | 80 000 |
| 110 | 12 | 1320 | 145 200 |
| 120 | 6 | 720 | 86 400 |
| 130 | 10 | 1300 | 169 000 |
| 140 | 8 | 1120 | 156 800 |
| 150 | 5 | 750 | 112 500 |
| 160 | 2 | 320 | 51 200 |
| 170 | 3 | 510 | 86 700 |
| 180 | 2 | 360 | 64 800 |
| 190 | 1 | 190 | 36 100 |
| 200 | 0 | 0 | 0 |
| 210 | 1 | 210 | 44 100 |
| | 70 | 8480 | 1104 600 |

Eq. (4-3): $\bar{x} = 8480/70 = 121.1$ kcycles

Eq. (4-6):

$$s = \left[\frac{1\ 104\ 600 - 8480^2/70}{69}\right]^{1/2}$$

s = 33.47 kcycles

4-3. (a)

| x | n | nx | $nx^2$ | f(x) | $n_i/232$ |
|---|---|---|---|---|---|
| 64 | 2 | 128 | 8 192 | 0.0057 | 0.0086 |
| 68 | 6 | 408 | 27 744 | 0.0175 | 0.0259 |
| 72 | 6 | 432 | 31 104 | 0.0388 | 0.0259 |
| 76 | 9 | 684 | 51 984 | 0.0597 | 0.0388 |
| 80 | 19 | 1520 | 121 600 | 0.0629 | 0.0819 |
| 84 | 10 | 840 | 70 560 | 0.0446 | 0.0431 |
| 88 | 4 | 352 | 30 976 | 0.0216 | 0.0172 |
| 92 | 2 | 184 | 16 928 | 0.0071 | 0.0086 |
| | 58 | 4548 | 359 088 | | |

$x = 4548/58 = 78.4$ kpsi

$$s = \left[\frac{359\ 088 - 4548^2/58}{57}\right]^{1/2} = 6.57 \text{ kpsi}$$

Eq. (4-13):

$$f(x) = \frac{1}{6.57\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - 78.4}{6.57}\right)^2\right]$$

Histographic PDF:

$$f_i(x) = n_i/(Nw) = n_i/(58[4]) = n_i/232$$

(b) From histogram:

$$F(72) = \frac{2 + 6 + 6/2}{58} = 0.189$$

From normal fit:

$$z = \frac{x - 78.4}{6.57} = \frac{72 - 78.4}{6.57} = -0.9741$$

From Table A-10:

$$F(72) = \Phi(-0.9741) = \alpha = 0.165$$

which is a better estimate of population CDF than sample information.

4-4.

| x | n | nx | $nx^2$ |
|---|---|---|---|
| 5.625 | 1 | 5.625 | 31.641 |
| 5.875 | 0 | 0. | 0. |
| 6.125 | 0 | 0. | 0. |
| 6.375 | 3 | 19.125 | 121.922 |
| 6.625 | 3 | 19.875 | 131.672 |
| 6.875 | 6 | 41.250 | 283.594 |
| 7.125 | 14 | 99.750 | 710.719 |
| 7.375 | 15 | 110.625 | 815.859 |
| 7.625 | 10 | 76.250 | 581.406 |
| 7.875 | 2 | 15.750 | 124.031 |
| 8.125 | 1 | 8.125 | 66.016 |
| | 55 | 396.375 | 2 866.859 |

$\bar{x} = 396.375/55 = 7.207$

$$s = \left[\frac{2\ 866.859 - 396.375^2/55}{54}\right]^{1/2}$$

$= 0.436$

$C_x = s/\bar{x} = 0.436/7.207 = 0.0605$

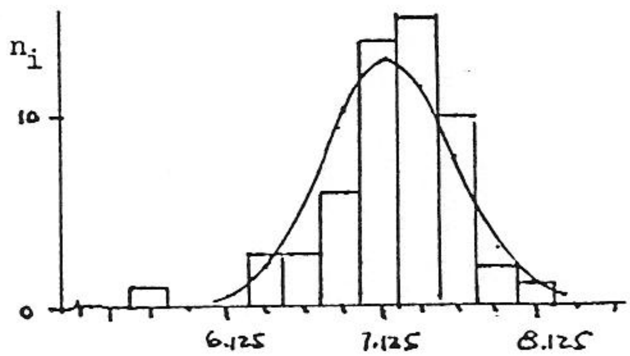
$$f(x) = \frac{1}{0.436\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - 7.207}{0.436}\right)^2\right]$$

Predicted class frequency:

$n_i = Nwf(x)$

$w = 5.875 - 5.625 = 0.25$ therefore

$n_i = 55(0.25)f(x) = 13.75f(x)$




4-6. Dimensions produced on parts by automatic machinery grow uniformly in size as the stream of parts accumulates. Dimensions become uniform random in distribution when mixed. For a uniform random dimension x in the interval a,b

$$\mu_x = \frac{a + b}{2} \qquad \hat{\sigma}_x = \frac{b - a}{2\sqrt{3}}$$

Solve simultaneously for a and b

$a = \mu - \sqrt{3}\hat{\sigma}$

$b = \mu + \sqrt{3}\hat{\sigma}$

Estimating population parameters $\mu$ and $\sigma$ using sample parameters $\bar{x}$ and s we obtain

$\hat{a} = 0.6241 - \sqrt{3}(0.000\ 581) = 0.6231$ in

$\hat{b} = 0.6241 + \sqrt{3}(0.000\ 581) = 0.6251$ in

We suspect that the dimensions were $\frac{0.625''}{0.623''}$.

---

4-7. Since F(x) is linear the distribution is uniform random. At x = a

$F(a) = 0 = 0.555a - 33$

$a = 33/0.555 = 59.46$ mm

At x = b

$F(b) = 1 = 0.555b - 33$

$b = (33 + 1)/0.555 = 61.26$ mm

Therefore

$$F(x) = \begin{cases} 0.555x - 33 & 59.46 \le x \le 61.26 \\ 0 & x < 59.46 \\ 1 & x > 61.26 \end{cases}$$

The PDF is the derivative of th CDF with respect to x, thus

$f(x) = dF(x)/dx = 0.555 \quad 59.46 \leq x \leq 61.26$

$= 0 \quad$ otherwise

From the range numbers

$= (a + b)/2 = (59.46 + 61.26)/2$

$= 60.36$ mm

$= (b - a)/2\sqrt{3} = (61.26 - 59.46)/2\sqrt{3}$

$= 0.520$ mm

---

4-9. The expression $\underset{\sim}{\epsilon} = \underset{\sim}{\delta}/\underset{\sim}{\ell}$ is of the form $\underset{\sim}{x}/\underset{\sim}{y}$. Now

$C_x = 0.000\ 092/0.0015 = 0.0613$

$C_y = 0.0018/2 = 0.000\ 90$

From Table 4-4

$$\mu_\epsilon = \frac{\mu_x}{\mu_y} = \frac{0.0015}{2} = 0.000\ 75$$

$C_\epsilon = (C_x^2 + C_y^2)^{1/2}$

$= (0.0613^2 + 0.0009^2)^{1/2} = 0.061\ 34$

$\hat{\sigma}_\epsilon = C_\epsilon \mu_\epsilon = 0.061\ 34(0.000\ 75)$

$= 0.000\ 046$

---

4-10. The equation $\underset{\sim}{\sigma} = \underset{\sim}{\epsilon}\ \underset{\sim}{E}$ is of the form $\underset{\sim}{xy}$.

$C_x = 0.000\ 034/0.0005 = 0.068$

$C_y = 0.885[10^6]/29.5[10^6] = 0.03$

From Table 4-4

$\mu_\sigma = \mu_e \mu_E = 0.0005(29.5[10^6])$

$= 14\ 800$ psi

$C_\sigma = (C_x^2 + C_y^2)^{1/2} = (0.068^2 + 0.03^2)^{1/2}$

$= 0.0743$

$\hat{\sigma}_\sigma = C_\sigma \mu_\sigma = 0.0743(14\ 800) = 1100$ psi

---

4-12. The form of the stress equation is $\underset{\sim}{x}/\underset{\sim}{y}$.

$C_x = 1350/15\ 000 = 0.09$

Using Table 4-4

$C_y = C_{d3} = 3C_d = 3(0.005)/2 = 0.0075$

$$\mu_\sigma = \frac{32\mu_x}{\pi \mu_{d3}} \doteq \frac{32(15\ 000)}{\pi\ 2^3} = 19\ 099 \text{ psi}$$

$C_\sigma = (C_x^2 + C_{d3}^2)^{1/2}$

$= (0.09^2 + 0.0075^2)^{1/2} = 0.0903$

$\hat{\sigma}_\sigma = C_\sigma \mu_\sigma = 0.0903(19\ 099) = 1725$ psi

---

4-13. The stochastic equation of interest is $\underset{\sim}{e} = \bar{r} - \underset{\sim}{r}$.

$\mu_e = \bar{r} - \mu_r$

$\hat{\sigma}_e = \hat{\sigma}_r$

$C_e = \hat{\sigma}_e/\mu_e$

For three digit display m = 3,

$r = 0.149(10^2)$

$\mu_r = 0.149(10^2)$

$$\hat{\sigma}_r = \frac{0.5(10^2)}{\sqrt{3}\ 10^3} = 0.0289$$

$\mu_e = 15 - 0.149(10^2) = 0.1$

$\hat{\sigma}_e = \hat{\sigma}_r = 0.0289$

$C_e = \hat{\sigma}_e/\mu_e = 0.0289/0.1 = 0.289 = C_\sigma$

Imprecision in e brings a coefficient of variation of 0.289 to the stress. For $m = 4$, $r = 0.1495(10^2)$ and

$$\mu_r = 0.1495(10^2)$$

$$\hat{\sigma}_r = \frac{0.5(10^2)}{\sqrt{3}\ 10^4} = 0.002\ 89$$

$$\mu_e = \bar{r} - \mu_r = 15 - 0.1495(10^2) = 0.05$$

$$\hat{\sigma}_e = \hat{\sigma}_r = 0.002\ 89$$

$$C_e = 0.002\ 89/0.05 = 0.0578 = C_\sigma$$

and the imprecision in stress has been reduced. For $m = 5$ $C_e = C_\sigma = 0.005\ 78$.

---

4-15. $C_F = 400/5000 = 0.08$. Call $\underset{\sim}{p}_{max}$ simply $\underset{\sim}{p}$. The COV of $\underset{\sim}{F}$ dominates. In Eq. (2-93) define h as

$$h = \frac{2}{\pi \ell} \frac{2(1-\nu^2)/E}{1/d_1 + 1/d_2}$$

$$= \frac{2}{\pi 3} \frac{2(1 - 0.30^2/30\ 10^6}{1/0.5 + 1/1} = 0.429\ 10^{-8}$$

then $b = \sqrt{hF}$. From Eq. (2-94)

$$p = \frac{2F}{\pi b \ell} = \frac{2F}{\pi h^{1/2} F^{1/2} \ell} = \frac{2F^{1/2}}{\pi h^{1/2} \ell}$$

$$= \frac{2F^{1/2}}{\pi(.0429\ 10^{-8})^{1/2}\ 3} = 3240F^{1/2}$$

$$\mu_p = 3240\,\mu_F^{1/2} = 3240(5000)^{1/2}$$

$$= 229\ 031 \text{ psi}$$

From Table 4-4 by analogy for powers of x, $C = C_x/2$ so

$$C_p = C_F/2 = 400/(2[5000]) = 0.04$$

$$\hat{\sigma}_p = C_p \mu_p = 0.04(229\ 031) = 9161 \text{ psi}$$

therefore

$$\underset{\sim}{p} = \underset{\sim}{p}_{max} = (229.0,\ 9.2) \text{ kpsi}$$

---

4-16. From Prob. 4-1 $\hat{\mu} = 121.1$ kcycles and $\hat{\sigma} = 33.47$ kcycles.

$$z_{10} = \frac{x_{10} - \hat{\mu}}{\hat{\sigma}} = \frac{x_{10} - 121.1}{33.47}$$

$$x_{10} = 121.1 + 33.47 z_{10}$$

| | | | |
|---|---|---|---|
| 0.1003 | 1.28 | Using Table A-10 |
| 0.1000 | a | and interpolat- |
| 0.0985 | 1.29 | ing gives |

$a = 1.281\underline{6}$

$z_{10} = -1.281\underline{6}$

$$x_{10} = 121.1 + 33.27(-1.281\underline{6})$$

$$= 78.2 \text{ kcycles}$$

---

4-17. From Table A-10 8.1% corresponds to $z_1 = -1.4$ and 5.5% corresponds to $z_2 = 1.6$ with position with respect to the mean noted.

$$k_1 = \mu_k + z_1 \hat{\sigma}_k$$

$$k_2 = \mu_k + z_2 \hat{\sigma}_k$$

from which

$$\mu_k = \frac{z_2 k_1 - z_1 k_2}{z_2 - z_1} = \frac{1.6(9) - (-1.4)11}{1.6 - (-1.4)}$$

$$= 0.9\underline{3}$$

$$\hat{\sigma}_k = \frac{k_2 - k_1}{z_2 - z_1} = \frac{11 - 9}{1.6 - (-1.4)} = 0.66\underline{6}$$

From Eq. (4-13)

$$f(k) = \frac{1}{0.66\underline{6}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{k - 9.9\underline{3}}{0.66\underline{6}}\right)^2\right]$$

This is the original density not the density after truncation.

---

4-19. (a)

$\mu = (a + b)/2 = (0.748 + 0.751)/2$

$= 0.7495$ in

$\sigma = (b - a)/2\sqrt{3} = (0.751 - 0.748)/2\sqrt{3}$

$= 0.000\ 8\underline{6}$ in

$f(x) = 1/(b - a) = 1/(0.751 - 0.748)$

$= 333.\underline{3}\ \text{in}^{-1}$

$F(x) = (x - 0.748)/(0.751 - 0.748)$

$= (x - 0.748)/\ 0.003$

(b) $F(x_1) = F(0.748) = 0$

$$F(x_2) = F(0.750) = \frac{0.750 - 0.748}{0.003}$$

$= 0.66\underline{6}$

If $g(x)$ is the truncated density, then from Prob. 4-18

$$g(x) = \frac{f(x)}{F(x_2) - F(x_1)} = \frac{333.3}{0.66\underline{6} - 0}$$

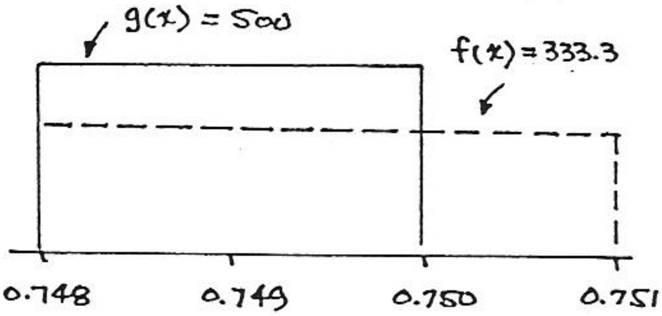
$= 500\ \text{in}^{-1}$

$= (a' + b')/2 = (0.748 + 0.750)/2$

$= 0.749$ in

$\hat{\sigma} = (b' - a')/2\sqrt{3} = (0.750 - 0.748)/2\sqrt{3}$

$= 0.000\ 577$ in




---

4-20. From the definition of a mean

$$\mu_{x,t} = E(x_t) = \int_{x_1}^{x_2} x\, g(x)\, dx$$

$$= \int_{x_1}^{x_2} \frac{x\, f(x)\, dx}{F(x_2) - f(x_1)} \quad \text{Ans.}$$

The variance is the difference between the expected square of the variate and the mean variate squared, or

$$\hat{\sigma}^2_{x,t} = E(x_t^{\,2}) - \mu_{x,t}^{\,2}$$

$$= \int_{x_1}^{x_2} x^2 g(x)\, dx - \mu_{x,t}^{\,2}$$

$$= \int_{x_1}^{x_2} \frac{x^2 f(x)\, dx}{F(x_2) - F(x_1)} - \mu_{x,t}^{\,2} \quad \text{Ans.}$$

The new CDF at $x_0$ is obtained by integrating the PDF between $x_1$ and $x_0$

$$G(x_0) = \int_{x_1}^{x_0} g(x) dx = \int_{x_1}^{x_0} \frac{f(x)\, dx}{F(x_2) - F(x_1)}$$

Ans.

---

4-21. From Prob. 4-18

$$a = \frac{1}{F(z_2) - F(z_1)} = \frac{1}{\phi(z_2) - \phi(z_1)}$$

$$= \frac{1}{0.9452 - 0.0808} = 1.156$$

$$g(k_t) = \frac{1.156}{0.666\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{k_t - 9.93}{0.666}\right)^2\right]$$

in the interval $9 \leq k \leq 11$ and zero otherwise. An alternative way of evaluating a avoiding interpolation is

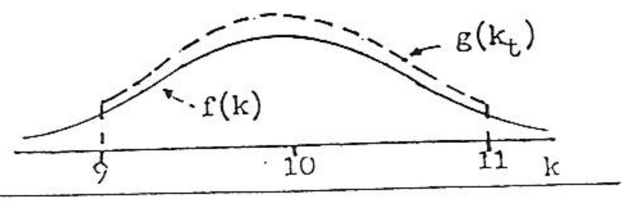
$$a = \frac{1}{1 - (\alpha + \beta)} = \frac{1}{1 - (0.081 + 0.055)}$$

$$= 1.157$$

| k | f(k) | $g(k_t)$ |
|---|---|---|
| 8.6 | 0.081 | 0 |
| 8.8 | 0.141 | 0 |
| 9.0 | 0.225 | 0.260 |
| 9.2 | 0.327 | 0.378 |
| 9.4 | 0.435 | 0.502 |
| 9.6 | 0.528 | 0.610 |
| 9.8 | 0.587 | 0.678 |
| 10.0 | 0.595 | 0.688 |
| 10.2 | 0.552 | 0.639 |
| 10.4 | 0.468 | 0.541 |
| 10.6 | 0.363 | 0.420 |
| 10.8 | 0.251 | 0.297 |
| 11.0 | 0.166 | 0.192 |
| 11.2 | 0.098 | 0 |
| 11.4 | 0.053 | 0 |

New mean and standard deviation can be found using Simpson's rule.

$\mu_{k_t} = 9.954$; $\hat{\sigma}_{k_t} = 0.5786$




4-22. Largest gap w exists when right tending vectors are largest and vice versa.

$$w_{max} = \sum(\bar{x}_i + t_i) - \sum(\bar{y}_j - t_j)$$
$$= \sum\bar{x}_i - \sum\bar{y}_j + \sum_{all} t_k = \bar{w} + \sum_{all} t_k$$

Smallest gap w exists when right-tending vectors smallest and vice versa.

$$w_{min} = \sum(\bar{x}_i - t_i) - \sum(\bar{y}_j + t_j)$$
$$= \sum\bar{x}_i - \sum\bar{y}_j - \sum_{all} t_k = \bar{w} - \sum_{all} t_k$$

The half-range bilateral tolerance $t_w$ on gap w is

$$t_w = \tfrac{1}{2}(w_{max} - w_{min})$$
$$= \tfrac{1}{2}[\bar{w} + \sum_{all} t_k - (\bar{w} - \sum_{all} t_k] = \sum_{all} t_k$$

4-23. $x_1 = d$, $y_2 = c$, $y_4 = b$, $y_6 = a$

(a) $\bar{w} = \sum\bar{x}_i - \sum\bar{y}_j = \bar{d} - \bar{c} - \bar{b} - \bar{a}$

$= 6.020 - 3.000 - 2.000 - 1.000$

$= 0.020$ in

$$t_w = \sum_{all} t_k = t_d + t_c + t_b + t_a$$
$$= 0.006 + 0.005 + 0.003 + 0.001$$
$$= 0.015 \text{ in}$$

$w = \bar{w} \pm t_w = 0.020 \pm 0.015$ in

(b) $\bar{w} = 0.020$ in as before. If the distributions are normal, natural tolerances given by

$$t_w = \sum_{all} t_k^2 = [t_d^2 + t_c^2 + t_b^2 + t_a^2]^{1/2}$$

$= [0.006^2 + 0.005^2 + 0.003^2 + 0.001^2]^{1/2}$

$= 0.008\ 426 = 0.009$ in

$w = \bar{w} \pm t_w = 0.020 \pm 0.009$ in

Dimensions produced by automatic machinery varying due to tool wear between setups exhibit uniform random distributions if mixed. Sums of uniform variates approach normality. With four elements in sum PDF is a bell-shaped, piecewise cubic and a normal distribution for gap w is a reasonable estimate.

With uniform random variates

$\hat{\sigma}_a = 0.001/\sqrt{3} = 0.000\ 577$ in

$\hat{\sigma}_b = 0.003/\sqrt{3} = 0.001\ 732$ in

$\hat{\sigma}_c = 0.005/\sqrt{3} = 0.002\ 887$ in

$\hat{\sigma}_d = 0.006/\sqrt{3} = 0.003\ 464$ in

From Table 4-4

$\hat{\sigma}_w = \sqrt{\sum \hat{\sigma}^2} = (0.000\ 577^2 + 0.001\ 732^2 + 0.002\ 887^2 + 0.003\ 464^2)^{\frac{1}{2}}$

$= 0.004\ 865$ in

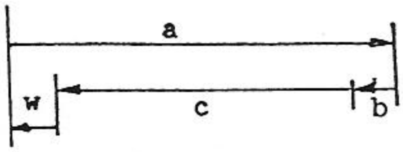
Since gap w is robustly normal in this case three standard deviations are used

$t_w = 3\sigma_w = 3(0.004\ 865) = 0.014\ 595$

$\doteq 0.015$ in

Note that this is larger than 0.009 in obtained when each element was normal. The standard deviations of each element under uniform hypothesis is about twice.

4-25. (a)




$w_{max} = 0.014$ in, $w_{min} = 0.004$ in

$\bar{w} = (w_{max} + w_{min})/2$

$= (0.014 + 0.004)/2 = 0.009$ in

$w = 0.009 \pm 0.005$ in

$\bar{w} = \sum \bar{x} - \sum \bar{y} = \bar{a} - \bar{b} - \bar{c}$

$0.009 = \bar{a} - 0.042 - 1.000$

$\bar{a} = 1.051$ in

$t_w = \sum_{all} t$

$0.005 = t_a + 0.002 + 0.002$

$t_a = 0.005 - 0.002 - 0.002 = 0.001$ in

Dimension a: $1.051 \pm 0.001$ in  Ans.

(b) Under normal hypothesis

$\bar{a} = 1.051$ as before

$t_w^2 = \sum_{all} t^2 = t_a^2 + 0.002^2 + 0.002^2$

$= 0.005^2$

$t_a = (0.005^2 - 0.002^2 - 0.002^2)^{1/2}$

$= 0.004\ 123 = 0.004$ in

Dimension a: $1.051 \pm 0.004$ in

---

4-26. The equation for gap $\underset{\sim}{w}$ is

$\underset{\sim}{w} = \underset{\sim}{a} - \underset{\sim}{b} - \underset{\sim}{c}$

$\bar{w} = \bar{a} - \bar{b} - \bar{c} = 1.051 - 1.000 - 0.042$

$= 0.009$ in

$\hat{\sigma}_w = (\hat{\sigma}_a^2 + \hat{\sigma}_b^2 + \hat{\sigma}_c^2)^{1/2}$

$= (0.0015^2 + 0.001^2 + 0.001^2)^{1/2}$

$\hat{\sigma}_w = 0.002\ 062$ in

$z = (w - \bar{w})/\hat{\sigma}_w$

$= (0.004 - 0.009)/0.002\ 062 = -2.4248$

Interpolating in Table A-10

| | | |
|---|---|---|
| 2.42 | 0.007 76 | |
| 2.4248 | $\alpha$ | $\alpha = 0.007\ 66$ |
| 2.43 | 0.007 55 | |

$F(z) = \Phi(-2.4248) = 0.007\ 66$

With centered normal processes the chance is less than one percent. Ans.

For chance of zero gap

$z = (0 - \bar{w})/\hat{\sigma}_w = (0 - 0.009)/0.002\ 062$

$= -4.3647$

Interpolating in Table A-10

| | |
|---|---|
| 4.3 | 0.000 008 54 |
| 4.3647 | $\alpha$ |
| 4.4 | 0.000 005 41 |

from which $\alpha = 0.000\ 006\ 515$

or about seven chances in 100 000. Ans.

---

4-28. Choose 15 mm as basic size, D, d. From Table 4-5 fit is designated as 15H7/h6. From Table A-11

$(t.g.)_{hole} = 0.018$ mm $= \Delta D$

$(t.g.)_{shaft} = 0.011$ mm $= \Delta d$

HOLE: Eqs. (4-16)

$D_{max} = D + \Delta D = 15 + 0.018 = 15.018$ mm

$D_{min} = D = 15.000$ mm

SHAFT: From Table A-12 fundamental deviation $\delta_F = 0$. From Eqs. (4-17)

$d_{max} = d + \delta_F = 15.000 + 0 = 15.000$ mm

$d_{min} = d + \delta_F - \Delta d$

$= 15.000 + 0 - 0.011 = 14.989$ mm

---

4-29. Choose 45 mm as basic size, D, d. From Table 4-5 fit is designated as 45H7/s6. From Table A-11 tolerance grades are

$\Delta D = 0.025$ mm, $\Delta d = 0.016$ mm

HOLE: Eqs. (4-16)

$D_{max} = D + \Delta D = 45.000 + 0.025$

$= 45.025$ mm

$D_{min} = D = 45.000$ mm

SHAFT: From Table A-12 fundamental deviation $\delta_F = +\ 0.043$ mm. Eqs. (4-18)

$d_{min} = d + \delta_F = 45.000 + 0.043$

$= 45.043$ mm

$d_{max} = d + \delta_F + \Delta d$

$= 45.000 + 0.043 + 0.016$

$= 45.059$ mm

---

4-31. Choose 1 in as basic size, D, d. From Table 4-5 fit is designated as (1.0 in)H8/f7. From Table A-13 tolerance grades are

$\Delta D = 0.0013$ in

$\Delta d = 0.008$ in

HOLE: Eqs. (4-16)

$D_{max} = D + \Delta D = 1.000 + 0.0013$

$= 1.0013$ in

$D_{min} = D = 1.0000$ in

SHAFT: From Table A-14 tolerance grade is $\delta_F = -0.0008$ in. Eqs. (4-17)

$d_{max} = d + \delta_F = 1.0000 + (-0.0008)$

$= 0.9992$ in

$d_{min} = d_{max} - \Delta d = 0.9992 - 0.0008$

$= 0.9984$ in.

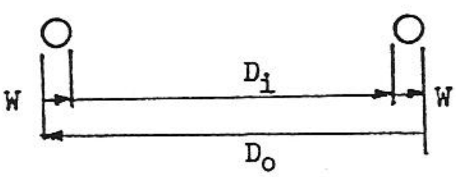
Expressed unilaterally

$$\text{bore} = 1.0000\,{}^{+0.0013}_{-0.0000}\ \text{in}$$

$$\text{journal} = 0.9992\,{}^{+0.0000}_{-0.0008}\ \text{in}$$ Ans.

---

4-32.




From figure the governing stochastic equation is

$\underset{\sim}{D_o} = \underset{\sim}{W} + \underset{\sim}{D_i} + \underset{\sim}{W}$

$\bar{D}_o = \bar{W} + \bar{D}_i + \bar{W}$

$= 0.139 + 3.734 + 0.139 = 4.012$ in

$t_{Do} = \sum_{all} t = 0.004 + 0.028 + 0.004$

$= 0.036$ in

$D_o = 4.012 \pm 0.036$ in

Alternatively

$\underset{\sim}{D_o} = \underset{\sim}{D_i} + 2\underset{\sim}{W}$

$\bar{D}_o = \bar{D}_i + 2\bar{W} = 3.734 + 2(0.139)$

$= 4.012$ in

$t_{Do} = \sum_{all} t = 0.028 + 2(0.004) = 0.036$ in

---

4-34. The stochastic equation is

$\underset{\sim}{D_o} = \underset{\sim}{D_i} + 2\underset{\sim}{W}$

$\bar{D}_o = \bar{D}_i + 2\bar{W} = 3.734 + 2(0.139)$

$= 4.012$ in

$t_{Do} = \sqrt{\sum_{all} t^2} = [t_{Do}{}^2 + (2t_W)^2]^{1/2}$

$= [0.028^2 + 2^2 0.004^2]^{1/2} = 0.029$ in

$D_o = 4.012 \pm 0.029$ in

$D_{omax} = 4.041$ in

$D_{omin} = 3.983$ in

---

4-35. The stochastic equation is

$\underset{\sim}{D_o} = \underset{\sim}{D_i} + 2\underset{\sim}{W}$

$\bar{D}_o = \bar{D}_i + 2\bar{W} = 208.92 + 2(5.33)$

$= 219.58$ mm

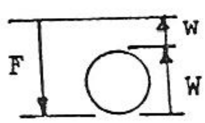
$t_{Do} = \sqrt{\sum_{all} t^2} = [1.30 + 2^2 0.13^2]^{1/2}$

$= 1.33$ mm

$D_o = 219.58 \pm 1.33$ mm

---

4-37. (a) Figure defines




$\underset{\sim}{w}$ as gap. $\underset{\sim}{w} = \underset{\sim}{F} - \underset{\sim}{W}$

$\bar{w} = \bar{F} - \bar{W}$

$= 4.32 - 5.33 = -1.01$ mm

$t_w = \sum_{all} t = t_F + t_W$

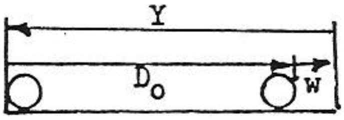
$= 0.13 + 0.13 = 0.26$ mm

$w_{max} = w + t_w = -1.01 + 0.26 = -0.75$ mm

$w_{min} = w - t_w = -1.01 - 0.26 = -1.27$ mm

All rings "squeezed" at least 0.75 mm.

(b)




$Y_{max} = D_o = 219.58$ mm

$Y_{min} = \max[0.99D_o, D_o - 1.52]$

$= \max[0.99(219.58), 219.58 - 1.52]$

$= 217.38$ mm

$Y = 218.48 \pm 1.10$ mm

From figure stochastic equation is

$\underset{\sim}{D_o} + \underset{\sim}{w} = \underset{\sim}{Y}$, or

$\underset{\sim}{w} = \underset{\sim}{Y} - \underset{\sim}{D_o}$

$\bar{w} = \bar{Y} - \bar{D}_o = 218.48 - 219.58 = -1.10$ mm

$t_w = \sum_{all} t = t_Y + t_{D_o} = 1.10 + 0.34$

$= 1.44$ mm

$w_{max} = \bar{w} + t_w = -1.10 + 1.44 = 0.34$ mm

$w_{min} = \bar{w} - t_w = -1.10 - 1.44 = -2.54$ mm

More rings are circumferentially compressed than free at assembly.

---

4-38. Prepare a table to find $\Sigma x$, $\Sigma x^2$, $\Sigma y$, $\Sigma xy$ conveniently.

| x<br>W | $(S_u)_W$ | y<br>$(S_u)_o/(S_u)_W$ | xy | $x^2$ |
|---|---|---|---|---|
| 0.0 | 43.0 | 1.000 | 0.000 00 | 0.00 |
| 0.1 | 48.0 | 0.896 | 0.089 60 | 0.01 |
| 0.2 | 53.0 | 0.811 | 0.162 20 | 0.04 |
| 0.3 | 60.0 | 0.717 | 0.215 10 | 0.09 |
| 0. | 70.9 | 0.606 | 0.242 40 | 0.16 |
| 1.0 | | 4.030 | 0.709 30 | 0.30 |

Writing Eqs. (4-8) and (4-9) differently

$$\hat{a} = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{n \Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{0.30(4.03) - 1.0(0.7093)}{5(0.30) - 1.0^2} = 0.9994$$

$$\hat{b} = \frac{n \Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{5(0.7093) - 1.0(4.03)}{5(0.30) - 1.0^2} = -0.9670$$

Therefore

$$\frac{(S_u)_o}{(S_u)_W} = a + bx = 0.9994 - 0.967W$$

which, for such a small sample, compares well to 1 - W.

---

4-39. Tabulate to conveniently find $\Sigma x$, $\Sigma y$, $\Sigma xy$, $\Sigma x^2$

| $\sigma$ | x<br>$\ln \sigma$ | y<br>$\varepsilon$ | xy | $x^2$ |
|---|---|---|---|---|
| 800 | 6.685 | 0.10 | 0.669 | 44.689 |
| 1000 | 6.908 | 0.15 | 1.036 | 47.720 |
| 1500 | 7.313 | 0.34 | 2.486 | 53.480 |
| 2000 | 7.601 | 0.47 | 3.572 | 57.775 |
| 2500 | 7.824 | 0.60 | 4.694 | 61.215 |
| 3000 | 8.006 | 0.70 | 5.604 | 64.096 |
| 3500 | 8.161 | 0.84 | 6.855 | 66.602 |
| | 52.498 | 3.20 | 24.916 | 395.577 |

Writing Eqs. (4-9) and (4-9) for the regression equation y = a + bx with independent expression for a and b

$$\hat{a} = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{n \Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{395.577(3.2) - 53.498(24.916)}{7(395.577) - 52.498^2}$$

$= -3.246$

$$= \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$= \frac{7(24.916) - 52.498(3.2)}{7(395.577) - 42.498^2} = 0.494$$

$= = -3.246 + 0.494 \ln \sigma$

$\varphi(\varepsilon) = \exp(-3.246)\exp(\ln \sigma^{0.494})$

$\quad = 0.0389\, \sigma^{0.494}$

r $\sigma = 715.1\exp(2.0243\varepsilon)$

-41. Stochastic equation is

$= \underset{\sim}{a} - \underset{\sim}{b} - \underset{\sim}{c} - \underset{\sim}{d}$

$= \bar{a} - \bar{b} - \bar{c} - \bar{d}$

$= 1.715 - 0.750 - 0.120 - 0.875$

$= -0.030$ in

$\underset{\sim}{\sigma}_w = [\underset{\sim}{\sigma}_a^{\,2} + \underset{\sim}{\sigma}_b^{\,2} + \underset{\sim}{\sigma}_c^{\,2} + \underset{\sim}{\sigma}_d^{\,2}]^{1/2}$

$= [(0.003/\sqrt{3})^2 + (0.001/\sqrt{3})^2$

$\quad + (0.005/\sqrt{3})^2 + (0.001/\sqrt{3})^2]^{1/2}$

$= 0.003\,46$ in

Compare these expextation with computer

simulation

| ials | $w_{max}$ | $w_{min}$ | $\bar{w}$ | $\underset{\sim}{\sigma}_w$ |
|---|---|---|---|---|
| 10 | −0.0236 | −0.0360 | −0.0295 | 0.003 82 |
| 100 | −0.0236 | −0.0376 | −0.0299 | 0.003 33 |
| 1000 | −0.0210 | −0.0391 | −0.0299 | 0.003 51 |
| 0 000 | −0.0208 | −0.0391 | −0.0299 | 0.003 45 |

rom Central Limit Theorem, addition of our non-normal distributions approaches ormality for sum. Expect a unimodal, ymmetric bell-shape distribution for hich the normal assumption is robust.

4.42. Tabulating for convenience

| x | n | nx | $nx^2$ |
|---|---|---|---|
| 93 | 19 | 1767 | 164 311 |
| 95 | 25 | 2375 | 225 625 |
| 97 | 38 | 3685 | 357 542 |
| 99 | 17 | 1683 | 166 617 |
| 101 | 12 | 1212 | 122 412 |
| 103 | 10 | 1030 | 106 090 |
| 105 | 5 | 525 | 55 125 |
| 107 | 4 | 428 | 45 796 |
| 109 | 4 | 436 | 47 524 |
| 111 | 2 | 222 | 24 624 |
| | 136 | 13364 | 1315 704 |

$\bar{x} = 13\,364/136$

$\quad = 98.26$ kpsi

$$s = \left(\frac{1\,315\,704 - 13\,364^2/136}{135}\right)^{1/2}$$

$\quad = 4.30$ kpsi

Under normal hypothesis

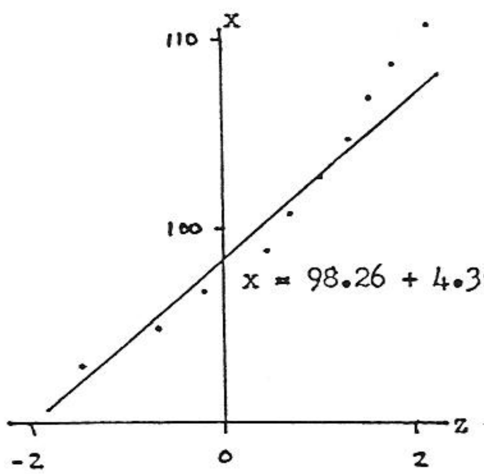
$z_{0.01} = (x_{0.01} - 98.26)/4.30$

$x_{0.01} = 98.26 + 4.30 z_{0.01}$

$\quad = 98.26 + 4.30(-2.326)$

$\quad = 88.26 \doteq 88.3$ kpsi

If distribution is normal then CDF of data plotted on xz-coordinates is rectified




Distribution not normal since data not random about normal fit.

4-43. From Prob. 4-42 $\hat{\mu}_x = 98.26$ kpsi and $\hat{\sigma}_x = 4.30$ kpsi. If $x \sim LN(\mu_x, \hat{\sigma}_x)$ then $y \sim N(\mu_y, \hat{\sigma}_y)$ and $y = \ln x$.

Eq. (4-21): $\mu_y = \ln \mu_x - C_x^2/2$

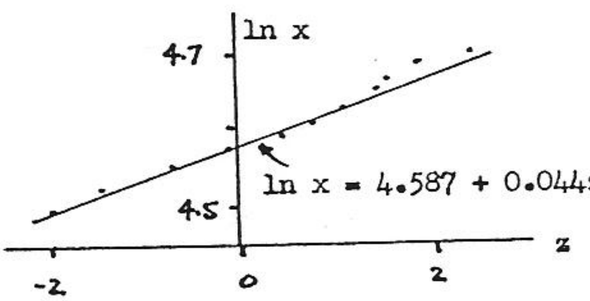
$\mu_y = \ln 98.26 - (4.30/98.26)^2/2 = 4.587$

Eq. (4-20): $\hat{\sigma}_y = C_x = 4.30/98.26 = 0.044$

| x | n | cum. n | $F_i^*$ | z | ln x |
|---|---|---|---|---|---|
| 93 | 19 | 19 | 0.070 | -1.47 | 4.533 |
| 95 | 25 | 44 | 0.232 | -0.73 | 4.554 |
| 97 | 38 | 82 | 0.463 | -0.09 | 4.575 |
| 99 | 17 | 99 | 0.665 | 0.43 | 4.595 |
| 101 | 12 | 111 | 0.772 | 0.75 | 4.615 |
| 103 | 10 | 121 | 0.853 | 1.05 | 4.635 |
| 105 | 5 | 126 | 0.908 | 1.33 | 4.654 |
| 107 | 4 | 130 | 0.941 | 1.56 | 4.673 |
| 109 | 4 | 134 | 0.971 | 1.90 | 4.691 |
| 111 | 2 | 136 | 0.993 | 2.45 | 4.710 |
| | 136 | | | | |

$^*F_i = \left(\sum_{j=1}^{i-1} n_j + n_i/2\right)/N$

Plot z vs. ln x




The CDF is closer to rectification under the lognormal hypothesis than under the normal hypothesis (see plot for Prob. 4-42). $z_{0.01} = (\ln x_{0.01} - 4.587)/0.044$

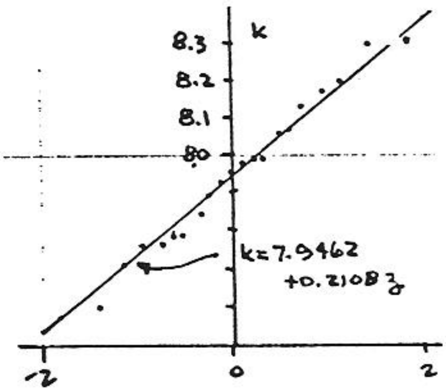
$\ln x_{0.01} = 4.587 + 0.044(-2.326) = 4.485$

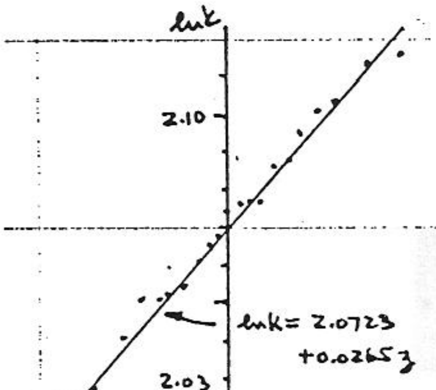
$x_{0.01} = \exp(4.485 = 88.6$ kpsi

compared to 88.3 kpsi under normal hyp.

4-44. Rank the data smallest to largest.

| i | k | ln k | $F_i$ | z |
|---|---|---|---|---|
| 1 | 7.58 | 2.0255 | 0.033 | -1.83 |
| 2 | 7.60 | 2.0281 | 0.079 | -1.41 |
| 3 | 7.71 | 2.0425 | 0.126 | -1.15 |
| 4 | 7.77 | 2.0503 | 0.173 | -0.94 |
| 5 | 7.77 | 2.0503 | 0.220 | -0.77 |
| 6 | 7.78 | 2.0516 | 0.266 | -0.62 |
| 7 | 7.79 | 2.0528 | 0.313 | -0.49 |
| 8 | 7.85 | 2.0605 | 0.360 | -0.36 |
| 9 | 7.90 | 2.0669 | 0.407 | -0.24 |
| 10 | 7.92 | 2.0694 | 0.453 | -0.12 |
| 11 | 7.96 | 2.0744 | 0.500 | 0 |
| 12 | 7.98 | 2.0769 | 0.547 | 0.12 |
| 13 | 7.99 | 2.0782 | 0.593 | 0.24 |
| 14 | 7.99 | 2.0782 | 0.640 | 0.36 |
| 15 | 8.07 | 2.0884 | 0.687 | 0.49 |
| 16 | 8.08 | 2.0894 | 0.734 | 0.62 |
| 17 | 8.14 | 2.0968 | 0.780 | 0.77 |
| 18 | 8.18 | 2.1017 | 0.827 | 0.94 |
| 19 | 8.20 | 2.1041 | 0.874 | 1.15 |
| 20 | 8.30 | 2.1163 | 0.921 | 1.41 |
| 21 | 8.31 | 2.1175 | 0.967 | 1.83 |



$\hat{\mu}_k = 7.9462$ lb/in, $\hat{\sigma}_k = 0.2108$ lb.in

$\mu_y = \ln 7.9462 - (0.2108/7.9462)^2/2$

$= 2.0723$ from Eq. (4-21)

$\hat{\sigma}_y = 0.2108/7.9462 = 0.0265$, Eq.(4-20).

Both fits are good, neither hypothesis can be rejected, so use either.

Normal:

$k_{0.05} = 7.9462 + 0.2108(-1.645)$

$= 7.60$ lb/in

Lognormal:

$k_{0.05} = \exp[2.0723 + 0.0265(-1.645)]$

$= 7.60$ lb/in

---

4-45. From Eq. (4-25) and a gamma function table for the Weibull

$\mu_n = n_0 + (\theta - n_0)\Gamma(1 + 1/b)$

$= 36.9 + (133.6 - 36.9)\Gamma(1 + 1/2.66)$

$= 36.9 + (133.6 - 36.9)0.888\ 86$

$= 122.6$ kcycles

From Eq. (4-26)

$\hat{\sigma}_n = (\theta - n_0)[\Gamma(1 + 2/b)\Gamma^2(1+1/b)]^{1/2}$

$= (133.6 - 36.9)[0.91\ 949 - 0.88\ 886^2]^{\frac{1}{2}}$

$= 34.8$ kcycles

From Eq. (4-24) the Weibull density is

$$f(n) = \frac{2.66}{96.7}\left(\frac{n - 36.9}{96.7}\right)^{1.66} \cdot \exp\left[-\left(\frac{n - 36.9}{96.7}\right)^2\right]$$

For the lognormal from Eq. (4-21)

$\mu_y = \ln \mu_n - C_n^2/2$

$= \ln 122.6 - (34.8/122.6)^2/2 = 4.769$

Use exact form of Eq. (4-20)

$\hat{\sigma}_y = \sqrt{\ln(1 + C_n^2)}$

$= \sqrt{\ln(1 + [34.8/122.6]^2)} = 0.278$

From Eq. (4-19) lognormal density is

$$f(n) = \frac{1}{0.278n\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln n - 4.769}{0.278}\right)^2\right]$$

Form a table of densities for plotting

| n | $f_W(n)$ | $f_{LN}(n)$ |
|---|---|---|
| 40 | 0.0001 | 0.0000 |
| 50 | 0.0010 | 0.0002 |
| 60 | 0.0025 | 0.0013 |
| 70 | 0.0044 | 0.0036 |
| 80 | 0.0064 | 0.0068 |
| 90 | 0.0083 | 0.0100 |
| 100 | 0.0098 | 0.0121 |
| 110 | 0.0108 | 0.0127 |
| 120 | 0.0110 | 0.0119 |
| 130 | 0.0105 | 0.0104 |
| 140 | 0.0093 | 0.0085 |
| 150 | 0.0078 | 0.0066 |
| 160 | 0.0061 | 0.0049 |
| 170 | 0.0045 | 0.0035 |
| 180 | 0.0031 | 0.0025 |
| 190 | 0.0020 | 0.0017 |
| 200 | 0.0012 | 0.0012 |
| 210 | 0.0007 | 0.0008 |
| 220 | 0.0003 | 0.0005 |

Weibull $B_{10}$ life from Eq. (4-23)

$n_{10} = n_0 + (\theta - n_0)[\ln(1/R)]^{1/b}$

$= 36.9 + (133.6 - 36.9)$

$\cdot [\ln(1/0.90)]^{1/2.66}$

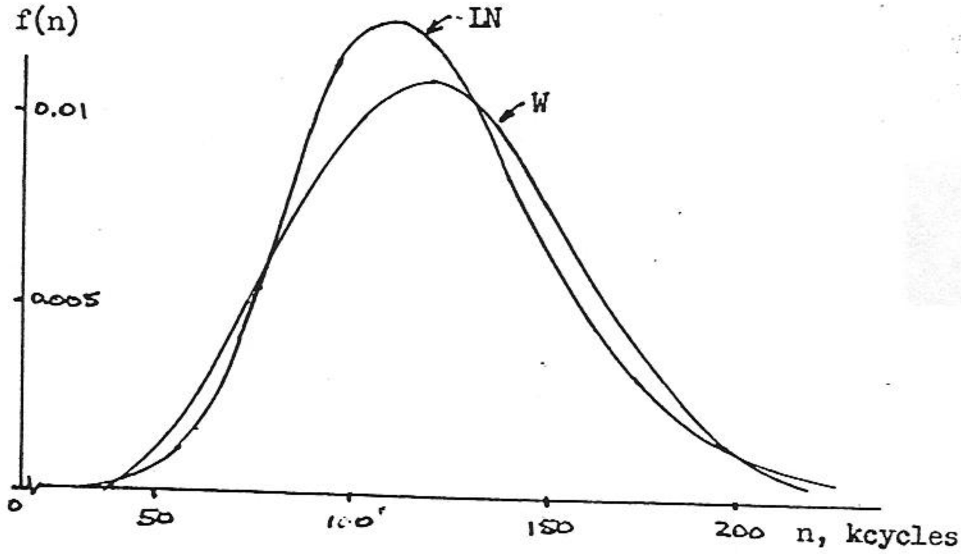
$= 78.4$ kcycles

Lognormal $B_{10}$ life

$\ln n_{10} = 4.769 + 0.278z$

$n = \exp[4.769 + 0.278(-1.28)] = 82.5$ kcy

4-45 con't.




4-47. Call variate $S_u$ the symbol $x$

Eq. (4-25) $\mu_x = x_0 + (\theta - x_0)\Gamma(1 + 1/b)$

$= 70.3 + (84.4 - 70.3)\Gamma(1 + 1/2.01)$

$= 70.3 + 14.1\Gamma(1.498)$

$= 70.3 + 14.1(0.886\ 17) = 82.8$ kpsi

Using Eq. (4-26)

$\hat{\sigma}_x = 14.1[\Gamma(1 + 2/2.01)$

$-\Gamma^2(1 + 1/2.01)]^{1/2}$

$=14.1[0.997\ 91 - 0.88\ 617^2]^{1/2}$

$= 6.62$ kpsi

$C_x = 6.62/82.8 = 0.080$

4-48. From Eqs. (4-25) and (4-26) form the quotient

$$\frac{\mu - x_0}{\hat{\sigma}} = \frac{\Gamma(1 + 1/b)}{[\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)]^{1/2}}$$

$$= \frac{\phi_1(b)}{\phi_2(b)} = \frac{49.0 - 33.8}{4.2} = 3.619$$

Find b by trials

| b | $\phi_1(b)$ | $\phi_2(b)$ | $\phi_1/\phi_2$ |
|---|---|---|---|
| 2.00 | 0.886 227 | 0.463 267 | 1.913 |
| 3.00 | 0.892 978 | 0.324 564 | 2.751 |
| 4.00 | 0.906 400 | 0.254 293 | 3.564 |
| 4.05 | 0.907 041 | 0.251 634 | 3.605 |
| 4.06 | 0.907 169 | 0.251 109 | 3.613 |
| 4.07 | 0.907 297 | 0.250 585 | 3.621 |

From Eq. (4-25)

$\theta = x_0 + (\mu - x_0)/\Gamma(1 + 1/b)$

$= 33.8 + (49.0 - 33.8)/\Gamma(1 + 1/4.07)$

$= 50.6$ kpsi

4-49. Call variate $S_y$ the symbol $x$

Eq. (4-25):

$\mu_x = 34.7 + (39.0 - 34.7)\Gamma(1 + 1/2.93)$

$= 34.7 + 4.3(0.892\ 08) = 38.5$ kpsi

Eq. (4-26):

$\hat{\sigma}_x = (39.0 - 34.7)[\Gamma(1 + 2/2.93)$

$-\Gamma^2(1 + 1/2.93)]^{1/2}$

$= 4.3[0.905\ 46 - 0.892\ 08^2]^{1/2}$

$= 1.42$ kpsi

$C_x = 1.42/38.5 = 0.037$

Long tail of distribution is to the right. Such skewness is termed right-skewed, or positively skewed.

---

4-53. When normally distributed data is plotted with x as ordinate and z as abscissa, the data string is rectified with the ordinate intercept as the mean and the slope equal to the standard deviation, $x = \mu + \hat{\sigma} z$, where z corresponds to the CDF. Seeking the mean value of the least bolt tension of a sample of six then

$$\bar{P}_1 = \mu + \hat{\sigma} z(F_1) = \mu + \hat{\sigma} z[1/(6+1)]$$
$$= \mu + \hat{\sigma} z(1/7) = 6667 + 600(-1.06758)$$
$$= 6026 \text{ lb}$$

The median value of the least bolt tension of a sample of six is

$$\tilde{P}_1 = \mu + \hat{\sigma} z(F_1)$$
$$= \mu + \hat{\sigma} z[(1 - 0.3)/(6 + 0.4)]$$
$$= \mu + \hat{\sigma} z(0.109375)$$
$$= 6667 + 600(-1.2311) = 5928 \text{ lb}$$ Ans.

Table 4-7 gives $\tilde{F}_1$ as 0.1091 which is more precise. Similarly, the median value of the largest bolt tension of a sample of six is

$$\tilde{P}_6 = 6667 + 600(1.2311) = 7406 \text{ lb}$$

Ans.

---

SIMULATION

Computer simulation is a powerful statistical tool. Fundamental to its use is the availability of random number generators (subprograms) for various distributions. Computing machinery vendors usually supply a uniform random number generator which provides a variate in the interval 0,1. Various seed integer schemes are utilized. Our generic routine will be RANDU(U) without seed integer arguments displayed. The steps will be shown in algebraic terms with translation into your computer language left to you.

(1) Uniform in the interval a,b

```
UNIF1(a,b,R)
Call RANDU(U)
R = a + (b - a)U
return; end
```

(2) Uniform with mean $\mu_x$, std. dev. $\hat{\sigma}_x$

UNIF2($\mu_x$, $\hat{\sigma}_x$,R)

Call RANDU(U)

$a = \mu_x - \sqrt{3}\,\hat{\sigma}_x$

$b = 2\sqrt{3}\,\hat{\sigma}_x$

R = a + (b - a)U

return; end

(3) Normal with mean $\mu_x$, std. dev. $\hat{\sigma}_x$

GAUSS($\mu_x$, $\hat{\sigma}_x$, G)

s = 0.

repeat twelve times

call RANDU(U)

s = s + U

$G = \mu_x + (s - 6.)\hat{\sigma}_x$

return; end

(4) Lognormal, mean $\mu_x$, std. dev., $\hat{\sigma}_x$

LGAUSS($\mu_x$, $\hat{\sigma}_x$,R)

$C_x = \hat{\sigma}_x/\mu_x$

$a = 1. + C_x^2$

$\mu_y = \ln(\mu_x/\sqrt{1. + C_x^2})$

$\hat{\sigma}_y = \sqrt{\ln(1. + C_x^2)}$

call GAUSS($\mu_y$, $\hat{\sigma}_y$,G)

R = exp(G)

return; end

(5) Weibull with parameters $x_0$, $\theta$, b

WEIBUL($x_0$,$\theta$,b,W)

call RANDU(U)

$W = x_0 + (\theta - x_0)[\ln(1/U)]^{1/b}$

return; end

The simplicity of WEIBUL reveals a useful relation for those distributions that have a closed-form relation for the cumulative density F, or the reliability R, and can be explicitly solved for the variate. The steps are

1. Solve the F or R equation for the variate x.
2. Substitute for F or R the uniform variate U in the interval 0,1.
3. x will be a random number from the desired distribution.

---

PROBLEM Timken tapered roller bearings have a two-parameter Weibull survival equation

$$R = \exp\left[-\left(\frac{x}{4.48}\right)^{3/2}\right]$$

where x is the life measure in multiples of a rating life of $90(10^6)$ revolutions.

(a) Estimate the mean, standard deviation, and coefficient of variation of life x.

(b) For a lognormal distribution with the same mean and standard deviation superpose the lognormal PDF(x) on the plot for part (a) and observe the differences.

---

PROBLEM When a part dimension is formed by a tool or a die which is subject to wear, each subsequent part is a little larger. If parts are small and the product stream is mixed, random selection is from a distribution that is called <u>uniform random</u>. If the lower bound of dimension x is a and the upper bound of x is b and all selections are equally likely, (a) what is the probability density, (b) mean, and (c) standard deviation?

---

## A NOTE ON PROGRAMMING

On pp. 590-601 of Raymond J. Roark and Warren C. Young,"Formulas for Stress and Strain," 5/e, McGraw-Hill, 1982, stress concentration factors are presented in equation form. This is an interesting idea for computer programming. However, many of the formulas are third order and require eight additional formulas to evaluate the constants. To avoid this it occurred to me (J.E.S.) that it might be possible to include an entire table of values, together with an interpolation routine, in a PC program. So I decided to test this idea using the gamma function. It worked! Here is the program in ZBASIC just as I first wrote it.

```
10 DIM A(101)
20 FOR I = 1 TO 101
30 READ A(I):NEXT I
40 DATA 1. ,1.01,1.02,1.03,1.04,1.05,1.06,1.07,1.08,1.09, 1.1,1.11,1.12,1.13
50 DATA 1.14,1.15,1.16,1.17,1.18,1.19,1.2,1.21,1.22,1.23,1.24,1.25,1.26,1.27
60 DATA 1.28,1.29,1.3,1.31,1.32,1.33,1.34,1.35,1.36,1.37,1.38,1.39,1.4,1.41
70 DATA 1.42,1.43,1.44,1.45,1.46,1.47,1.48,1.49,1.5,1.51,1.52,1.53,1.54,1.55
80 DATA 1.56,1.57,1.58,1.59,1.6,1.61,1.62,1.63,1.64,1.65,1.66,1.67,1.68,1.69
90 DATA 1.7,1.71,1.72,1.73,1.74,1.75,1.76,1.77,1.78,1.79,1.8,1.81,1.82,1.83
100 DATA 1.84,1.85,1.86,1.87,1.88,1.89,1.9,1.91,1.92,1.93,1.94,1.95,1.96,1.97
110 DATA 1.98,1.99,2.
120 DIM B(101)
130 FOR I = 1 TO 101
140 READ B(I):NEXT I
150 DATA 1,.99433,.98884,.98355,.97844,.9735,.96874,.96415,.95973,.95546,.95135
160 DATA .94739,.94359,.93993,.93642,.93304,.9298,.9267,.92373,.92088,.91817
170 DATA .91558,.91311,.91075,.90852,.9064,.9044,.9025,.90072,.89904,.89747
180 DATA .896,.89464,.89338,.89222,.89115,.89018,.88931,.88854,.88785,.88726
190 DATA .88676,.88636,.88604,.8858,.88565,.8856,.88563,.88575,.88595,.88623
200 DATA .88659,.88704,.88757,.88818,.88887,.88964,.89049,.89142,.89243,.89352
210 DATA .89468,.89592,.89724,.89864,.90012,.90167,.9033,.905,.90678,.90864
220 DATA .91057,.91258,.91466,.91683,.91906,.92137,.92376,.92623,.92877,.93138
230 DATA .93408,.93685,.93969,.94261,.94561,.94869,.95184,.95507,.95838,.96177
240 DATA .96523,.96878,.9724,.9761,.97988,.98374,.98768,.99171,.99581,1.
250 PRINT "THIS PROGRAM IS USED TO COMPUTE THE GAMMA FUNCTION"
260 PRINT "LET GAMMA = FUNCTION OF M"
270 PRINT "ENTER ANY VALUE OF M BETWEEN 1 AND 3"
280 INPUT "M = ";M
290 IF M<1 THEN 300 ELSE 310
300 PRINT "M OUT OF RANGE":STOP
310 IF M>3 THEN 300 ELSE 320
320 IF M>2 THEN 330 ELSE 350
330 L = M-1:GOSUB 420
340 GAMMA = L*G:GOTO 370
350 L = M: GOSUB 420
360 GAMMA = G
370 PRINT "GAMMA = ";GAMMA
380 PRINT "DO YOU WANT TO SOLVE ANOTHER PROBLEM? (Y OR N)"
390 INPUT D$
400 IF D$ = "Y" THEN 280 ELSE 410
410 STOP
420 I = 1
430 IF L = A(I) THEN 440 ELSE 450
440 G = B(I): RETURN
450 I = I+1
460 IF L<A(I) THEN 470 ELSE 430
470 G = ((B(I)-B(I-1))*(L-A(I-1))/(A(I)-A(I-1)))+B(I-1)
480 RETURN
```

# THE UNIFORM DISTRIBUTION

A Supplement to Mechanical Engineering Design, 5/e

The uniform random distribution is fundamental to statistics because all elements of the population are equally likely to be selected. In the interval a, b the PDF is

$$f(x) = \begin{cases} \dfrac{1}{b - a} & a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

The CDF is

$$F(x) = \begin{cases} 0 & x < a \\ \displaystyle\int_{-\infty}^{x} f(x)\,dx = \dfrac{x - a}{b - a} & a \leq x \leq b \\ 1 & x > b \end{cases}$$

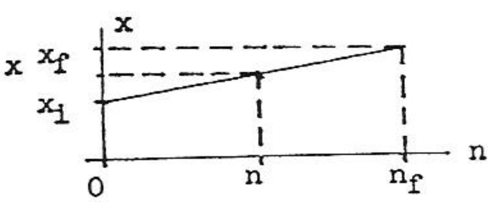
Mean: $\mu_x = \dfrac{a + b}{2}$

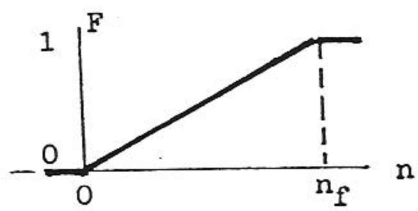
Std. deviation: $\hat{\sigma}_x = \dfrac{b - a}{2\sqrt{3}}$

Also: $f(x) = \dfrac{1}{2\sqrt{3}\hat{\sigma}_x}$

And: $F(x) = \dfrac{x - (\mu_x - \sqrt{3}\hat{\sigma}_x)}{2\sqrt{3}\hat{\sigma}_x} \quad a \leq x \leq b$

The uniform distribution arises in manufacturing when a part dimension is mass produced and the dimension gradually changes due to tool wear between setups. If n is the part number and $n_f$ is the last part prior to setup, then the dimension x is




predictable from the part number n by

$$x = x_i + (x_f - x_i)\frac{n}{n_f}$$

The CDF is given by the equation $F = n/n_f$ which may be substituted in the above relation giving

$$F(x) = \frac{x - x_i}{x_f - x_i} \qquad x_i \leq x \leq x_f$$

The form of the CDF is that of the uniform random distribution. A uniform distribution exists when the parts are thoroughly mixed, which happens less and less as parts become heavier and heavier, so be on your guard. A uniform distribution is a 2-parameter distribution. If the parameters used are the range numbers a, b the distribution is described as $x \sim U[a, b]$. Note the use of square brackets to denote range numbers. If the parameters are chosen are the mean and standard deviation, then the distribution is described as $x \sim U(\mu_x, \hat{\sigma}_x)$. Note here the use of parentheses.

# A NOTE ON REGRESSION ANALYSIS

A supplement to MECHANICAL ENGINEERING DESIGN, 5/e, by Shigley and Mischke

In describing regression it is useful to distinguish between coordinates of data points and coordinates of points on the regression line. We use $x_i$, $y_i$ as coordinates of data point i, and $\hat{y}_i$ as ordinate of the regression line. Equation (a) of Sec. 4-4 comes from analytical geometry and displays a traditional notation of m for slope and b for ordinate intercept. In regression the model is often a polynomial of the form

$$\hat{y}_i = a + bx_i + cx_i^2 + \ldots$$

and the linear form is displayed as $\hat{y}_i = a + bx_i$. An observation $y_i$ at some abscissa $x_i$ is a random variable composed of the prediction of the regression model $\hat{y}_i$ (deterministic) and a random variable

$$\mathbf{y}_i = \hat{y}_i + bx_i + \boldsymbol{\xi} \tag{4-33}$$

where the letter $\xi$ is the 14th character of the Greek alphabet, called xi. In ordinary regression it is normally distributed with a mean of 0 and a standard deviation of $\hat{\sigma}_{y \cdot x}$. The wavy underlines indicate a bold-face character in print, either a random variable or a vector. In a linear model

$$\mathbf{y}_i = a + bx_i + N(0, \hat{\sigma}_{y \cdot x}) \tag{4-34}$$

Regression procedures are for the purpose of estimating a and b as well as $\hat{\sigma}_{y \cdot x}$ from a sample of the population. As the sample size approaches infinity, $\hat{a}$ approaches its true value a, $\hat{b}$ approaches b, and $s_{y \cdot x}$ approaches $\hat{\sigma}_{y \cdot x}$. The nature of $\hat{a} + \hat{b}x$ approaches deterministic, but $N(0, \hat{\sigma}_{y \cdot x})$ is always there and $\mathbf{y}_i$ remains stochastic, a subtlety lost by many engineers. OUr task will be to estimate a, b, and $\hat{\sigma}_{y \cdot x}$ from the data and to provide confidence bounds on $\hat{a}$, $\hat{b}$ and $\mathbf{y}_i$ if needed.

The least squares method of obtaining estimates of a and b consists of minimizing the squares of the deviations of the regression ordinates and the data ordinates, that is

$$e = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} (y_i - a - bx_i{}^2 \tag{4-35}$$

is differentiated partially with $\partial e/\partial a = 0$, and $\partial e/\partial b = 0$. Simultaneous solution of the resulting equations yields

$$\hat{a} = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{n\Sigma x^2 - (\Sigma x)^2} \qquad \hat{b} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} \tag{4-36}$$

which are the independent versions of Eqs. (4-8) and (4-9). The estimate of $\hat{\sigma}_{y \cdot x}$ is obtained from

$$s_{y \cdot x} = \left( \frac{\Sigma y^2 + n\hat{a}^2 + \hat{b}^2\Sigma x^2 - 2\hat{a}\Sigma y - 2\hat{b}\Sigma xy + 2\hat{a}\hat{b}\Sigma x}{n - 2} \right)^{\frac{1}{2}} \quad (4\text{-}37)$$

Take care in these computations, providing sufficient computational precision, not using rounded values. This applies also to a convenient form obtained by expanding Eq. (4-37)

$$s_{y \cdot x} = \left( \frac{\Sigma y^2 + n\hat{a}^2 + \hat{b}^2\Sigma x^2 - 2\hat{a}\Sigma y - 2\hat{b}\Sigma xy + 2\hat{a}\hat{b}\Sigma x}{n - 2} \right)^{\frac{1}{2}} \quad (4\text{-}38)$$

which uses summations developed in estimating a and b.

Equations can be developed (See J. B. Kennedy and A. M. Neville, Basic Statistical Methods for Engineers and Scientists, Harper & Row, 3/e, 1986, Chap. 17, for example) for the standard deviation in the estimate of a, $s_a$, the standard deviation in the estimate of b, $s_b$, and the standard deviation in estimate of a future observation (a prediction by the regression equation) $y_i$. These are

$$s_a = s_{y \cdot x} \left( \frac{1}{n} + \frac{\bar{x}^2}{\Sigma (x_i - \bar{x})^2} \right)^{\frac{1}{2}} \quad (4\text{-}39)$$

$$s_b = \frac{s_{y \cdot x}}{[\Sigma (x_i - \bar{x})^2]^{\frac{1}{2}}} \quad (4\text{-}40)$$

$$s_{yi} = s_{y \cdot x} \left[ 1 + \frac{1}{n} + \frac{(x_i - \bar{x})^2}{\Sigma (x_i - \bar{x})^2} \right]^{\frac{1}{2}} \quad (4\text{-}41)$$

where $\bar{x}$ is the abscissa centroid of the data points. The distribution of a, b, and $y_i$ are normal but the scarcity of data, often less than 30 points, requires recognition that in small samples these quantities are t-distributed.

The t distribution is normal-like, that is symmetrical about a mean of zero and with a larger standard deviation than the normal. As the number of elements in the sample increase, the t distribution approaches the unit normal N(0, 1).

TABLE The t statistic for two-sided confidence level of 0.95

| ν | t | ν | t | ν | t | ν | t |
|---|---|---|---|---|---|---|---|
| 1 | 12.706 | 11 | 2.201 | 21 | 2.080 | 40 | 2.021 |
| 2 | 4.303 | 12 | 2.179 | 22 | 2.074 | 60 | 2.000 |
| 3 | 3.182 | 13 | 2.160 | 23 | 2.069 | 120 | 1.980 |
| 4 | 2.776 | 14 | 2.145 | 24 | 2.064 | ∞ | 1.960 |
| 5 | 2.571 | 15 | 2.131 | 25 | 2.060 | | |
| 6 | 2.447 | 16 | 2.120 | 26 | 2.056 | | |
| 7 | 2.365 | 17 | 2.110 | 27 | 2.052 | | |
| 8 | 2.306 | 18 | 2.101 | 28 | 2.048 | | |
| 9 | 2.262 | 19 | 2.093 | 29 | 2.045 | | |
| 10 | 2.228 | 20 | 2.086 | 30 | 2.042 | | |

The degrees of freedom $\nu$ is the number of data points less the number of parameters in the regression model, two for the $y = a + bx$ model.

The 0.95 two-sided confidence interval on a is $\hat{a} \pm ts_a$, on b is $\hat{b} \pm ts_b$, and on $y_i$ is $y_i \pm ts_{yi}$. Note that in the case of the parameter a as the sample size grows without bound $s_a$ approaches zero and a has been found exactly. The same is true of parameter b. In the case of the prediciton $y_i$, $s_{y \cdot x}$ approaches $\hat{\sigma}_{y \cdot x}$ and the radical in Eq. (4-41) approaches unity.

---

EXAMPLE 4-11. Solve Prob. 5-4 and find

(a) The regression constants $\hat{a}$ and $\hat{b}$.

(b) The standard deviations $s_a$, $s_b$, and $s_{y \cdot x}$.

(c) The 0.95 confidence interval on a and what this says about the tangent modulus at zero stress level.

(d) The 0.95 confidence interval on the tangent modulus at the 20 kpsi stress level.

Solution (a) Interpreting the stress $\sigma$ as x and the quotient $\varepsilon/\sigma$ as y construct the table

| x | y |
|---|---|
| $\sigma$ | $\varepsilon/\sigma$ |
| 5 000 | $4.0(10^{-8})$ |
| 10 000 | 4.40 |
| 16 000 | 5.00 |
| 19 000 | 5.26 |
| 26 000 | 5.77 |
| 32 000 | 6.25 |
| 40 000 | 7.00 |
| 46 000 | 7.39 |
| 49 000 | 8.16 |
| 54 000 | 10.0 |

Construct the necessary sums:

$\Sigma x = 297\ 000 \qquad \Sigma y = 0.632\ 300(10^{-6})$

$\bar{x} = 29\ 700 \qquad \bar{y} = 0.632\ 300(10^{-7})$

$\Sigma x^2 = 0.114\ 750(10^{-11})$

$\Sigma y^2 = 0.430\ 581(10^{-13})$

$\Sigma xy = 0.215\ 374(10^{-1})$

$\Sigma(x_i - \bar{x})^2 = 0.265\ 410(10^{10})$

$\Sigma(y_i - \bar{y})^2 = 0.307\ 774(10^{-14})$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 0.275\ 809(10^{-2})$

$\Sigma(y_i - \hat{y}_i)^2 = 0.211\ 587(10^{-15})$

From Eqs. (4-36)

$$\hat{a} = \frac{0.114\ 750(10^{-11})[0.6323(10^{-6})] - 297\ 000(0.021\ 537\ 4)}{10[0.114\ 750(10^{-11})] - (297\ 000)^2} = 0.323\ 663(10^{-7})$$

$$\hat{b} = \frac{10(0.021\ 537) - 297\ 000[0.6323(10^{-6})]}{10[0.114\ 750(10^{-11})] - (297\ 000)^2} = 0.103\ 918(10^{-11})$$

The sums and answers depend on computational precision.

(b) From Eq. (4-37)

$$s_{y \cdot x} = \left[\frac{0.211\ 587(10^{-15})}{10 - 2}\right]^{\frac{1}{2}} = 0.514\ 280(10^{-8})$$

From Eq. (4-39)

$$s_a = 0.514\ 280(10^{-8})\left[\frac{1}{10} + \frac{(29\ 700)^2}{0.265\ 410(10^{10})}\right]^{\frac{1}{2}} = 0.338\ 156(10^{8})$$

From Eq. (4-40)

$$s_b = \frac{0.514\ 280(10^{-8})}{[0.265\ 410(10^{10})]^{\frac{1}{2}}} = 0.998\ 253(10^{-13})$$

(c) The regression line is $y = \hat{a} + \hat{b}x = \varepsilon/\sigma = \hat{a} + \hat{b}\sigma$ or

$$\varepsilon = \hat{a}\sigma + \hat{b}\sigma^2$$

The modulus of elasticity is $E = \sigma/\varepsilon$ and at $\sigma = 0$

$$E\Big|_{\sigma=0} = \frac{1}{\hat{a} + \hat{b}\sigma}\Big|_{\sigma=0} = \frac{1}{\hat{a}} = \frac{1}{0.323\ 663(10^{-7})} = 30.9 \text{ Mpsi}$$

which is an estimate of the mean value. The 0.95 confidence half-interval on a is

$$ts_a = 2.306(0.338\ 156)(10^{-8}) = 0.779\ 788(10^{-8})$$

The upper bound on a at the 0.95 confidence level is

$$a^+ = 0.401\ 642(10^{-7})$$

and the lower bound is

$$a^- = 0.245\ 685(10^{-7})$$

So the upper and lower bounds on E are, respectively

$$E^+ = 1/a^- = 40.7 \text{ Mpsi} \qquad E^- = 1/a^+ = 24.9 \text{ Mpsi}$$

This means that the tangent modulus of elasticity at zero stress level has a 95 percent chance of lying between 24.9 and 40.7 Mpsi. This is very different than saying the modulus is 30.9 Mpsi. The role of statistics is to tell the investigator what one can dare believe and the chances of being right or wrong. This is very frustrating to those who wish to believe more. The frustration is good because it leads us to conclude that there is far too little experimental data to say anything meaningful about the tangent modulus! The remedy is to take significantly more data.

For the tangent modulus at the 20 000 psi stress level it is necessary to make a prediciton from the regression equaiton. Using Eq. (4-41) we get

$$s_{yi} = 0.514\ 280(10^{-8})\left[1 + \frac{1}{10} + \frac{(x_i - 29\ 700)^2}{0.265\ 410(10^{10})}\right]^{\frac{1}{2}}$$

Here, at $x_i = 20\ 000$, we get $s_{yi} = 0.548\ 00(10^{-8})$

The half-range of the tolerance interval at the 0.95 confidence interval is

$$ts_{yi} = 2.306(2.306[0.548(10^{-8})] = 0.126\ 370(10^{-7})$$

Note that $t = 2.306$ corresponding to a $\nu = 10 - 2 = 8$ degrees of freedom.

The expected value of $y_i$ from the regression equaiton is

$y_i = \hat{a} + \hat{b}x = 0.323\ 663(10^{-7}) + 0.103\ 918(10^{-11})(20\ 000) = 0.531\ 500(10^{-7})$

The upper and lower limits are

$y_i^+ = y_i + ts_{yi} = 0.5315(10^{-7}) + 0.126\ 370(10^{-7}) = 0.657\ 869(10^{-7})$

$y_i^- = y_i - ts_{yi} = 0.5315(10^{-7}) - 0.126\ 370(10^{-7}) = 0.405\ 130(10^{-7})$

So the tangent modulus of elasticity E at a stress of 20 000 psi and the upper and lower 0.95 confidence bounds, respectively, are

$E = 1/y_i = 1/0.5315(10^{-7}) = 18.8$ Mpsi

$E^+ = 1/y_i^- = 1/0.405\ 130(10^{-7}) = 24.7$ Mpsi

$E^- = 1/y_i^+ = 1/0.657\ 869(10^{-7}) = 15.2$ Mpsi

Again, we are 95 percent confident that the value of E at 20 000 psi lies between 15.2 and 24.7 Mpsi with the most likely value of 18.8 Mpsi.

It is interesting to learn what the results would have been if the number of data points n were infinite. This presumes that the values of a and b are exact, which is not so. The procedure is the same as before except t is replaced by z with z = 1.96 for one bound and −1.96 for the other (see Table A-10). For $\sigma = 0$ the results are

$E^- = 23.6$ Mpsi  $\bar{E} = 30.9$ Mpsi  $E^+ = 44.9$ Mpsi

and, for $\sigma = 20\ 000$ psi,

$E^- = 15.8$ Mpsi  $\bar{E} = 18.8$ Mpsi  $E^+ = 23.2$ Mpsi

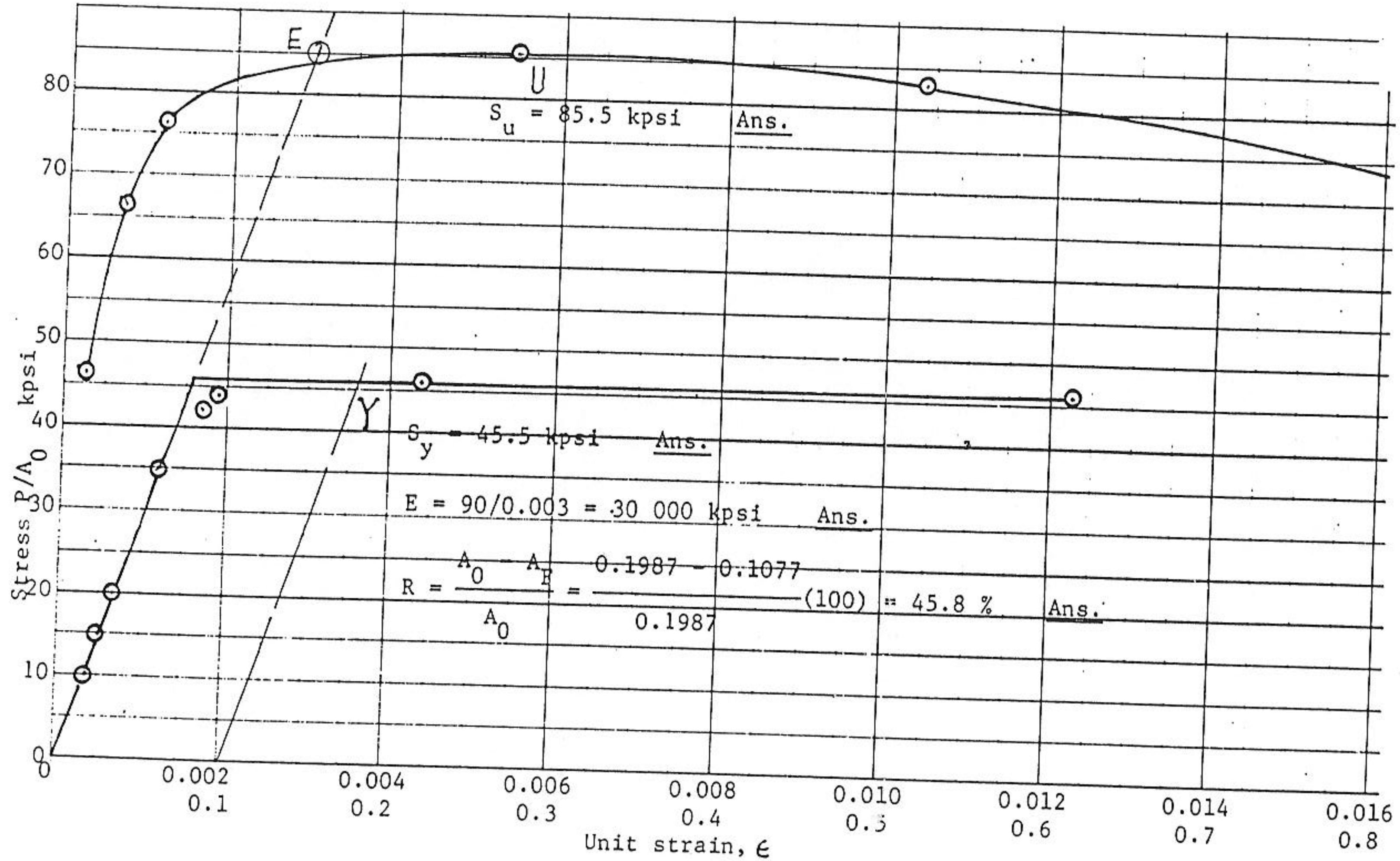
Comparing these results with those for an infinite number of data points should provide more insight into the problem.

////

There is considerable merit in writing a computer program for your programmable calculator, PC or timeshared mainframe computer to do the number crunching associated with a linear regression. If you have a graphics capability plot the data as soon as you enter it to study before proceeding. You want the data string to be rectified. The steps are then:

1. Initialize all counters for forming the necessary running sums.
2. Enter the number of data pairs n.
3. Enter $x_i$ and $y_i$ for i = 1 to n.
4. Form the sums $\Sigma x$, $\Sigma y$, $\Sigma x^2$, $\Sigma y^2$, $\Sigma xy$.
5. Find $\bar{x}$ and $\bar{y}$ from $\Sigma x/n$ and $\Sigma y/n$.
6. Form the sums $\Sigma(x - \bar{x})^2$, $\Sigma(y - \bar{y})^2$, and $\Sigma(x - \bar{x})(y - \bar{y})$.
7. Estimate $\hat{a}$ and $\hat{b}$ from Eqs. (4-36).
8. Form the sum $\Sigma(y_i - \hat{y}_i)^2$.
9. Find $s_{y\cdot x}$ from Eq. (4-37).
10. Find $s_a$ from Eq. (4-39).
11. Find $s_b$ from Eq. (4-40).
12. From a subprogram which contains the t table recover t from $\nu = n - 2$.
13. Report $s_{y\cdot x}$, $\hat{a}$, $\hat{a} + ts_a$, and $\hat{a} - ts_a$.
14. Report $\hat{b}$, $\hat{b} + ts_b$, and $\hat{b} - ts_b$.
15. Report correlation coefficient r from Eq. (4-10) forming $s_x$ from $\Sigma(x-\bar{x})^2$.
16. Allow Eq. (4-41) to be evaluated with user supplying $x_i$. Compute $y_i = a + bx_i$, $s_{yi}$ from Eq. (4-41) and report $y_i$, $\bar{y}_i + ts_{yi}$, and $y_i - ts_{yi}$.

5-1



E
U
$S_u$ = 85.5 kpsi Ans.
Y
$S_y$ = 45.5 kpsi Ans.
E = 90/0.003 = 30 000 kpsi Ans.
$R = \frac{A_0 - A_F}{A_0} = \frac{0.1987 - 0.1077}{0.1987}(100) = 45.8\ \%$ Ans.
Stress $P/A_0$ kpsi
0
10
20
30
40
50
60
70
80
0
0.002
0.1
0.004
0.2
0.006
0.3
0.008
0.4
0.010
0.5
0.012
0.6
0.014
0.7
0.016
0.8
Unit strain, ε

5-1 and 5-2

Meaning of calculator printouts are: P = P, DELT = $\delta$, A = $A_i$, EPSI = $\epsilon$, E = $\mathcal{E}$, P/A = $P/A_0$, SIGM = $\sigma$, LOGS = Log $\sigma$, LOGE = Log $\mathcal{E}$

ELASTIC PORTION:

| | |
|---|---|
| 1000. | P |
| 0.0004 | DELT |
| 0.0002 | EPSI |
| 0.00019998 | E |
| .1986730693 | A |
| 5032.388353 | P/A |
| 5033.39483 | SIGM |
| 3.701860999 | LOGS |
| -3.69901343 | LOGE |

| | |
|---|---|
| 2000. | P |
| 0.0006 | DELT |
| 0.0003 | EPSI |
| 0.000299955 | E |
| 0.198653208 | A |
| 10064.77671 | P/A |
| 10067.79614 | SIGM |
| 4.002934413 | LOGS |
| -3.522943881 | LOGE |

| | |
|---|---|
| 3000. | P |
| 0.001 | DELT |
| 0.0005 | EPSI |
| 0.000499875 | E |
| .1986134972 | A |
| 15097.16506 | P/A |
| 15104.71364 | SIGM |
| 4.179112496 | LOGS |
| -3.301138547 | LOGE |

| | |
|---|---|
| 4000. | P |
| 0.0013 | DELT |
| 0.00065 | EPSI |
| .0006497888 | E |
| .1985837245 | A |
| 20129.55341 | P/A |
| 20142.63762 | SIGM |
| 4.30411634 | LOGS |
| -3.187227751 | LOGE |

| | |
|---|---|
| 7000. | P |
| 0.0023 | DELT |
| 0.00115 | EPSI |
| .0011493393 | E |
| .1984845467 | A |
| 35226.71847 | P/A |
| 35267.2292 | SIGM |
| 4.54737134 | LOGS |
| -2.939551759 | LOGE |

| | |
|---|---|
| 8400. | P |
| 0.0028 | DELT |
| 0.0014 | EPSI |
| .0013990209 | E |
| .1984349949 | A |
| 42272.06216 | P/A |
| 42331.24305 | SIGM |
| 4.626661022 | LOGS |
| -2.854175793 | LOGE |

| | |
|---|---|
| 8800. | P |
| 0.0036 | DELT |
| 0.0018 | EPSI |
| .0017983819 | E |
| .1983557635 | A |
| 44285.0175 | P/A |
| 44364.73054 | SIGM |
| 4.647037848 | LOGS |
| -2.745118067 | LOGE |

| | |
|---|---|
| 9200. | P |
| 0.0089 | DELT |
| 0.00445 | EPSI |
| 0.00440128 | E |
| .1978324495 | A |
| 46297.97285 | P/A |
| 46503.99882 | SIGM |
| 4.667490299 | LOGS |
| -2.352604507 | LOGE |

5-1 and 5-2 (Continued)

PLASTIC PORTION:

| | |
|---|---|
| 8800. | P |
| 0.1984 | A |
| .0015753911 | E |
| .0015766327 | EPSI |
| 44285.0175 | P/A |
| 44354.83871 | SIGM |
| 4.646941004 | LOGS |
| -2.802611614 | LOGE |

| | |
|---|---|
| 9200. | P |
| 0.1978 | A |
| .0046041668 | E |
| .0046147822 | EPSI |
| 46297.97285 | P/A |
| 46511.62791 | SIGM |
| 4.66756154 | LOGS |
| -2.336848955 | LOGE |

| | |
|---|---|
| 9100. | P |
| 0.1963 | A |
| .0122164847 | E |
| .0122914107 | EPSI |
| 45794.73401 | P/A |
| 46357.61589 | SIGM |
| 4.666121093 | LOGS |
| -1.913053746 | LOGE |

| | |
|---|---|
| 13200. | P |
| 0.1924 | A |
| .0322840477 | E |
| .0328108312 | EPSI |
| 66427.52626 | P/A |
| 68607.06861 | SIGM |
| 4.836368864 | LOGS |
| -1.491012019 | LOGE |

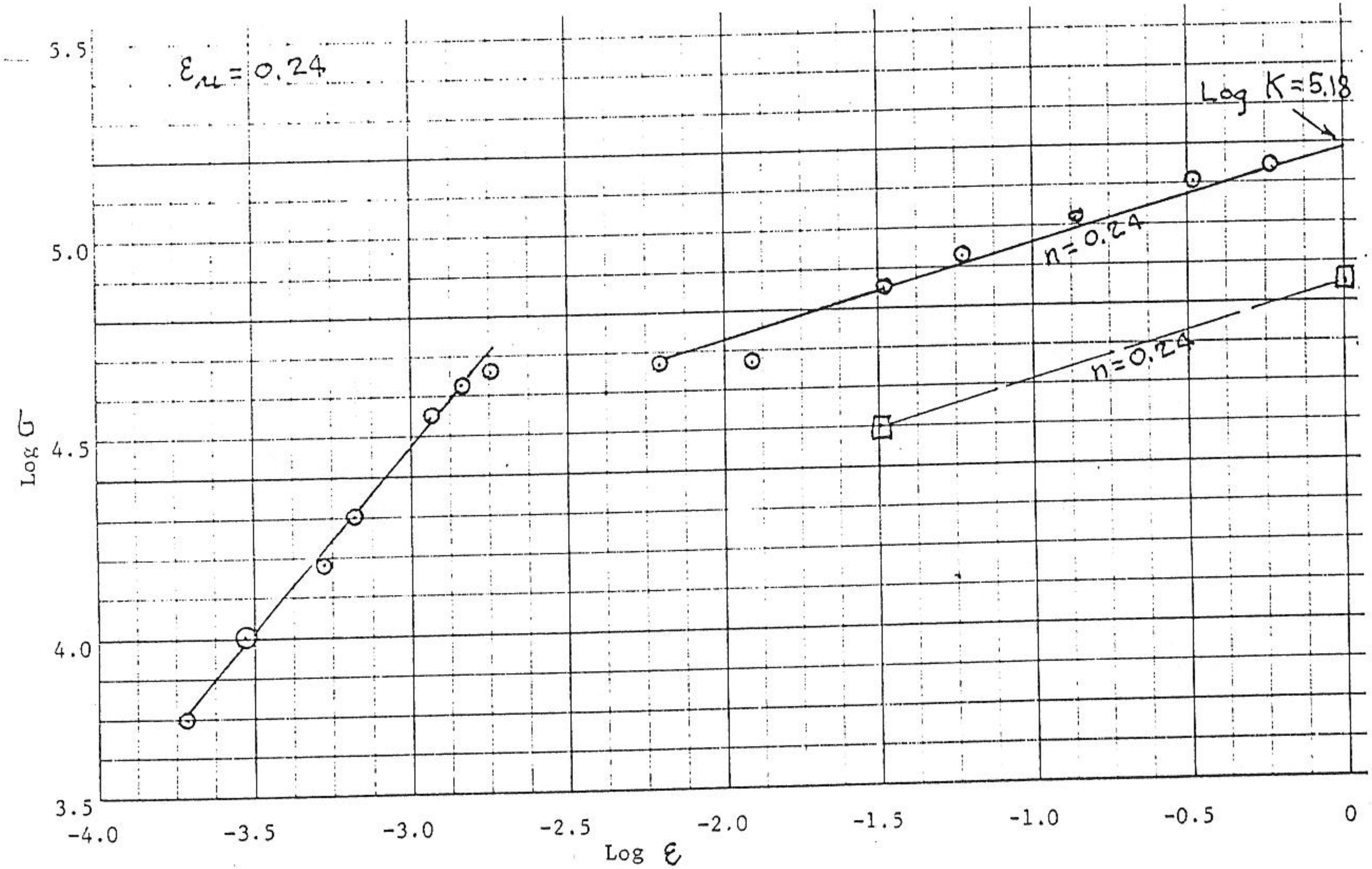
| | |
|---|---|
| 15200. | P |
| 0.1875 | A |
| .0580817405 | E |
| .0598016209 | EPSI |
| 76492.30296 | P/A |
| 81066.66667 | SIGM |
| 4.908842316 | LOGS |
| -1.235960378 | LOGE |

| | |
|---|---|
| 17000. | P |
| 0.1563 | A |
| .2400833485 | E |
| .2713551115 | EPSI |
| 85550.602 | P/A |
| 108765.1951 | SIGM |
| 5.036489943 | LOGS |
| -.6196379603 | LOGE |

| | |
|---|---|
| 16400. | P |
| 0.1307 | A |
| .4189559653 | E |
| .5203734042 | EPSI |
| 82531.16898 | P/A |
| 125478.1943 | SIGM |
| 5.09856826 | LOGS |
| -.3778316215 | LOGE |

| | |
|---|---|
| 14800. | P |
| 0.1077 | A |
| .6125110018 | E |
| .8450585322 | EPSI |
| 74479.34762 | P/A |
| 137418.7558 | SIGM |
| 5.138046012 | LOGS |
| -.2128861062 | LOGE |

$n = \varepsilon_u = 0.24$. Corresponding to some point removed (see p )
say $\log \sigma = 4.5$, $\log \varepsilon = -1.5$, we have, since $\sigma = K\varepsilon^n$,
$\log \sigma = \log K + n \log \varepsilon$ ; so
$\log K = \log \sigma - n \log \varepsilon = 4.5 - 0.24(-1.5) = 4.86$ (See p. )


$\varepsilon_u = 0.24$
Log K=5.18
n=0.24
n=0.24
Log σ
Log ε
3.5
4.0
4.5
5.0
5.5
-4.0
-3.5
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0

5-1 and 5-2 (Concluded)

These two points define the slope, so draw a line having the same slope through points to get log K = 5.18 as shown. This gives K = 151 000 psi, and so

$\sigma = 151\ 000\ \varepsilon^{0.24}$ Ans.

After 20% cold work, the new area is

$$A_i' = A_0(1 - W) = 0.1987(1 - 0.20) = 0.1590\ in^2$$

Therefore the true strain caused by the coldworking is

$$\varepsilon_i = \ln \frac{A_0}{A_i} = \ln \frac{0.1987}{0.1590} = 0.223$$

From Eq. (4-13) we have

$$S_y' = K\varepsilon_i^{\ n} = 151(10)^3(0.223)^{0.24} = 105.3(10)^3 \text{ psi}$$ Ans.

The new value of $S_u$ is, from Eq. (4-14)

$$S_u' = \frac{S_u}{1 - W} = \frac{85.5}{1 - 0.20} = 106.9 \text{ kpsi}$$ Ans.

---

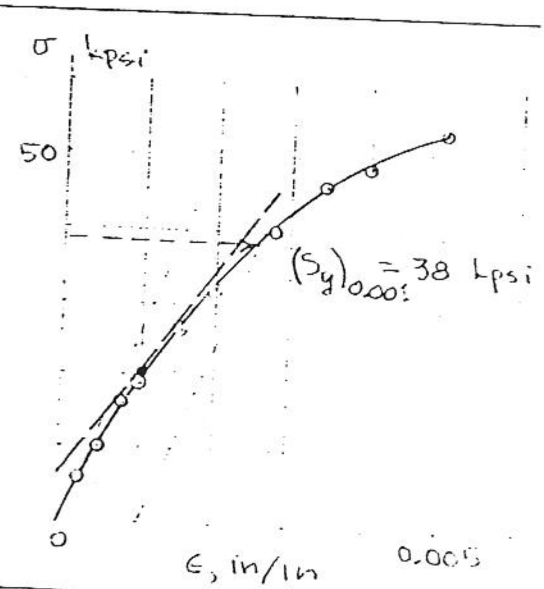
5-3 The tangent modulus at $\sigma = 0$ is

$$E = \frac{\sigma}{\varepsilon} \cong \frac{5000 - 0}{0.00020 - 0} = 25(10^6) \text{ psi}$$

The tangent modulus at $\sigma = 20$ kpsi is

$$E = \frac{\text{rise}}{\text{run}} = \frac{40\ 000 - 5000}{0.0025 - 0} = 14(10^6) \text{ psi}$$

0.1% $S_y$: construct a line parallel to the tangent at origin and containing the intercept of 0.001 permanent strain. It intersects the stress-strain diagram at 38 kpsi.




---

5-4

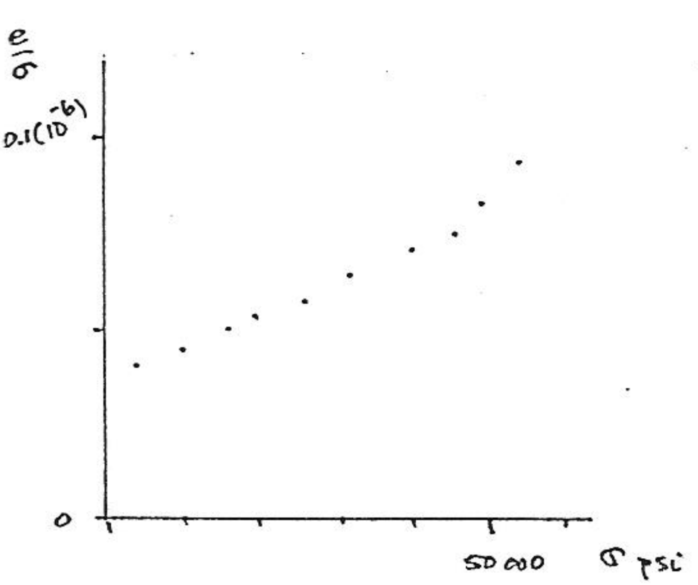
| σ, kpsi | 5 | 10 | 16 | 19 | 26 | 32 | 40 | 46 | 49 | 54 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\varepsilon/\sigma \times 10^6$ | 0.040 | 0.044 | 0.050 | 0.053 | 0.058 | 0.063 | 0.070 | 0.074 | 0.082 | 0.093 |

From graph on next page, the first eight points look linear, and are used.

$a = 0.362(10^{-7})$, $0.373(10^{-7})$, $0.352(10^{-7})$; $s_{y \cdot x} = 0.583(10^{-9})$, $s_{y \cdot x} = 0.206(10^{-9})$;

$s_a = 0.424(10^{-9})$; $s_b = 0.153(10^{-13})$

5-4 (Continued)




The regression line is

$$\frac{\varepsilon}{\sigma} = 0.362(10^{-7}) + 0.835(10^{-12})\sigma$$

$$\left.\frac{\varepsilon}{\sigma}\right|_{\sigma=0} = \frac{1}{E|_{\sigma=0}} = 0.362(10^{-7}) + 0$$

So $E|_{\sigma=0} = 1/0.362(10^{-7}) = 27.6\ (10^6)$ psi Ans.

$$\left.\frac{\varepsilon}{\sigma}\right|_{\sigma=20\ 000} = \frac{1}{E|_{\sigma=20\ 000}}$$

$$= 0.362(10^{-7}) + 0.835(10^{-12})(20\ 000)$$

$$= 18.9(10^{-6})\text{ psi}$$ Ans.

The error in this value of E is in the regression constant a.

$0.352(10^{-7}) < a < 0.373(10^{-7})$ and so

$$\frac{1}{0.373(10^{-7})} < E < \frac{1}{0.352(10^{-7})}$$

Thus $26.8(10^6) < E < 28.4(10^6)$ at 95% confidence level Ans.

The unloading line with 0.001 permanent strain is $\sigma = E|_{\sigma=0}(\ - 0.001)$ (1)

The regression line can then be written

See also page 73

$$\varepsilon = 0.362(10^{-7})\sigma + 0.835(10^{-12})\sigma^2 = \frac{\sigma}{E|_{\sigma=0}} + 0.835(10^{-12})\sigma^2$$

Substituting for ε from Eq. (1) gives

$$\frac{\sigma}{E|_{\sigma=0}} + 0.001 = \frac{\sigma}{E|_{\sigma=0}} + 0.835(10^{-12})\sigma^2;\ \text{from which } \sigma = \sqrt{\frac{0.001}{0.835(10^{-12})}} = 34.6\text{ kpsi}$$ Ans.

---

5-5

| | | | | | | |
|---|---|---|---|---|---|---|
| True stress σ', kpsi | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 |
| True strain ε, in/in | 0.15 | 0.32 | 0.46 | 0.60 | 0.70 | 0.82 |
| log σ' | 3.000 | 3.176 | 3.301 | 3.398 | 3.477 | 3.544 |
| log ε | -0.824 | -0.495 | -0.337 | -0.222 | -0.155 | -0.086 |

In general, for a regression model with base 10 logarithms

$\log\sigma' = a + b\log\varepsilon + (0, s_{y\cdot x})$; or $10\exp(\log\sigma') = 10\exp[a + \log\varepsilon^b + (0, s_{y\cdot x})]$

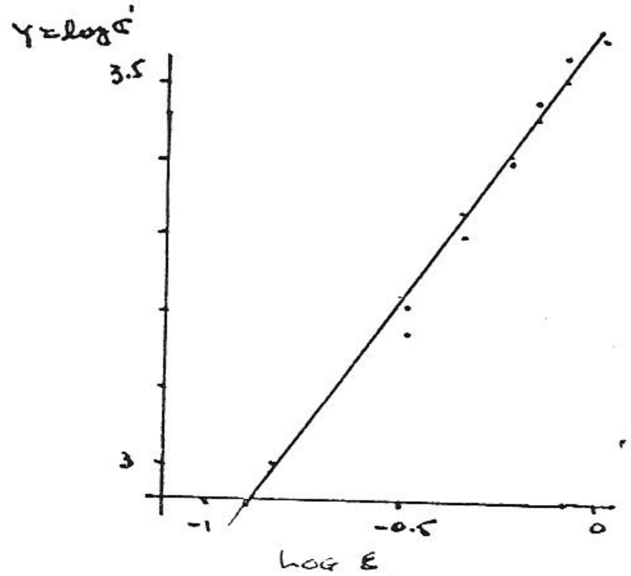
$\sigma' = 10^a\varepsilon^b 10^{(0,s_{y\cdot x})}$; $\mu_{\sigma'} = 10^a\varepsilon^b 10^{s_{y\cdot x}}$; $\mu'_{\sigma'} + \hat\sigma_{\sigma'} = 10^a\varepsilon^b 10^{s_{y\cdot x}}$

From regression, a = 3.576, b = 0.735, r = 0.989, and $S_{y\cdot x} = 0.0330$; we then have

$\mu_{\sigma'} = 10^{3.576}\varepsilon^{0.735} = 3767\varepsilon^{0.735}$, and $\hat\sigma_{\sigma'} = 297^{0.735}$

The estimate of the strain strengthening coefficient is $\sigma_0 = 3767$ psi and the strain strengthening exponent is m = 0.735. See next page for graph.

5-5 (Concluded)




5-6 OMITTED

5-7 (a) From Table A-22, $S_y = 32.0$ kpsi, $S_u = 49.5$ kpsi, $\sigma_0 = 90$ kpsi, $m = 0.25$, $\varepsilon_f = 1.05$

$$|\varepsilon_i| = \varepsilon_o = \ell n \left[1 + \frac{0.1094}{0.125}\right]^{\frac{1}{2}}$$

$= 0.314$ in/in the cold work strain, $\varepsilon_w$.

(b) $S'_u = \sigma_0(\varepsilon_w)^m$

$$= 90(0.314)^{0.25} = 67.4 \text{ kpsi}$$

or, since

$$\varepsilon + 1 = \frac{1}{1 - W}$$

$$W = 1 - \exp(-\varepsilon) = 1 - \exp(-0.314)$$

$$= 0.269$$

$$S'_u = \frac{S_u}{1 - W} = \frac{49.5}{1 - 0.269} = 67.8 \text{ kpsi}$$

(c) Datsko reports the significant strain for yield strength determination is

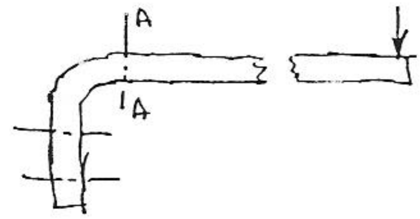
$$S'_y = \sigma_0\left(\frac{\varepsilon_w}{1 + 0.2\varepsilon_w}\right) = 90\left[\frac{0.314}{1 + 0.2(0.314)}\right]$$

$= 66.4$ kpsi

But Eq. (5-17) gives

$$S'_y = S'_u = \sigma_0 \varepsilon_i{}^m = 67.8 \text{ kpsi}$$

Although no distinctions have been made in Chap 5 these strengths are in the same sense as the plastic strains, i. e., compression on inside and tension outside. Strength in opposite senses are found differently. See STD HDBK OF MACH DES p. 8.6. Note also the strain strengthening is localized to the curve. For a bending load, nearly the full bending moment is felt at A-A which has original properties, hence the load-carrying capability is not really enhanced.



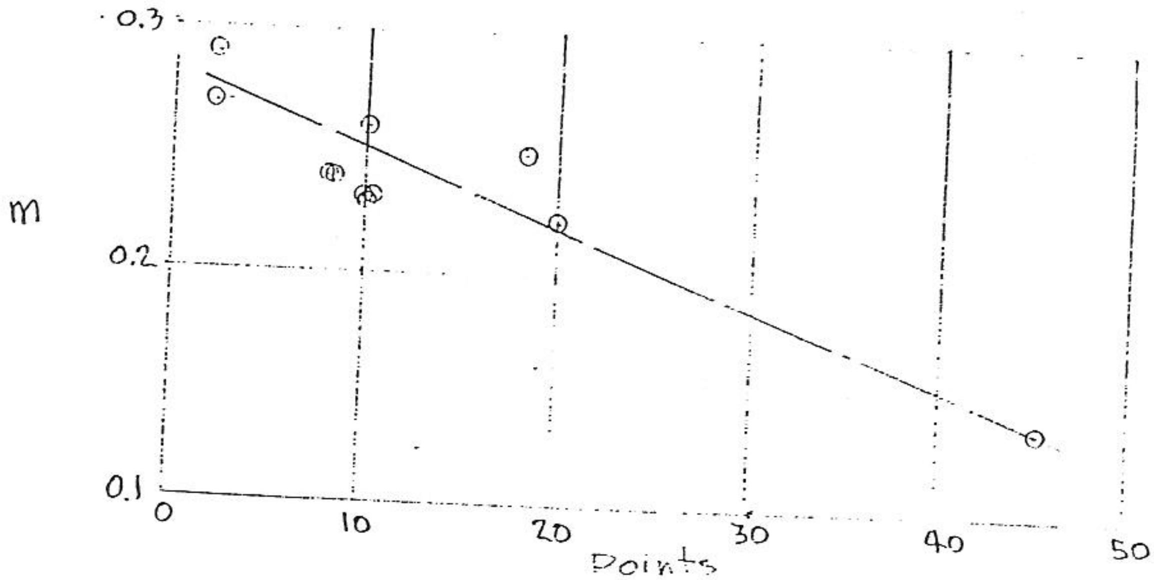

PROBLEM In table, $S_u$ in kpsi, $H_B$ from 500 kg load with 10-mm ball.

| $S_u$ | $H_B$ |
|---|---|
| 24 | 45 |
| 28 | 70 |
| 28 | 70 |
| 30 | 80 |
| 35 | 100 |
| 37 | 115 |
| 48 | 140 |
| 36 | 105 |
| 47 | 125 |
| 32 | 100 |
| 40 | 105 |
| 47 | 110 |
| 37 | 75 |
| 40 | 90 |
| 39 | 80 |
| 27 | 60 |

Tests of some aluminum alloys gave results as shown. Do you think that a formula similar to Eqs. (5-23) or (5-24) could be developed from this data. How much value would it be? Give reasons for your decision.

QUIZ Using Fig. 5-11 and Table A-20 estimate the ASTM minimum ultimate and yield strengths of an SAE 1020 cold-drawn steel bar used at a temperature of 300°C.

5-8 This eyeball solution indicates a YES answer.




5-9 and 5-10 OMITTED

**6-1**

| Term: | Part a | Part b | Part c | Part d |
|---|---|---|---|---|
| $\sigma_1$ | 9 | 12.7 | 0 | 11.14 |
| $\sigma_2$ | 0 | 0 | -0.91 | 3.86 |
| $\sigma_3$ | -5 | 0.71 | -12.09 | 0 |
| $\tau_{max}$ | 7 | 6.71 | 6.05 | 5.57 |
| $\sigma'$ | 12.29 | 12.37 | 11.66 | 9.80 |
| Theory: f. s. | | | | |
| Max. norm. str. | 4.89 | 3.46 | 3.64 | 3.95 |
| Max. shear str. | 3.14 | 3.28 | 3.64 | 3.95 |
| Dist. ener. | 3.58 | 3.56 | 3.77 | 4.49 |
| Coul. Mohr | 3.14 | 3.46 | 3.64 | 3.95 |

**6-3**

| Term: | Part a | Part b | Part c | Part d |
|---|---|---|---|---|
| $\sigma_1$ | | 176 | 86.5 | |
| $\sigma_2$ | | 0 | 0 | |
| $\sigma_3$ | | -36.3 | -166.5 | |
| $\tau_{max}$ | | 106.3 | 126.5 | |
| $\sigma'$ | | 196.7 | 223 | 346 |
| Theory: f. s. | | | | |
| Max. norm. str. | 2.33 | 2.39 | 2.52 | 2.10 |
| Max. shear str. | 2.33 | 1.98 | 1.66 | 1.05 |
| Dist. ener. | 2.33 | 2.14 | 1.89 | 1.21 |
| Coul. Mohr | 2.33 | 1.98 | 1.66 | 1.05 |

**6-4**

| Theory: f. s. | Part a | Part b | Part c | Part d |
|---|---|---|---|---|
| Max. norm. str. | 4.4 | 4.4 | 4.4 | 4.4 |
| Max. shear str. | 4.4 | 4.4 | 2.2 | 4.4 |
| Dist. ener. | 4.4 | 5.08 | 2.54 | 4.4 |
| Coul. Mohr | 4.4 | 4.4 | 2.2 | 4.4 |

**6-5**

| Theory: f. s. | Part a | Part b | Part c | Part d |
|---|---|---|---|---|
| Max. norm. str. | 1.5 | 2 | 1.25 | 1.5 |
| Coul. Mohr | 1.5 | 1.54 | 1.25 | 1.30 |

6-6 At A $\sigma_x = 95.5$ MPa, $\tau_{xz} = 19.1$ MPa

Since $S_y = 330$ MPa and

$\sigma' = [(95.5)^2 + 3(19.1)^2]^{\frac{1}{2}} = 101$ MPa

$n = 330/101 = 3.27$ Ans.

At B $\tau'_{xy} = 19.1$ MPa, $\tau'_{xy} = 2.3$ MPa,

so $\tau_{xy} = 21.4$ MPa; $\sigma_x = 25.5$ MPa, and

$\sigma' = [(25.5)^2 + 3(21.4)^2]^{\frac{1}{2}} = 45.0$ MPa

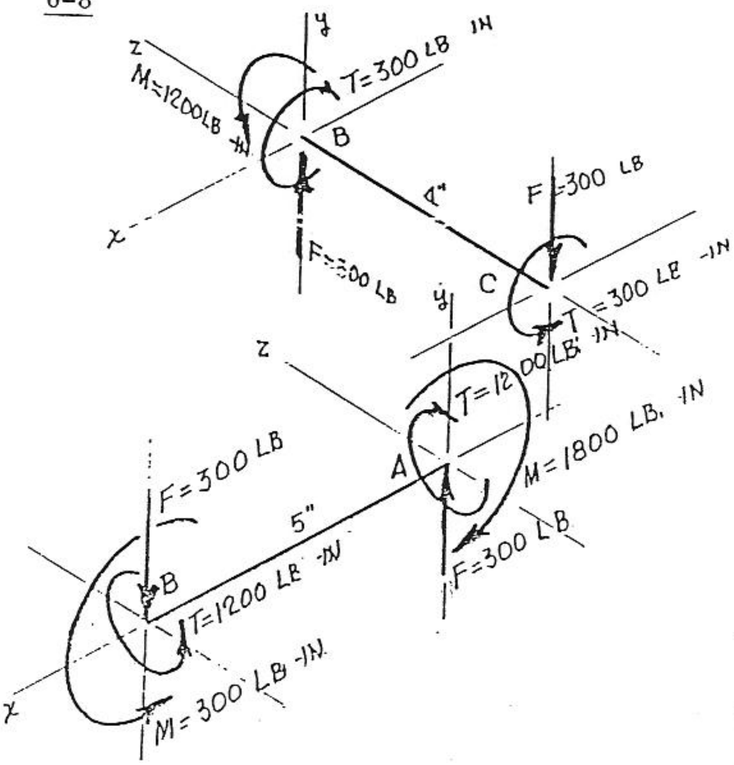
$n = 330/45.0 = 7.33$ Ans.

---

6-7 OMITTED

---

6-8




$$\frac{I}{c} = \frac{\pi d^3}{32} = \frac{\pi(0.75)^3}{32} = 0.0414 \text{ in}^3$$

$$\sigma_x = \frac{M}{I/c} = \frac{1800}{0.0415} = 43\ 500 \text{ psi}$$

$$J/c = (0.0415)(2) = 0.0830 \text{ in}^3$$

$$\tau_{xz} = \frac{T}{J/c} = \frac{1200}{0.0830} = 14\ 500 \text{ psi}$$

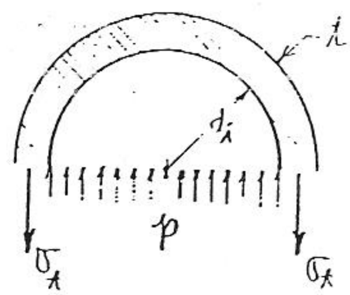
$S_y = 32$ kpsi; $S_{sy} = 0.5(32) = 16$ kpsi

$$\tau_{max} = [(43.5/2)^2 + (14.5)^2]^{\frac{1}{2}} = 26.1 \text{ kpsi}$$

$n = S_{sy}/\tau_{max} = 16/26.1 = 0.613$ FAILURE!

---

6-9




$$p\frac{\pi d_i^2}{4} = \sigma_t \pi(d_i + t)t$$

or

$$pd_i^{\,2} = 4t(d_i + t)$$

Since t is small, $d_i \cong d_i + t$, and so

$$\sigma_t \cong \frac{p(d_i + t)}{4t}$$

Also $\sigma_\ell = \sigma_t$ and $\sigma_r = -p$

These are principal stresses and so the distortion-energy theory [Eq. (6-11)] applies. Thus we have

$$\sigma' = S_y - \left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}\right]^{\frac{1}{2}}$$

Upon substituting, we find

$$S_y = \left[\frac{2(\sigma_t - \sigma_r)^2}{2}\right]^{\frac{1}{2}} = \sigma_t - \sigma_r$$

Then

$$S_y = \sigma_t - \sigma_r = \frac{p(d_i + t)}{4t} + p$$

and so

$$p = \frac{S_y}{\frac{d_i + t}{4t} + 1}$$

Here $S_y = 54$ kpsi, $S_{ut} = 64$ kpsi, and $t = 0.05$ in. Then

$$p = \frac{54\ 000}{\frac{8 + 0.05}{4(0.05)} + 1} = 1309 \text{ psi} \quad \text{Ans.}$$

Use max. norm. str. theory for bursting.

$$p = \frac{4tS_{ut}}{d_i + t} = \frac{4(0.05)(64\ 000)}{8 + 0.05}$$

$$= 1590 \text{ psi} \quad \text{Ans.}$$

6-10 For carbon steel $w = 0.282\ \text{lb/in}^3$. Also $S_y = 30$ kpsi, and $\nu = 0.292$. Then $\rho = w/g = 0.282/386\ \text{lb}\cdot\text{s}^2/\text{in}$; $r_i = 3$ in; $r_o = 5$ in; $r_i^2 = 9$; $r_o^2 = 25$; $3 + \nu = 3.292$; $1 + 3\nu = 1.876$.

Eq. (2-56) for $r = r_i$ becomes

$$\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left[2r_o^2 + r_i^2\left(1 - \frac{1+3\nu}{3+\nu}\right)\right]$$

So

$$\frac{S_y}{\omega^2} = \frac{0.282}{386}\left(\frac{3.292}{8}\right)\left[50 + 9\left(1 - \frac{1.876}{3.292}\right)\right]$$

$$= 0.0162$$

$$\text{So } \omega = \left(\frac{30\ 000}{0.0162}\right)^{\frac{1}{2}} = 1361 \text{ rad/s}$$

$$\text{or } n = 60\omega/2\pi = 60(1361)/(2\pi)$$

$$= 13\ 000 \text{ rev/min}$$

Now check stresses at $r = (r_o r_i)^{\frac{1}{2}}$, or $r = [5(3)]^{\frac{1}{2}} = 3.873$ in

Here

$$\sigma_r = \rho\omega^2\left(\frac{3+\nu}{8}\right)(r_o - r_i)^2$$

$$= \frac{0.282\omega^2}{386}\left(\frac{3.292}{8}\right)(5-3)^2$$

$$= 0.001\ 203\omega^2$$

Eq. (2-56) for $\sigma_t$ becomes

$$\sigma_t = \omega^2\left(\frac{0.282}{386}\right)\left(\frac{3.292}{8}\right)\left[9 + 25 + \frac{9(25)}{15} - \frac{1.876(15)}{3.292}\right] = 0.012\ 16\omega^2$$

Using the distortion-energy theory

$$\sigma' = \omega^2(\sigma_t^2 - \sigma_r\sigma_t + \sigma_r^2) = 0.011\ 60\omega^2$$

$$\text{Then } \omega = \left(\frac{30\ 000}{0.011}\right)^{\frac{1}{2}} = 1608 \text{ rad/s}$$

So the inner radius governs and

$n = 13\ 000$ rev/min  Ans.

6-11 OMITTED

6-12 Table A-20 gives $S_y = 320$ MPa. The maximum significant stress occurs at $r_i$ where $\sigma_r = \sigma_1 = 0$, $\sigma_2 = 0$, and $\sigma_3 = \sigma_t$. From Eq. (2-50)

$$\sigma_t = -\frac{2r_o^2 p_o}{r_o^2 - r_i^2} = -\frac{2(150)^2 p_o}{(150)^2 - 100)^2}$$

$$= -3.6p_o$$

Now equate this with the yield strength and solve for $p_o$. Thus

$$p_o = -\frac{320}{-3.6} = 88.9 \text{ MPa} \qquad \text{Ans.}$$

6-13 $S_{ut} = 30$ kpsi, $w = 0.260\ \text{lb/in}^3$, $\nu = 0.211$, $3 + \nu = 3.211$, $1 + 3\nu = 1.633$

At inner radius (see Prob. 6-10),

$$\frac{\sigma_t}{\omega^2} = \rho\left(\frac{3+\nu}{8}\right)\left(2r_o^2 + r_i^2 - \frac{1+3\nu}{3+\nu}\,r_i^2\right)$$

Here $r_o^2 = 25$, $r_i^2 = 9$, and so

$$\frac{\sigma_t}{\omega^2} = \frac{0.260}{386}\left(\frac{3.211}{8}\right)\left[50 + 9 - \frac{1.633(9)}{3.211}\right]$$

$$= 0.0147$$

Since $\sigma_r$ is the same sign, use the maximum normal stress theory. Thus

$$\omega = \left(\frac{30\ 000}{0.0147}\right)^{\frac{1}{2}} = 1428 \text{ rad/s}$$

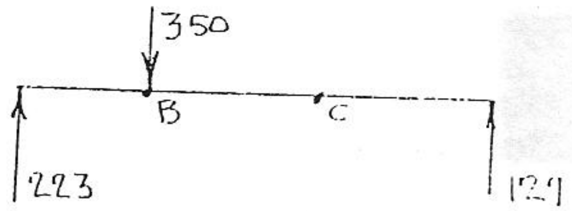
and so

$$n = 60\omega/2\pi = 60(1428)/(2\pi)$$

$$= 13\ 600 \text{ rev/min} \qquad \text{Ans.}$$

6-14 $T_C = (360 - 27)(3) = 1000$ lb·in

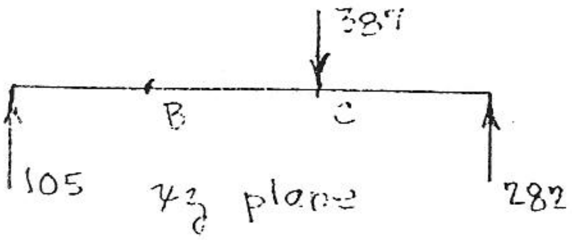
$T_B = (300 - 50)(4) = 1000$ lb·in




xy plane

6-14 (Continued)

In xy plane $M_B = 1784$ lb·in, and $M_C = 762$ lb·in.




In xz plane, $M_B = 840$ lb·in, and $M_C = 1692$ lb·in. The resultants are

$$M_B = [(1784)^2 + (840)^2]^{\frac{1}{2}} = 1972 \text{ lb·in}$$

$$M_C = [(1692)^2 + (762)^2]^{\frac{1}{2}} = 1889 \text{ lb·in}$$

So point B governs and the stresses are

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi d^3} = \frac{5100}{d^3} \text{ psi}$$

$$\sigma_x = \frac{32M_B}{\pi d^3} = \frac{32(1972)}{\pi d^3} = \frac{20\ 100}{d^3} \text{ psi}$$

Then

$$\sigma_1, \sigma_2 = \frac{\sigma_x}{2} \pm \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{\frac{1}{2}}$$

$$d^3\sigma = \frac{20.1}{2} \pm \left[\left(\frac{20.1}{2}\right)^2 + (5.1)^2\right]^{\frac{1}{2}}$$

$$= 10.05 \pm 11.27 \text{ kpsi}$$

Then

$$\sigma_1 = \frac{10.05 + 11.27}{d^3} = \frac{21.32}{d^3} \text{ kpsi}$$

and

$$\sigma_2 = \frac{10.05 - 11.27}{d^3} = -\frac{1.22}{d^3} \text{ kpsi}$$

For this state of stress the Coulomb-Mohr theory applies. Here we use $S_{ut}(\min) = 25$ kpsi, $S_{uc}(\min) = 97$ kpsi, and Eq. (6-14) to get

$$\frac{21.32}{25d^3} - \frac{-1.22}{97d^3} = \frac{1}{2.8}$$

Solving gives d = 1.34 in.

So use d = 1 3/8 in  Ans.

Note that this has been solved as a statics problem. Fatigue will be considered in the next chapter.

---

6-15 As in Prob. 6-14 we will assume this to be statics problem. Since the proportions are unchanged the bearing reactions will be the same as in Prob. 6-14. Thus

xy plane: $M_B = 223(4) = 892$ lb·in

xz plane: $M_B = 105(4) = 420$ lb·in

So

$$M_{max} = [(892)^2 + (420)^2]^{\frac{1}{2}} = 986 \text{ lb·in}$$

$$\sigma_x = \frac{32M_B}{\pi d^3} = \frac{32(986)}{\pi d^3} = \frac{10\ 000}{d^3} \text{ psi}$$

The torsional stress is unchanged. So

$$\tau_{xz} = 5.1/d^3 \text{ kpsi}$$

$$\sigma_1, \sigma_2 = \frac{1}{d^3}\left(\frac{10.0}{2}\right) \pm \left[\left(\frac{10.0}{2}\right)^2 + (5.1)^2\right]^{\frac{1}{2}}$$

$$\sigma_1 = 12.14/d^3 \text{ and } \sigma_2 = -2.14/d^3$$

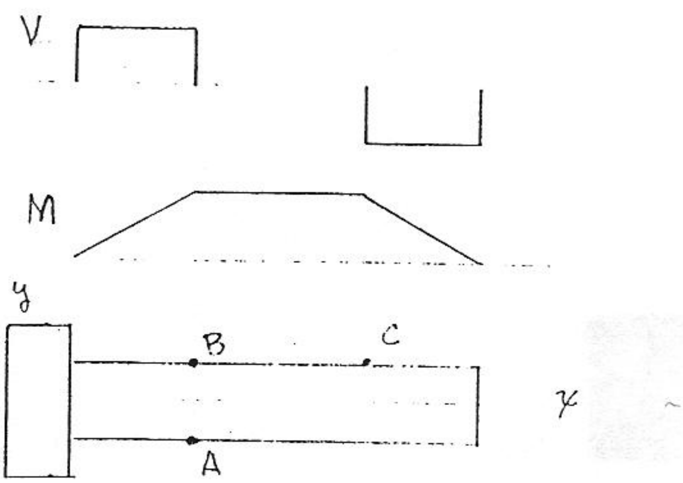
Using the Coulomb-Mohr theory as in Prob. 6-14 gives

$$\frac{12.14}{25d^3} - \frac{12.14}{97d^3} = \frac{1}{2.8}$$

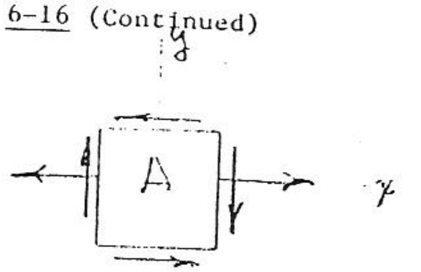
Solving and rounding gives d = 1 1/8 in  Ans.

---

6-16



6-16 (Continued)



(a) $M_{max} = \frac{F}{2}\left(\frac{a}{2} + \frac{b}{4}\right) = \frac{4.4}{2}(6 + 4.5)$

$= 23.1$ N·m

For a stress element at A:

$\sigma_x = \frac{32M}{\pi d^3} = \frac{32(23.1)(10^3)}{\pi(12)^3} = 136.2$ MPa

The direct shear at A is

$\tau_{xy} = \frac{4(F/2)}{\pi d^2} = \frac{4(4.4/2)(10^3)}{\pi(12)^2}$

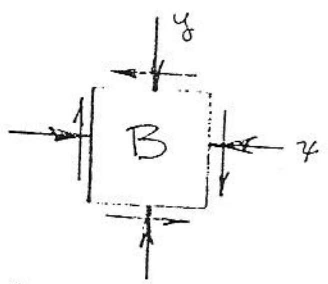
$= 19.45$ MPa

$\tau_{max} = \left[\left(\frac{136.2}{2}\right)^2 + (19.45)^2\right]^{\frac{1}{2}} = 70.8$ MPa

Since $S_y = 220$ MPa, $S_{sy} = 220/2 = 110$ MPa

and

$\bar{n} = \frac{S_{sy}}{\tau_{max}} = \frac{110}{70.8} = 1.55$ Ans.



AT point B we have surface compression too. So

$\sigma_y = -\frac{F}{A} = -\frac{F}{bd} = -\frac{4.4(10^3)}{18(12)} = -20.4$ MPa

Also, $\sigma_x = -136.2$ MPa, $\tau_{xy} = 19.45$ MPa

From a Mohr's circle, find $\tau_{max} = 69.7$ MPa

Then $n = \frac{110}{69.7} = 1.58$ Ans.

(b) Same bending stress, but at C; so there is no difference.

---

6-17 (a) $\sigma_1 = 138.9$, $\sigma_2 = 0$, $\sigma_3 = 2.72$

By dist. ener. n = 1.57 Ans.

(b) $\sigma_1 = 0$, $\sigma_2 = -17.2$, $\sigma_3 = -139$

n = 1.67 Ans.

---

6-18 (a) $\underset{\sim}{M} = 14\underset{\sim}{F}$

$\underset{\sim}{\sigma}_x = \frac{32\underset{\sim}{M}}{\pi d^3} = \frac{32(14)\underset{\sim}{F}}{\pi(1)^3} = 142.6\underset{\sim}{F}$

$\underset{\sim}{\tau}_{xy} = \frac{16\underset{\sim}{T}}{\pi d^3} = \frac{16(15)\underset{\sim}{F}}{\pi(10^3} = 76.4\underset{\sim}{F}$

The coefficient of variation of F is

$C_F = \frac{\hat{\sigma}}{\mu} = \frac{45}{410} = 0.110$

At A

$\bar{\sigma}_x = 142.6\bar{F} = 142.6(410) = 58\,500$ psi

$\hat{\sigma}_{\sigma x} = C_F\bar{\sigma}_x = 0.110(58\,500) = 6430$ psi

$\bar{\tau}_{xy} = 76.4\bar{F} = 76.4(410) = 31\,300$ psi

$\hat{\sigma}_{\tau xy} = C_F\bar{\tau}_{xy} = 0.110(31\,300) = 3440$ psi

Then, in kpsi, we have

$\bar{\sigma}_1, \bar{\sigma}_2 = \frac{58.5}{2} \pm \left[\left(\frac{58.5}{2}\right)^2 + (31.3)^2\right]^{\frac{1}{2}}$

$= 72.1, -13.6$ kpsi

The standard deviations are

$\hat{\sigma}_{\sigma 1} = 0.110(72.1) = 7.93$ kpsi

$\hat{\sigma}_{\sigma 2} = |0.110(-13.6)| = 1.50$ kpsi

Thus $\underset{\sim}{\sigma}_1 \sim N(72.1, 7.93)$ kpsi Ans.

$\underset{\sim}{\sigma}_2 \sim N(-13.6, 1.50)$ kpsi Ans.

(b) Let $\sigma_2 = k\sigma_1$ so that the mean von Mises stress is

$\bar{\sigma}' = (\bar{\sigma}_1^2 - \bar{\sigma}_1\bar{\sigma}_2 + \bar{\sigma}_2^2)^{\frac{1}{2}} = \bar{\sigma}_1(1 - k + k^2)^{\frac{1}{2}}$

with a standard deviation of

6-18 (Continued)

$\hat{\sigma}_{\sigma'} = \hat{\sigma}_{\sigma 1}(1 - k + k^2)^{\frac{1}{2}}$

Since $k = \sigma_2/\sigma_1 = -13.6/72.1 = -0.189$

$\bar{\sigma}' = 72.1[1 - (-0.189) + (-0.189)^2]^{\frac{1}{2}}$
$= 79.8$ kpsi

$\hat{\sigma}_{\sigma'} = C_F\bar{\sigma}' = 0.110(79.8) = 8.78$ kpsi

and so $\underset{\sim}{\sigma}' \sim N(79.8, 8.78)$ kpsi Ans.

---

6-19 (a) First convert the data to radial dimensions to agree with the formulations of Fig. 2-25. Thus

$r_o = 0.5625 \pm 0.001$ in

$r_i = 0.1875 \pm 0.001$ in

$R_o = 0.375 \pm 0.0002$ in

$R_i = 0.376 \pm 0.0002$ in

The stochastic nature of the dimensions affects the $\underset{\sim}{\delta} = |\underset{\sim}{R}_i| - |\underset{\sim}{R}_o|$ relation in Eq. (2-60), but not the others. Set $R = \frac{1}{2}(R_i + R_o) = 0.3755$ in. From Eq. (2-60)

$$\underset{\sim}{p} = \frac{E\underset{\sim}{\delta}}{R}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{2R^2(r_o^2 - r_i^2)}\right]$$

Substituting and solving with E = 30 Mpsi gives

$\underset{\sim}{p} = 18.70(10^6)\underset{\sim}{\delta}$

Since $\underset{\sim}{\delta} = \underset{\sim}{R}_i - \underset{\sim}{R}_o$

$\mu_\delta = \mu_{Ri} - \mu_{Ro} = 0.376 - 0.375 = 0.001$ in

and

$$\hat{\sigma}_\delta = \left[\left(\frac{0.0002}{4}\right)^2 + \left(\frac{0.0002}{4}\right)^2\right]^{\frac{1}{2}}$$
$= 0.000\ 0707$ in

Then

$$C_\delta = \frac{\hat{\sigma}_\delta}{\mu_\delta} = \frac{0.000\ 0707}{0.001} = 0.0707$$

The tangential inner cylinder stress at the shrink-fit surface is given by Eq. (2-57) as

$$\underset{\sim}{\sigma}_{it} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2}$$

$$= -18.70(10^6)\underset{\sim}{\delta}\,\frac{0.3755^2 + 0.1875^2}{0.3755^2 - 0.1875^2}$$

$= -31.1(10^6)\underset{\sim}{\delta}$

$\bar{\sigma}_{it} = -31.1(10^6)\mu_\delta = -31.1(10^6)(0.001)$
$= -31.1(10^3)$ psi

Note, here $\bar{\sigma}_{it}$ means the same as $\mu_{\sigma it}$. Also

$\hat{\sigma}_{\sigma it} = |C_\delta\bar{\sigma}_{it}| = |0.0707[-31.1(10^3)]|$
$= 2899$ psi

(b) The tangential stress for the outer cylinder at the shrink-fit sur-surface is given by Eq. (2-58).

$$\underset{\sim}{\sigma}_{ot} = \underset{\sim}{p}\frac{r_o^2 + R^2}{r_o^2 - R^2}$$

$$= 18.70(10^6)\underset{\sim}{\delta}\,\frac{0.5625^2 + 0.3755^2}{0.5625^2 - 0.3755^2}$$

$= 48.76(10^6)\underset{\sim}{\delta}$ psi

$\bar{\sigma}_{ot} = 48.76(10^6)(0.001) = 48.76(10^3)$ psi

$\hat{\sigma}_{\sigma ot} = C_\delta\bar{\sigma}_{ot} = 0.0707(48.76)(10^3)$
$= 3445$ psi

---

6-20 From Prob. 6-19, at the fit surface $\underset{\sim}{\sigma}_{ot} \sim N(48.8, 3.45)$ kpsi

The radial stress is the fit pressure which was found to be

$\underset{\sim}{p} = 18.70(10^6)\underset{\sim}{\delta}$

$\mu_p = 18.70(10^6)(0.001) = 18.7(10^3)$ psi

$\hat{\sigma}_p = C_\delta\mu_p = 0.0707(18.70)(10^3)$
$= 1322$ psi

6-20 (Continued)

and so

$\underset{\sim}{p} \sim N(18.7,\ 1.32)$ kpsi

and

$\underset{\sim}{\sigma}_{or} \sim -(18.7,\ 1.32)$ kpsi

These are the principal stresses

$\sigma_A = 48.8$ kpsi, $\sigma_B = -18.7$ kpsi

$k = \sigma_B/\sigma_A = -18.7/48.8 = -0.383$

$$\sigma' = \sigma_A(1 - k + k^2)^{\frac{1}{2}}$$

$$= 48.8[1 - (-0.383) + (-0.383)^2)]^{\frac{1}{2}}$$

$$= 60.4 \text{ kpsi}$$

$$\hat{\sigma}_{\sigma'} = C_p\mu_{\sigma'} = 0.0707(60.4) = 4.27 \text{ kpsi}$$

Using the interference equation

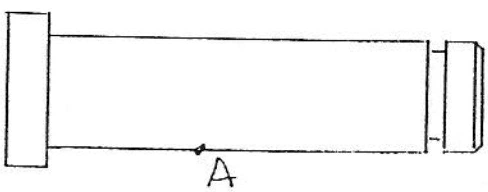
$$z = -\frac{\mu_S - \mu_\sigma}{(\hat{\sigma}_S^2 + \hat{\sigma}_\sigma^2)^{\frac{1}{2}}}$$

$$= -\frac{95.5 - 60.4}{[(6.59)^2 + (4.27)^2]^{\frac{1}{2}}} \cong -4.5$$

$p_f = \alpha = 0.000\ 003\ 40$,

or about 3 chances in a million. Ans.

6-21



For point A

$$\underset{\sim}{M}_{max} = \frac{\underset{\sim}{F}}{2}\left(\frac{a}{2} + \frac{b}{4}\right)$$

$$= \frac{1500(1,\ 0.1)}{2}\left(\frac{0.5}{2} + \frac{0.75}{4}\right)$$

$$= 329(1,\ 0.1) \text{ lb}\cdot\text{in}$$

$$\underset{\sim}{\sigma}_x = \frac{32\underset{\sim}{M}}{\pi d^3} = \frac{32[329(1,\ 0.1)]}{\pi(0.5)^3}$$

$$= 26.7(1,\ 0.1) \text{ kpsi}$$

For a brittle material in uniaxial tension $S_{ut}$ is the governing strength.

$$z = \frac{\mu_S - \mu_\sigma}{(\hat{\sigma}_S^2 + \hat{\sigma}_\sigma^2)^{\frac{1}{2}}}$$

$$= -\frac{44.5 - 26.7}{[(4.34)^2 + (2.67)^2]^{\frac{1}{2}}} = -3.49$$

$$R = 1 - \Phi(z) = 1 - \Phi(3.49)$$

$$= 1 - 0.000\ 233 = 0.9998$$

(b) For Weibull strength

$$R = \exp\left[-\left(\frac{x - x_0}{\theta - x_0}\right)^b\right]$$

where $x = x_0 + (\theta - x_0)\left(\ln\frac{1}{R}\right)^{1/b}$

For the normal stress

$$z_2 = \frac{x - \mu_\sigma}{\hat{\sigma}_\sigma}$$

Continued on next page.

6-22

$$\underset{\sim}{\sigma}_t = \frac{\underset{\sim}{p}d}{2t} = \frac{6000(1,\ 0.08\underline{3})(0.75)}{2(0.125)}$$

$$= 18(1,\ 0.08\underline{3}) \text{ kpsi}$$

Note that $0.08\underline{3}$ means 0.0833333 . . .

$$\underset{\sim}{\sigma}_\ell = \frac{\underset{\sim}{p}d}{4t} = \frac{6000(1,\ 0.08\underline{3})(0.75)}{4(0.125)}$$

$$= 9(1,\ 0.08\underline{3}) \text{ kpsi}$$

$$\underset{\sim}{\sigma}_r = -\underset{\sim}{p} = -6000(1,\ 0.08\underline{3}) \text{ kpsi}$$

These three stresses are principal stresses whose variability is due to the loading. From Eq. (6-11) we find the von Mises stress to be

$$\sigma' = \left\{\frac{(18 - 9)^2 + [9 - (-6)]^2 + (-6-18)^2}{2}\right\}^{\frac{1}{2}}$$

$$= 21.0 \text{ kpsi}$$

$$\hat{\sigma}_{\sigma'} = C_p\bar{\sigma}' = 0.08\underline{3}(21.0) = 1.75 \text{ kpsi}$$

6-21 (Continued) The area on the $R_1R_2$ diagram is in the lower right-hand corner.

| $R_1$ | Cursor x, kpsi | $z_2$ | $R_2$ | Mult. |
|---|---|---|---|---|
| 0.980 | 35.2 | 3.18 | 0.000 743 | 1 |
| 0.982 | 35.0 | 3.11 | 0.000 9 | 4 |
| 0.984 | 34.8 | 3.04 | 0.001 16 | 2 |
| 0.986 | 34.6 | 2.96 | 0.001 54 | 4 |
| 0.988 | 34.4 | 2.87 | 0.002 05 | 2 |
| 0.990 | 34.1 | 2.76 | 0.002 86 | 4 |
| 0.992 | 33.8 | 2.64 | 0.004 15 | 2 |
| 0.994 | 33.4 | 2.50 | 0.006 21 | 4 |
| 0.996 | 32.9 | 2.31 | 0.010 4 | 2 |
| 0.998 | 32.1 | 2.00 | 0.021 7 | 4 |
| 1.000 | 27.7 | 0.375 | 0.354 | 1 |

0.523 103

$$I = \frac{0.002}{3}[0.523\ 103] = 0.000\ 348\ 7$$

$$R = 1 - 0.000\ 348\ 7 = 0.9997$$

---

6-22 (Concluded)

$$z = -\frac{\mu_S - \mu_\sigma}{(\hat{\sigma}_S^2 + \hat{\sigma}_{\sigma'}^2)^{\frac{1}{2}}}$$

$$= \frac{50 - 21.0}{(4.1^2 + 1.75^2)^{\frac{1}{2}}} = -6.5$$

The reliability is very high

$R = 1 - \Phi(6.5) = 1 - 0.0^{10}402 \cong 1$ Ans.

---

6-23, 24, 25 OMITTED

---

6-26 (a) Convert to radial dimensions.

$r_i = 0.500 \pm 0.001$ in

$R_i = 1.000 \pm 0.0002$ in

$R_o = 0.9995 \pm 0.0002$ in

$r_o = 1.500 \pm 0.002$ in

The dimensions can be represented by their midrange values

$r_i = 0.500$ in    $R_i = 1.000$ in

$r_o = 1.500$ in    $R_o = 0.9995$ in

$$\delta = |R_i| - |R_o| = |1.000| - |0.9995| = 0.0005 \text{ in}$$

Substituting these values with E = 30 Mpsi, R = 0.99975 into Eq. (2-60) gives

$p = 3517$ psi

Then the four stresses at radius R are:

$$\sigma_{it} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -3517\frac{0.99975^2 + 0.5^2}{0.99975^2 - 0.5^2} = -5864 \text{ psi}$$

$$\sigma_{ir} = -p = -3517 \text{ psi}$$

$$\sigma_{ot} = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 3517\frac{1.5^2 + 0.99975^2}{1.5^2 - 0.99975^2} = 9140 \text{ psi}$$

$$\sigma_{or} = -p = -3517 \text{ psi}$$

(b) $r_i = 0$ in, $r_o = 1.5$ in, $R_i = 1.000$ in $R_0 = 0.9995$ in, $\delta = 0.0005$ in. Using the same approach gives the stresses at R as: $\sigma_{it} = -p = -4169$ psi,

6-26 (Concluded)

$\sigma_{ir} = -p = -4169$ psi, $\sigma_{ot} = 10\,833$ psi, and $\sigma_{or} = -p = -4169$ psi

---

6-27 Here $r_i = 12.5 \pm 0.025$ mm, $R_i = 25 \pm 0.005$ mm, $R_o = 24.99 \pm 0.005$, $r_o = 37.5 \pm 0.050$ mm

$$\delta = |R_i| - |R_o| = |25| - |24.99| = 0.01 \text{ mm}$$

Also, $E = 207$ GPa, and $R = 24.995$ mm

Then

$$p = \frac{E\delta}{R}\,\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{2R^2(r_o^2 - r_i^2)} = 19.4 \text{ MPa}$$

The stresses at radius R are:

$$\sigma_{it} = -p\,\frac{R^2 + r_i^2}{R^2 - r_i^2} = -32.4 \text{ MPa}$$

$$\sigma_{ir} = -p = -19.4 \text{ MPa}$$

$$\sigma_{ot} = p\,\frac{r_o^2 + R^2}{r_o^2 - R^2} = 50.5 \text{ MPa}$$

$$\sigma_{or} = -p = -19.4 \text{ MPa}$$

Inner tube:

$$\sigma' = (\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2)^{\frac{1}{2}}$$

$$= [(-32.4)^2 + (-19.4)^2 - (-32.4)(-19.4)]^{\frac{1}{2}} = 28.4 \text{ MPa}$$

Outer tube:

$$\sigma' = [(50.5)^2 + (-19.4)^2 - (50.5)(-19.4)]^{\frac{1}{2}}$$

$$= 62.5 \text{ MPa}$$

---

6-28 $r_i = 0$ in, $R_o = 0.9995 \pm 0.0002$ in

$r_o = 2.00 \pm 0.0625$ in

$R_i = 1.000 \begin{matrix} +0.0000 \\ -0.0002 \end{matrix}$ in

With bilateral tolerance

$R_i = 0.9999 \pm 0.0001$ in

$$\delta = |R_i| - |R_o| = |0.9999| - |0.9995| = 0.0004 \text{ in} \quad \text{mean}$$

$$\delta = |1.000| - |0.9993| = 0.0007 \text{ largest}$$

From Eq. (2-59)

$$p = \frac{\delta}{\dfrac{R}{E_o}\left(\dfrac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \dfrac{R}{E_i}(1 - \nu_i)}$$

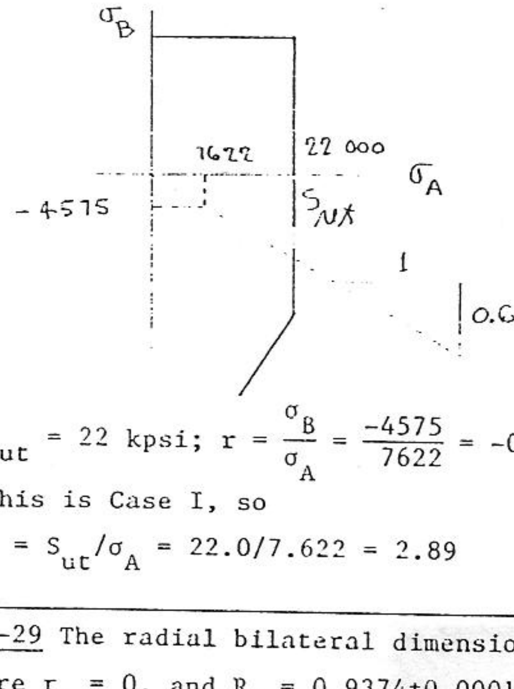
Now use $E_o = 14.5$ Mpsi, $E_i = 30$ Mpsi, $\nu_o = 0.211$, $\nu_i = 0.292$, and $\delta = 0.0007$ in to get $p = 4575$ psi

Then, at R

$$\sigma_{ot} = p\,\frac{r_o^2 + R^2}{r_o^2 - R^2}$$

$$= 4575\,\frac{2^2 + 0.9997^2}{2^2 - 0.9997^2} = 7622 \text{ psi}$$

$$\sigma_{or} = -p = -4575 \text{ psi}$$




$$S_{ut} = 22 \text{ kpsi}; \; r = \frac{\sigma_B}{\sigma_A} = \frac{-4575}{7622} = -0.600$$

This is Case I, so

$$n = S_{ut}/\sigma_A = 22.0/7.622 = 2.89$$

---

6-29 The radial bilateral dimensions are $r_i = 0$, and $R_i = 0.9374 \pm 0.0001$ in for the shaft. For the hub $R_o = 0.935 \pm 0.001$ in and $r_o = 1.625 \pm 1/16$in

Then

$$\bar{\delta} = |\bar{R}_i| - |\bar{R}_o| = |0.9374| - |0.935|$$

29 (Continued) From previous page

= 0.0024 in. We now estimate

$_i = \frac{0.0001}{3} = 0.000\ 033$ in and

$_o = \frac{0.001}{3} = 0.000\ 333$ in

ien $\hat{\sigma}_\delta = [0.000\ 033)^2 + (0.000\ 333)^2]^{\frac{1}{2}}$

$= 0.000\ 335$ in

and so

$$C_\delta = \frac{0.000\ 335}{0.0024} = 0.139$$

ow, using R = 0.9362 in, $\nu_o = 0.211$,

$_i = 30$ Mpsi, $\nu_i = 0.292$, and the

quation for p in last solution, gives

= 16.7 kpsi

nen, at radius R

$$_{ot} = \bar{p}\,\frac{r_o^2 + R^2}{r_o^2 - R^2}$$

$$= 16.7\,\frac{1.625^2 + 0.9362^2}{1.625^2 - 0.9362^2} = 33.3 \text{ kpsi}$$

$_{or} = -p = -16.7$ kpsi

$_{\sigma ot} = C_\delta\,\bar{\sigma}_{ot} = 0.139(33.3) = 4.63$ kpsi

Jsing the modified Mohr

$r = \frac{33.3}{-16.7} = -1.99$ therefore case II

$$\bar{S}_A = \frac{\bar{S}_{ut}\bar{S}_{uc}}{\bar{S}_{uc} - (1 + r)\bar{S}_{ut}}$$

$$= \frac{1}{\frac{1}{\bar{S}_{ut}} - \frac{1 + r}{\bar{S}_{uc}}} = \frac{1}{\frac{1}{44.5} - \frac{1 - 1.99}{140}}$$

$$= 33.8 \text{ kpsi}$$

$$\hat{\sigma}_{SA} = \frac{1}{\frac{1}{44.5 + 4.34} - \frac{1 - 1.99}{140 + 13.1}} - 33.8$$

$$= 3.32 \text{ kpsi}$$

Now, interfere $\sigma_A$ with $S_A$, where

$\sigma_A = (33.3,\ 4.63)$ kpsi

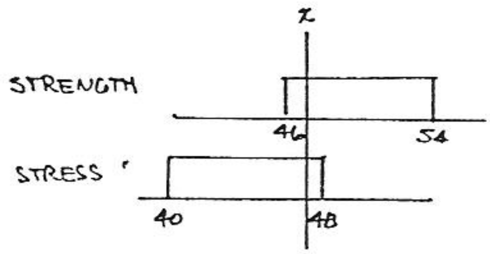
$$z = -\frac{\bar{S}_A - \bar{\sigma}_A}{(\hat{\sigma}_{SA}^2 + \hat{\sigma}_{\sigma A}^2)^{\frac{1}{2}}}$$

$$= -\frac{33.8 - 33.3}{(3.32^2 + 4.63^2)^{\frac{1}{2}}} = -0.088$$

which corresponds to a reliability of 0.54

---

6-30




Strength:

$x < 46$ $\quad R_1 = 1$

$46 < x < 54$ $\quad R_1 = \frac{x - 46}{54 - 46} = \frac{54 - x}{8}$

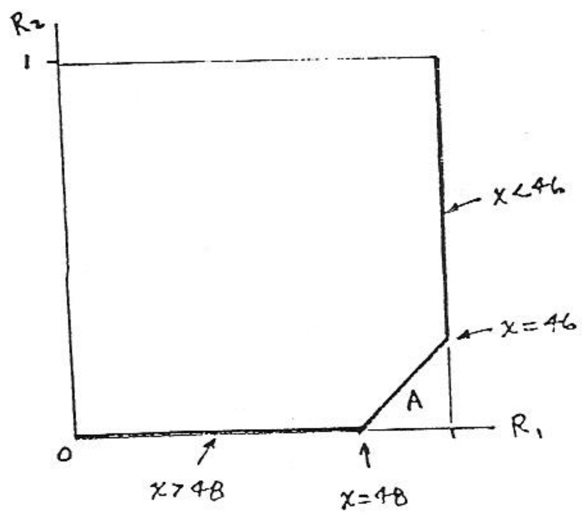
$x > 54$ $\quad R_1 = 0$

Stress:

$x < 40$ $\quad R_2 = 1$

$40 < x < 48$ $\quad R_2 = 1 - \frac{x - 40}{48-40} = \frac{48 - x}{8}$

$x > 48$ $\quad R_2 = 0$



**6-30** (Concluded)

Tabulate $R_1$, $R_2$

| x | $R_1$ | $R_2$ |
|---|---|---|
| 40 | 1 | 1 |
| 46 | 1 | 0.25 |
| 47 | 0.875 | 0.125 |
| 48 | 0.75 | 0 |
| 50 | 0.50 | 0 |
| 54 | 0 | 0 |

Now relate $R_2$ to $R_1$ through x

$$54 - 8R_1 = x = 48 - R_2$$

Then $R_2 = R_1 - 0.75$ (note that $R_2 = 0$, when $R_1 < 0.75$)

Using Eq. (6-29) gives

$$R = 1 - \int_0^1 R_2\, dR_1 = 1 - \int_{0.75}^{1} (R_1 - 0.75)\, dR_1$$

$$= 1 - \frac{R_1^{\,2}}{2}\Bigg|_{0.75}^{1} + 0.75R_1\Bigg|_{0.75}^{1}$$

$$R = 1 - 0.218\ 75 + 0.75(1 - 0.75)$$

$$= 0.968\ 75$$

Check from graph

$$R = 1 - A = 1 - \tfrac{1}{2}(0.25)(1 - 0.75)$$

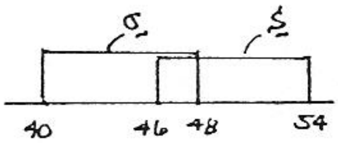
$$= 0.968\ 75$$

---

**6-31** $\mu_S = (54 + 46)/2 = 50$ kpsi

$$\hat{\sigma}_S = \frac{54 - 46}{2\sqrt{3}}$$

$$= 2.31 \text{ kpsi}$$




$$\mu_\sigma = \frac{48 + 40}{2}$$

$$= 44 \text{ kpsi}$$

$$\hat{\sigma}_\sigma = \frac{48 - 40}{2\sqrt{3}} = 2.31 \text{ kpsi}$$

The range numbers for m are

$$m_{large} = 54 - 40 = 14 \text{ kpsi}$$

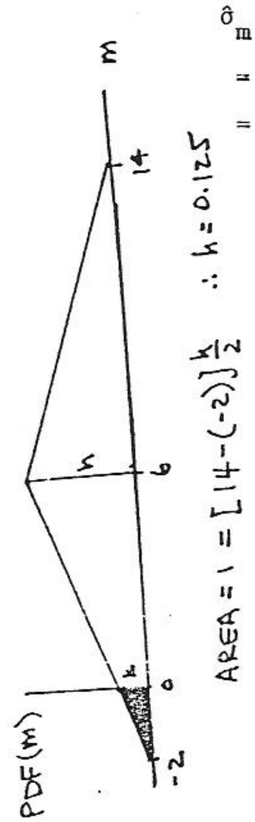
$$m_{small} = 46 - 48 = -2 \text{ kpsi}$$

$$\mu_m = \mu_S - \mu_\sigma = 50 - 44 = 6 \text{ kpsi}$$

$$\hat{\sigma}_m = (\hat{\sigma}_S^2 + \hat{\sigma}_\sigma^2)^{\frac{1}{2}}$$

$$= (2.31^2 + 2.31^2)^{\frac{1}{2}}$$

$$= 3.27 \text{ kpsi}$$




Reliability = area to right of m = 0 = 1 − [shaded area]

$$\frac{k}{0 - (-2)} = \frac{h}{6 - (-2)} = \frac{0.125}{8}$$

Therefore k = 0.03125

$$\text{Reliability} = 1 - [0 - (-2)](k/2)$$

$$= 1 - 2\,\frac{0.03125}{2}$$

$$= 0.96875 \quad \underline{\text{Ans.}}$$

Next, $S_{min} = \mu_S - 3\hat{\sigma}_S = 50 - 3(2.31)$

$$= 43.07 \text{ kpsi}$$

$$\sigma_{max} = \mu_\sigma + 3\hat{\sigma}_\sigma = 44 + 3(2.31)$$

$$= 50.93 \text{ kpsi}$$

$$n = \frac{S_{min}}{\sigma_{max}} = \frac{43.07}{50.93} = 0.846 \quad \text{No safety!}$$

---

INTERFERENCE BY NUMERICAL ANALYSIS

A supplement to Shigley and Mischke MECHANICAL ENGINEERING DESIGN, 5/e

Here we are going to integrate areas, similar to those of Fig. 6-32, using Simpson's rule, to obtain the reliability. The first step is to specify a set of values for $R_1$ and then to find the corresponding values of $R_2$. This will define the $R_1$-$R_2$ relation. But to find $R_2$ we first require a set of cursor positions x corresponding to each value of $R_1$ we have selected. This depends upon how the strength is distributed. The procedure for the normal, the lognormal, and the Weibull distributions is as follows.

<u>Normal strength.</u> $\underset{\sim}{S} \sim N(\mu_S, \hat{\sigma}_S)$

1. With $R_1$ given enter Table A-10 with tabulation = $R_1$ and read $z_1$.
2. Solve for x from the equation

$$z_1 = \frac{x - \mu_S}{\hat{\sigma}_S} \qquad (6\text{-}32)$$

<u>Lognormal strength.</u> $\underset{\sim}{S} \sim LN(\mu_S, \hat{\sigma}_S)$

1. With $R_1$ given enter Table A-10 with tabulation = $R_1$ and read $z_1$.
2. Compute

$$\hat{\sigma}_{\ell n\, S} = \frac{\hat{\sigma}_S}{\mu_S} = C_S \qquad (6\text{-}33)$$

$$\mu_{\ell n\, S} = \ell n\, \mu_S - \frac{C_S^2}{2} \qquad (6\text{-}34)$$

3. Solve the equation

$$z_1 = \frac{\ell n\, x - \mu_{\ell n\, S}}{\hat{\sigma}_{\ell n\, S}} \qquad (6\text{-}35)$$

to get

$$x = \ell n^{-1}\, (z_1 \hat{\sigma}_{\ell n\, S} + \mu_{\ell n\, S}) \qquad (6\text{-}36)$$

<u>Weibull strength.</u> $\underset{\sim}{S} \sim W(x_{OS}, \theta_S, b_S)$

1. $R_1$ is given; solve Eq. (4-22) for x.

$$x = x_{OS} + (\theta_S - x_{OS})\left[\ell n\, \frac{1}{R_1}\right]^{1/b_S} \qquad (6\text{-}37)$$

We have now found the cursor positions x corresponding to all values of $R_1$ from the strength distributions. The next step is to find the value of $R_2$ corresponding to each cursor position from the distribution of stress. We note very particularly here that the stresses need not be distributed the same as the strengths. Thus any distribution of strength can be used with any other distribution of stress. The procedure for each of the stress distributions is as follows.

Normal stress. $\underset{\sim}{\sigma} \sim N(\mu_\sigma, \hat{\sigma}_\sigma)$

1. Find $z_2$ from the equation

$$z_2 = \frac{x - \mu_\sigma}{\hat{\sigma}_\sigma} \tag{6-38}$$

2. Enter Table A-10 with $z_2$ and read $R_2$ = tabulation.

Lognormal stress. $\underset{\sim}{\sigma} \sim LN(\mu_\sigma, \hat{\sigma}_\sigma)$

1. Compute

$$\hat{\sigma}_{\ell n\ \sigma} = \frac{\hat{\sigma}_\sigma}{\mu_\sigma} = C_\sigma \tag{6-39}$$

$$\mu_{\ell n\ \sigma} = \ell n\ \mu_\sigma - \frac{C_\sigma^2}{2} \tag{6-40}$$

2. Solve the following equation

$$z_2 = \frac{\ell n\ x - \mu_{\ell n\ \sigma}}{\hat{\sigma}_{\ell n\ \sigma}} \tag{6-41}$$

3. Enter $z_2$ in Table A-10 and read $R_2$ = tabulation.

Weibull stress. $\underset{\sim}{\sigma} \sim W(x_{0\sigma}, \theta_\sigma, b_\sigma)$

1. Find $R_2$ from Eq. (4-22) as

$$R_2 = \exp\left[-\left(\frac{x - x_{0\sigma}}{\theta_\sigma - x_{0\sigma}}\right)^{b_\sigma}\right] \quad 6\text{-}42)$$

For high reliabilities, say over 0.95, most of the area under the $R_1$-$R_2$ curves of Fig. 6-32 is at the far right of the interval. To integrate such an area by Simpson's rule could easily require one hundred or more ordinates. Since the work of estimating a "zero" ordinate is as much as that for a significant one, we are tempted to take the following approach:

1. For $0 \leq R_1 \leq 0.9$ in steps of 0.1 find the largest value of $R_1$ for which the ordinate is still zero. Call this $R_1^*$ and begin the integration at this value.

2. Select an interval of $0.8(1 - R_1^*)$ and integrate using 9 ordinates. If the right-hand ordinate in the integration interval is beginning to show some size, say 0.005 or greater, go to step 4.

3. If the right-hand ordinate is not showing sufficient size, make the next interval 0.8 of the remaining interval, integrate using 9 ordinates, then go to step 2.

4. Place 9 ordinates in the remaining interval and integrate.

EXAMPLE 6-7

In Prob. 7-17 it is found that the shear strength (torsional endurance limit) is $\underset{\sim}{S} \sim LN(21.15, 4.06)$ kpsi and the corresponding torsional stress is $\underset{\sim}{\tau} \sim N(9.98, 1.30)$ kpsi. Estimate the reliability.

Solution

First we find $R_1^* = 0.9$. The first interval is $0.8(1 - 0.9) = 0.08$ and $h_1 = 0.08/8 = 0.01$ (see Prob. 1-9). The computations of Table 1, for example, are explained as follows.

TABLE 1

| $R_1$ | $z_1$ | x | $z_2$ | $R_2$ | w |
|---|---|---|---|---|---|
| 0.9 | -1.2817 | 16.23 | 4.8083 | 0 | 1 |
| 0.91 | -1.3406 | 16.05 | 4.6679 | 0 | 4 |
| 0.92 | -1.4053 | 15.85 | 4.5155 | 0 | 2 |
| 0.93 | -1.4757 | 15.64 | 4.3518 | 0 | 4 |
| 0.94 | -1.5550 | 15.40 | 4.1701 | 0 | 2 |
| 0.95 | -1.6450 | 15.14 | 3.9671 | 0 | 4 |
| 0.96 | -1.7511 | 14.83 | 3.7323 | 0.0001 | 2 |
| 0.97 | -1.8814 | 14.47 | 3.4504 | 0.0003 | 4 |
| 0.98 | -2.0540 | 13.99 | 3.0877 | 0.0010 | 1 |

$\Sigma_1 wR_2 = 0.0024$

Col. 1. Enter $R_1$ values.

Col. 2. Enter Table A-10 with tabulation = $R_1$ and read $z_1$. Interpolation is usually necessary so use the program you developed for Prob. 1-2.

Col. 3. Solve Eqs. (6-33), (6-34), and (6-36) to obtain x.

$$\hat{\sigma}_{\ell n\, S} = \frac{\hat{\sigma}_S}{\mu_S} = C_S = \frac{4.06}{21.15} = 0.192$$

$$\mu_{\ell n\, S} = \ell n\, \mu_S - \frac{C_S{}^2}{2} = \ell n\, (21.15) - \frac{0.192)^2}{2} = 3.033$$

For $R_1 = 0.98$

$$x = \ell n^{-1}(z_1\hat{\sigma}_{\ell n\, S} + \mu_{\ell n\, S}) = \ell n^{-1}[-2.0540(0.192) + 3.033]$$
$$= 13.994\ 046\ 65$$

Col. 4. Find $z_2$ from Eq. (6-38).

$$z_2 = \frac{x - \mu_\tau}{\hat{\sigma}_\tau} = \frac{13.994\ 046\ 65 - 9.98}{1.30} = 3.0877$$

Note in this example that $z_2$ was found by program without first rounding off the value of x.. There is a slight difference.

Col. 5. Enter Table A-10 with $z_2$ and read $R_2$ = tabulation.

Col. 6. These are the weightings for Simpson's rule.

The results of Table 1 indicate that we should proceed to step 3. The remaining interval is 1 - 0.98 = 0.02. So Table 2 is prepared using the interval 0.8(0.02) = 0.016. Thus $h_2$ = 0.016/8 = 0.002.

TABLE 2

| $R_1$ | $z_1$ | x | $z_2$ | $R_2$ | w |
|---|---|---|---|---|---|
| 0.98 | -2.0540 | 13.99 | 3.0877 | 0.0010 | 1 |
| 0.982 | -2.0975 | 13.88 | 2.9982 | 0.0014 | 4 |
| 0.984 | -2.1450 | 13.75 | 2.9013 | 0.0018 | 2 |
| 0.986 | -2.1875 | 13.64 | 2.8153 | 0.0024 | 4 |
| 0.988 | -2.2467 | 13.49 | 2.6967 | 0.0035 | 2 |
| 0.99 | -2.3267 | 13.28 | 2.5386 | 0.0056 | 4 |
| 0.992 | -2.4091 | 13.07 | 2.3783 | 0.0087 | 2 |
| 0.994 | -2.5124 | 12.82 | 2.1808 | 0.0146 | 4 |
| 0.996 | -2.6518 | 12.48 | 1.9205 | 0.0274 | 1 |

$\Sigma_2 wR_2 = 0.1524$

Since $R_2$ is now greater than 0.005 the next is the last interval. Thus last interval = 1 - 0.996 = 0.004 and $h_3$ = 0.004/8 = 0.0005.

TABLE 3

| $R_1$ | $z_1$ | x | $z_2$ | $R_2$ | w |
|---|---|---|---|---|---|
| 0.996 | -2.6518 | 12.48 | 1.9205 | 0.0274 | 1 |
| 0.9965 | -2.6970 | 12.37 | 1.8375 | 0.0331 | 4 |
| 0.997 | -2.7478 | 12.25 | 1.7452 | 0.0405 | 2 |
| 0.9975 | -2.8075 | 12.11 | 1.6378 | 0.0507 | 4 |
| 0.998 | -2.8683 | 11.97 | 1.5297 | 0.0630 | 2 |
| 0.9985 | -2.9680 | 11.74 | 1.3551 | 0.0877 | 4 |
| 0.999 | -3.0916 | 11.47 | 1.1433 | 0.1264 | 2 |
| 0.9995 | -3.2917 | 11.03 | 0.8109 | 0.2087 | 4 |
| 1.0000 | | | | 1.0000 | 1 |

$\Sigma_3 wR_2 = 3.008$

$$A = \frac{h_1}{3}\Sigma_1 wR_2 + \frac{h_2}{3}\Sigma_2 wR_2 + \frac{h_3}{3}\Sigma_3 wR_2 = \frac{0.01}{3}(0.0024) + \frac{0.002}{3}(0.1524) + \frac{0.0005}{3}(3.008)$$

$$= 0.000\ 611$$

So the reliability is $R = 1 - A = 1 - 0.000\ 611 \cong 0.9994$ Ans.

////

7-1 From Table A-20, $S_{ut} = 55$ kpsi

$S'_e = 0.504S_{ut} = 0.504(55) = 27.7$ kpsi

$$a = \frac{[0.9(55)]^2}{27.7} = 88.46$$

$$b = -\frac{1}{3}\log\frac{0.9(55)}{27.7} = -0.084$$

$$S_f = aN^b = 88.46(12\ 500)^{-0.084}$$

$$= 40.05 \text{ kpsi} \quad \text{Ans.}$$

$$N = \left(\frac{36}{88.46}\right)^{1/-0.084} = 44.5 \text{ kcycles} \quad \text{Ans.}$$

---

7-2 and 7-3 OMITTED

---

7-4 $\underset{\sim}{S}_{ut} \sim N(734, 42.4)$ MPa

$= 734(1, 0.0578)$ MPa

From Eq. (7-9)

$\underset{\sim}{S}'_e = 0.504(1, 0.146)(734)$

$= 370(1, 0.146)$ MPa  Ans.

$C_{Se} = 0.146, \; C_{Sut} = 0.0578$

$$a = \frac{[0.9(734)]^2}{370} = 1179$$

$$b = -\frac{1}{3}\log\frac{0.9(734)}{370} = -0.0839$$

From Eq. (7-10)

$$C_{Sf} = 2S_{Su} - C_{Se} + \frac{1}{3}(C_{Se} - C_{Su})\log N$$

$$= 2(0.0578) - 0.146$$

$$+ \frac{1}{3}(0.146 - 0.0578)\log 130\ 000$$

$$= 0.1200$$

$$\bar{S}_f = aN^b = 1179(130\ 000)^{-0.0839}$$

$$= 439 \text{ MPa}$$

$$\sigma_{Sf} = C_{Sf}\bar{S}_f = 0.1200(439) = 52.7 \text{ MPa}$$

So $\underset{\sim}{S}_f = 439(1, 0.1200)$ MPa  Ans.

---

7-5 From Eq. (5-20) we have

$S_u = 0.45H_B = 0.45(490) = 220.5$ kpsi

From Eq. (7-4) $S'_e = 100$ kpsi

From Table 7-4 $a = 1.34$, $b = -0.085$

$$k_a = 1.34(220.5)^{-0.085} = 0.847$$

From Eq. (7-15)

$$k_b = \left(\frac{0.1875}{0.3}\right)^{-0.1133} = 1.055$$

Since the remaining factors are unity

$S_e = 0.847(1.055)(100) = 89.4$ kpsi Ans.

---

7-6 From Eq. (5-23) we have

$\underset{\sim}{S}_u = (0.5, 0.022)\underset{\sim}{H}_B = 0.5(1, 0.044)\underset{\sim}{H}_B$

$\bar{S}_u = 0.5(490) = 245$ kpsi

Table 4-4: $C_{xy} = (C_x{}^2 + C_y{}^2)^{\frac{1}{2}}$

So $C_{Su} = [(0.03)^2 + (0.044)^2]^{\frac{1}{2}} = 0.0533$

$\underset{\sim}{S}_u = 245(1, 0.0533)$ kpsi

Eq. (7-4): $\underset{\sim}{S}_e = 100(1, 0.146)$ kpsi

Tables 7-4, 7-6: $a = 1.34$, $b = -0.085$,

$C = 0.13$

$$\underset{\sim}{k}_a = a(1, C)\bar{S}_u{}^b$$

$$= 1.34(1, 0.13)(245)^{-0.085}$$

$$= 0.840(1, 0.13)$$

$k_b = 1.055$ as in Prob. 7-5

We now have

$$\underset{\sim}{S}_e = [0.840(1, 0.13)](1.055)[100(1,0.146)]$$

So $\bar{S}_e = 0.840(1.055)(100) = 88.6$ kpsi

$C_{Se} = [(0.13)^2 + (0.146)^2]^{\frac{1}{2}} = 0.196$

$\underset{\sim}{S}_e = 88.6(1, 0.196)$ kpsi  Ans.

---

7-7 Eq. (7-4): $S'_e = 710(0.504)$

$= 357.8$ MPa

Table 7-4: $a = 4.51$, $b = -0.265$

Eq. (7-14): $k_a = 4.51(357.8)^{-0.265}$

$= 0.949$

Eq. (7-15): $k_b = (d/7.62)^{-0.1133}$

$= (32/7.62)^{-0.1133}$

$= 0.850$

7-7 (Concluded)

All other factors are unity, and so

$S_e = 0.949(0.850)(357.8) = 288.6$ MPa Ans

---

7-8 OMITTED

---

7-9 Table A-20: $S_{ut} = 440$ MPa,

$S_{yt} = 370$ MPa. Therefore

$S'_e = 0.504(440) = 221.76$ MPa

Table 7-4: $a = 4.51$, $b = -0.265$

$k_a = 4.51(440)^{-0.265} = 0.899$

Eq. (7-16): $k_b = 1$

Eq. (7-22): $k_c = 0.923$

Eq. (7-13): $S_e = 0.899(1)(0.923)(221.76)$

$= 184$ MPa

Table A-15-1: $d/w = 12/60 = 0.2$, $K_t = 2.5$

Fig. 5-16: $q = 0.78$ (end of chart)

$K_f = 1 + 0.78(2.5 - 1) = 2.17$

$$\sigma = K_f \frac{F}{A} = 2.17 \frac{F}{10(60-12)} = \frac{F}{221} \text{ MPa}$$

Then $\frac{S_e}{n_d} = \sigma$ or $\frac{184}{1.8} = \frac{F}{221}$

So $F = 22.6$ kN Ans.

Based on yielding

$$\frac{S_y}{n_d} = \frac{F}{A} \text{ or } \frac{370}{1.8} = \frac{F}{10(60-12)}$$

or $F = 98.7$ kN

---

7-10 Table A-21: $\bar{S}_{ut} = 1090$ MPa,

$\bar{S}_{yt} = 793$ MPa

Eq. (7-9): $\underset{\sim}{S}'_e = 0.504(1, 0.146)(1090)$

$= 549(1, 0.146)$ MPa

Tables 7-4 and 7-6: $a = 1.58$, $b = -0.085$

$C = 0.13$

$\underset{\sim}{k}_a = 1.58(1, 0.13)(549)^{-0.085}$

$= 0.924(1, 0.13)$

Eq. (7-19): $d_e = 0.808[20(10)]^{1/2}$

$d_e = 11.43$ mm

Eq. (7-15): $k_b = \left(\frac{11.43}{7.62}\right)^{-0.1133}$

$= 0.955$

The remaining factors are unity and so

Eq. (7-27):

$\underset{\sim}{S}_e = 0.924(1, 0.13)(0.955)[549(1, 0.146)]$

$\bar{S}_e = 484.4$ MPa

$C_S = [(0.13)^2 + (0.146)^2]^{1/2} = 0.196$

Table A-15-6: $D/d = 30/20 = 1.5$,

$r/d = 1/20 = 0.05$, $K_t = 2.4$

Fig. 5-16: $q = 0.85$

Table 5-5: $C = 0.08$

$\bar{K}_f = 1 + 0.85(2.4 - 1) = 2.19$

or $\underset{\sim}{K}_f = 2.19(1, 0.08)$

$$\frac{I}{c} = \frac{bh^2}{6} = \frac{1(2)^2}{6} = 0.667 \text{ cm}^3$$

$$\underset{\sim}{\sigma} = \underset{\sim}{K}_f \frac{F\ell}{I/c} = 2.19(1, 0.08)\left(\frac{75F}{0.667}\right)$$

$= 246F(1, 0.08)$ MPa

where F is in kN.

With $R = 0.5$, $z = 0$, and the coupling equation is simply $\mu_\sigma = \mu_{Se}$.

Or $246F = 484.4$ from which $F = 1.97$ kN. Any C. O. V. in $\underset{\sim}{F}$ results in 50 percent reliability as long as the mean of $\underset{\sim}{F}$ is 1.97 kN.

---

7-11 See diagrams on next page. The most dangerous section is at the fillet joining the 40- and 48-mm sections at C. Here $M_C = 2.9(200) + 0.8(62.5)$

$= 630$ N·m

$S'_e = 0.504(610) = 307$ MPa

**7-12** (Concluded)

$$b = -\frac{1}{3}\log\frac{0.9(89)}{18.2} = -0.2145$$

$$\text{Eq. (7-7): } N = \left(\frac{46.29}{352.5}\right)^{1/-0.2145}$$

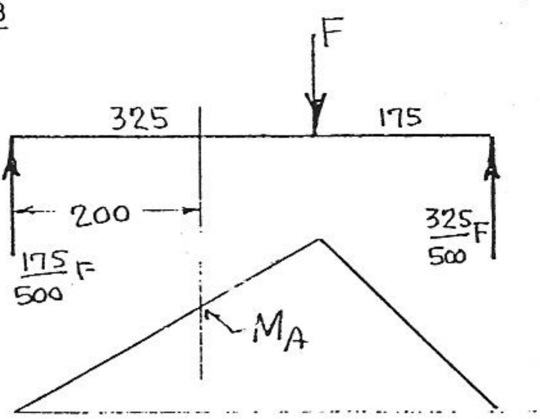
$$= 12\,893 \text{ cycles}$$

At 1720 rev/min

$$\text{life } t = \frac{12\,893}{1720} = 7.5 \text{ min} \quad \underline{\text{Ans.}}$$

---

**7-13**




$$\text{Eq. (7-9): } \underset{\sim}{S}'_e = 0.504(1,\ 0.146)(550)$$

$$= 277.2(1,\ 0.146) \text{ MPa}$$

Table 7-4: $a = 4.51$, $b = -0.265$

Table 7-6" $C = 0.06$

$$\underset{\sim}{k}_a = 4.51(1,\ 0.06)(550)^{-0.265}$$

$$= 0.847(1,\ 0.06)$$

$$\text{Eq. (7-15): } k_b = \left(\frac{35}{7.62}\right)^{-0.1133} = 0.841$$

The remaining factors are unity and so

$$\underset{\sim}{S}_e = 0.847(1,\ 0.06)(0.841)[277.2(1,.146)]$$

$$\bar{S}_e = 197.46 \text{ MPa}$$

$$C_S = [(0.06)^2 + (0.146)^2]^{\frac{1}{2}} = 0.1581$$

and so $\underset{\sim}{S}_e = 197.46(1,\ 0.1581)$ MPa

$$\underset{\sim}{M} = \frac{175}{500}(200)\underset{\sim}{F};\quad \underset{\sim}{F} = 6(1,\ 0.01667) \text{ kN}$$

$$\underset{\sim}{M} = 420(1,\ 0.01667) \text{ N·m}$$

Fig. A-15-9: $K_t = 1.72$

Fig. 5-16: $q = 0.8$

$$K_f = 1 + 0.8(1.72 - 1) = 1.58$$

Table 5-5: $C_K = 0.08$

$$\underset{\sim}{K}_f = 1.58(1,\ 0.08)$$

$$\frac{I}{c} = \frac{\pi(3.5)^3}{32} = 4.209 \text{ cm}^3$$

$$\underset{\sim}{\sigma} = \underset{\sim}{K}_f\frac{\underset{\sim}{M}}{I/c}$$

$$= 1.58(1,\ 0.08)\frac{420(1,\ 0.01667)}{4.209}$$

$$\bar{\sigma} = \frac{1.58(420)}{4.209} = 157.7 \text{ MPa}$$

$$C_\sigma = [(0.08)^2 + (0.01667)^2]^{\frac{1}{2}} = 0.0817$$

So $\underset{\sim}{\sigma} = 158(1,\ 0.0817)$ MPa

Eq. (6-32): $\hat{\sigma}_{\ln S} = C_S = 0.1581$

$$\hat{\sigma}_{\ln\sigma} = C_\sigma = 0.0817$$

Eq. (6-33):

$$\hat{\mu}_{\ln S} = \ln\hat{\mu}_S - \frac{1}{2}C_S^{\,2}$$

$$= \ln 197.46 - \frac{(0.1581)^2}{2} = 5.2730$$

$$\hat{\mu}_{\ln\sigma} = \ln\hat{\mu}_\sigma - \frac{C_\sigma^2}{2}$$

$$= \ln 158 - \frac{0.0817)^2}{2} = 5.059$$

Eq. (6-34):

$$z = -\frac{5.2730 - 5.0592}{[(0.1581)^2 + (0.0817)^2]^{\frac{1}{2}}}$$

$$= -1.2014$$

Table A-10:

| | | | |
|---|---|---|---|
| $z$ | 1.20 | 1.2014 | 1.21 |
| $\Phi(z)$ | 0.1151 | <u>0.1148</u> | 0.1131 |

$$R = 1 - 0.1148 = 0.8852 \quad \underline{\text{Ans.}}$$

---

**7-14** $S'_e = 0.504(120) = 60.5$ kpsi

Table 7-4: $a = 2.70$, $b = -0.265$

$$k_a = 2.70(120)^{-0.265} = 0.759$$

<u>1st trial</u> Use $d = 2$ in

$$k_b = \left(\frac{2}{0.3}\right)^{-0.1133} = 0.807$$

Fig. A-15-9: $D/d = 1.5$, $r/d = 0.10$,

$K_t = 1.68$

7-14 (Concluded)

Fig. 5-16: $q = 0.86$ (at end of chart)

Eq. (5-26): $K_f = 1 + 0.86(1.68 - 1)$

$= 1.58$

$k_c = k_d = 1;\ k_e = 1/1.58 = 0.633$

$S_e = 0.759(0.807)(0.633)(60.5) = 23.5$ kpsi

Life: $N = 75(1150) = 86\ 250$ cycles

Eq. (7-6): $a = \dfrac{[0.9(120)]^2}{23.5} = 496$

$b = -\dfrac{1}{3}\log\dfrac{0.9(120)}{23.5} = -0.221$

Eq. (7-5): $S_f = 496(86\ 250)^{-0.221}$

$= 40.2$ kpsi

$\dfrac{I}{c} = \dfrac{\pi d^3}{32} = 0.09817d^3$

At the central shoulder

$M = \dfrac{6}{24}(10)(12) = 30$ kip·in

$\sigma = \dfrac{M}{I/c} = \dfrac{30}{0.09817d^3} = \dfrac{305.6}{d^3}$

Using $S_f/n = \sigma$ we have

$\dfrac{40.2}{1.60} = \dfrac{305.6}{d^3}$ or $d = 2.30$ in

For the next trial, choose $d = 2\ 3/8$ in

$k_b = \left(\dfrac{2.375}{0.3}\right)^{-0.1133} = 0.791$

$r = 0.2375;\ q = 0.86$ at end of chart

$K_f = 1 + .86(1.68 - 1) = 1.58$

$k_e = 1/1.58 = 0.633$

$S_e = 0.759(0.791)(0.633)(60.5) = 23.0$ kpsi

$a = \dfrac{[0.9(120)]^2}{23.0} = 507$

$b = -\dfrac{1}{3}\log\dfrac{0.9(120)}{23.0} = -0.224$

$S_f = 507(86\ 250)^{-0.224} = 39.8$ kpsi

$\sigma = \dfrac{305.6}{(2.375)^3} = 22.8$ kpsi

$n = \dfrac{39.8}{22.8} = 1.75$ which is ok.

---

7-15 Rotation assumed. Eq. (7-9):

$\mathbf{S}'_e = 0.504(1,\ 0.146)(110)$

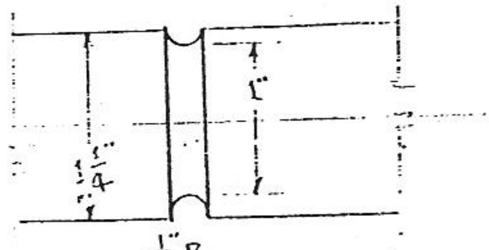
$= 55.44(1,\ 0.146)$ kpsi

Tables 7-4 and 7-6: $a = 2.7$, $b = -0.265$,

$C = 0.06$

$k_a = 2.7(110)^{-0.265} = 0.777$

$\mathbf{k}_a = 0.777(1,\ 0.06)$




Base $k_b$ on $D = 1.25$ in

Eq. (7-15): $k_b = \left(\dfrac{1.25}{0.3}\right)^{-0.1133} = 0.851$

$\mathbf{S}_e = 0.777(1,\ 0.06)(0.851)[55.44(1,\ .146)]$

$\bar{S}_e = 36.7$ kpsi

$C_S = [(0.06)^2 + (0.146)^2] = 0.158$

$\hat{\sigma}_{Se} = 0.158(36.7) = 5.80$ kpsi

So $\mathbf{S}_e = 36.7(1,\ 0.158)$ kpsi

Fig. A-15-14: $D/d = 1.25$, $r/d = 0.125$

$K_t = 1.70$

Fig. 5-16: $q = 0.84$

Eq. (5-26): $K_f = 1 + 0.84(1.70 - 1)$

$= 1.59$

Table 5-5: $C_K = 0.13$

So $\mathbf{K}_f = 1.59(1,\ 0.13)$

$\boldsymbol{\sigma} = \mathbf{K}_f \dfrac{32M}{\pi d^3}$

$= 1.59(1,\ 0.13)\left[\dfrac{32(1400)}{\pi(1)^3}(10^{-3})\right]$

7-15 (Concluded)

$\underset{\sim}{\sigma} = 22.7(1,\ 0.13)$ kpsi

$\hat{\sigma}_\sigma = 22.7(0.13) = 2.95$ kpsi

$$z = -\frac{\mu_S - \mu_\sigma}{(\hat{\sigma}_S^2 + \hat{\sigma}_\sigma^2)^{\frac{1}{2}}}$$

$$= -\frac{36.7 - 22.7}{[(5.80)^2 + (2.95)^2]^{\frac{1}{2}}}$$

$= -2.1515$

From Table A-10 by interpolation

$F(z) = 0.157$

So $R = 1 - 0.0157 = 0.9843$ Ans.

No rotation

---

7-16 $\underset{\sim}{S}'_e = 0.504(1,\ 0.146)(58)$

$= 29.2(1,\ 0.146)$ kpsi

Tables 7-4 and 7-6: $a = 14.4$,

$b = -0.718$, $C = 0.11$

$\underset{\sim}{k}_a = 14.4(1,\ 0.11)(58)^{-0.718}$

$= 0.780(1,\ 0.11)$

Eq. (7-18): $d_e = 0.370(1.25) = 0.462$ in

Eq. (7-15): $k_b = \left(\frac{0.462}{0.3}\right)^{-0.1133} = 0.952$

$\underset{\sim}{S}_e = 0.780(1,\ 0.11)(0.952)[29.2(1,\ 0.146)]$

$\bar{S}_e = 21.7$ kpsi

$C_S = [(0.11)^2 + (0.146)^2]^{\frac{1}{2}} = 0.183$

$\hat{\sigma}_{Se} = 21.7(0.183) = 3.97$ kpsi

Table A-16: $d/D = 0$, $a/D = 0.1$; $A = 0.83$

$K_t = 2.27$, $C_K = 0.11$

Fig. 5-16: $q = 0.69$

$K_f = 1 + 0.69(2.27 - 1) = 1.88$

Eq. (5-26): $\underset{\sim}{K} = 1.88(1,\ 0.11)$

Table A-16:

$$Z = \frac{\pi A D^3}{32} = \frac{\pi(0.83)(1.25)^3}{32} = 0.159 \text{ in}^3$$

$$\underset{\sim}{\sigma} = \underset{\sim}{K}_f \frac{M}{Z} = 1.88(1,\ 0.11)\left(\frac{1.6}{0.159}\right)$$

$= 18.9(1,\ 0.11)$ kpsi

$\hat{\sigma}_\sigma = 18.9(0.11) = 2.08$ kpsi

$$z = -\frac{21.7 - 18.9}{[(3.97)^2 + (2.08)^2]^{\frac{1}{2}}} = -0.6247$$

Table A-10:

| | | | |
|---|---|---|---|
| z | 0.62 | 0.6247 | 0.63 |
| f(z) | 0.2676 | 0.2660 | 0.2643 |

$R = 1 - 0.2660 = 0.734$ Ans.

---

7-17 Eq. (7-29): $\underset{\sim}{k}_c = 0.577(1,\ 0.11)$

Both $\underset{\sim}{k}_a$ and $k_b$ are unchanged from Prob. 7-15, and so

$\bar{S}_{se} = k_a k_b k_c S'_e$

$= 0.777(0.851)(0.577)(55.44)$

$= 21.15$ kpsi

and $C_S = [0.06)^2 + (0.146)^2 + (0.11)^2]^{\frac{1}{2}}$

$= 0.192$

$\hat{\sigma}_{se} = 21.15(0.192) = 4.06$ kpsi

Note that there are three variates in $\underset{\sim}{S}_{se}$. This means that the distribution tends toward lognormal. See Sec. 6-14.

Fig. A-15-15: $D/d = 1.25$, $r/d = 0.125$,

$K_{ts} = 1.40$

Table 5-5: $C_K = 0.13$

So $\underset{\sim}{K}_{ts} = 1.40$

Note, from Fig. 5-17, that $q \cong 1$.

So $\underset{\sim}{K}_{ts} = 1.40(1,\ 0.13)$

Then

$$\underset{\sim}{\tau} = \underset{\sim}{K}_{ts} \frac{16T}{\pi d^3}$$

$$= 1.40(1,\ 0.13)\ \frac{16(1.4)}{\pi(1)^3}$$

$= 9.98(1,\ 0.13)$ kpsi

$\hat{\sigma}_\tau = 9.98(0.13) = 1.30$ kpsi

Since the stress has only one variate, it is likely to have a normal distribution. We therefore have a LN-N interference problem. (See Table 18-2).

7-17 (Concluded)

Summarizing:

$\underset{\sim}{S}_{se} \sim LN(21.15, 4.06)$ kpsi

$\underset{\sim}{\tau} \sim N(9.98, 1.30)$ kpsi

The solution begins on page 97. It is found that R = 0.9994 Ans.

---

7-18 and 7-19 OMITTED

---

7-20 Table A-20: $S_{ut} = 64$ kpsi, $S_{yt} = 54$ kpsi

$A = 0.375(1 - 0.25) = 0.281\ in^2$

$$\sigma = \frac{F_{max}}{A} = \frac{3000(10^{-3})}{0.281} = 10.67 \text{ kpsi}$$

For yielding

$n = 54/10.67 = 5.06$ Ans.

$S'_e = 0.504(64) = 32.3$ kpsi

Table 7-4: a = 2.7, b = -0.265

$k_a = 2.7(64)^{-0.265} = 0.897$

Eq. (7-16: $k_b = 1$

Eq. (7-22(: $k_c = 0.923$

$S_e = 0.897(1)(0.923)(32.3) = 26.7$ kpsi

Fig. A-15-1: w = 1 in, d = ¼ in,

d/w = 0.25, $K_t = 2.45$

Fig. 5-16: q = 0.78 (end of chart)

$K_f = 1 + 0.78(2.45 - 1) = 2.13$

$$\sigma_a = K_f \frac{F_{max} - F_{min}}{2A}$$

$$= 2.13 \frac{3.000-0.800}{2(0.281)} = 8.34 \text{ kpsi}$$

$$\sigma_m = \frac{F_{max} + F_{min}}{2A}$$

$$= \frac{3.000 + .800}{2(0.281)} = 6.76 \text{ kpsi}$$

These two components vary in the same ratio because they both result from the same force. Therefore Eq. (7-39) applies, and

$$n = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} = \frac{S_e S_{ut}}{\sigma_a S_{ut} + \sigma_m S_e}$$

$$= \frac{26.7(64)}{8.34(64) + 6.76(26.7)} = 2.39 \quad \text{Ans.}$$

---

7-21

$$\sigma_a = K_f \frac{F_{max} - F_{min}}{2A}$$

$$= 2.13 \frac{3000 - (-800)}{2(0.281)}(10^{-3}) = 14.4 \text{ kpsi}$$

$$\sigma_m = \frac{F_{max} + F_{min}}{2A}$$

$$= \frac{3000 + (-800)}{2(0.281)}(10^{-3}) = 3.91 \text{ kpsi}$$

Based on fatigue:

$$n = \frac{26.7(64)}{14.4(64) + 3.91(26.7)} = 1.67 \quad \text{Ans.}$$

Based on yielding:

$n = 54/10.67 = 5.06$ Ans.

---

7-22

$$\sigma_a = K_f \frac{F_{max} - F_{min}}{2A}$$

$$= 2.13 \frac{800 - (-3000)}{2(0.281)}(10^{-3})$$

$$= 14.4 \text{ kpsi}$$

$$\sigma_m = \frac{F_{max} + F_{min}}{2A}$$

$$= \frac{800 + (-3000)}{2(0.281)}(10^{-3}) = -3.91 \text{ kpsi}$$

This is a compressive mean stress and so

$$n = \frac{S_e}{\sigma_a} = \frac{26.7}{14.4} = 1.85 \quad \text{Ans.}$$

---

7-23 $S'_e = 0.504(1400) = 706$ MPa

Table 7-4: a = 272, b = -0.995

$k_a = 272(1400)^{-0.995} = 0.201$

Eq. (7-18): $d_e = 0.808(hb)^{\frac{1}{2}}$

$= 0.808[75(18)]^{\frac{1}{2}} = 29.7$

7-23 (Continued) Eq. (7-23):

$$k_b = \left(\frac{29.7}{7.62}\right)^{-0.1133} = 0.857$$

All other k's are unity, and so

$$S_e = 0.201(0.857)(706) = 122 \text{ MPa}$$

Fig. A-15-2: $K_t = 2.18$

Also q = 0.95 (end of chart)

$$K_f = 1 + 0.95(2.18 - 1) = 2.12$$

For the stresses

$$\sigma_0 = \frac{6M}{h^2(w-d)} = \frac{6M}{(1.8)^2(7.5-1)} = 0.285M \text{ MPa}$$

where M is in N·m, and is

$$M = (F/2)(150) = 75F$$

The preload stress is

$$\sigma_p = 0.285(75)(9.36) = 200 \text{ MPa}$$

The maximum (nominal) stress is

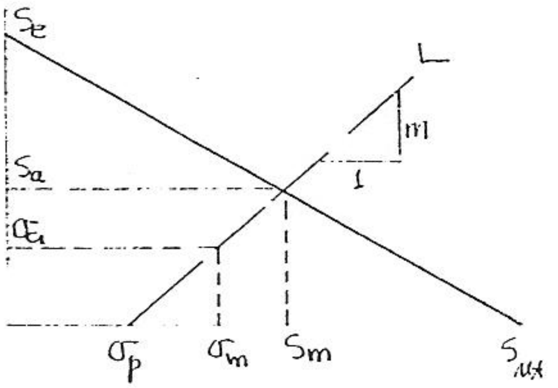
$$\sigma_{max} = 0.285(75)(10.67) = 228 \text{ MPa}$$

The alternating component is

$$\sigma_a = K_f \frac{\sigma_{max} - \sigma_p}{2} = 2.12\, \frac{228 - 200}{2} = 29.7 \text{ MPa}$$

The mean is $\sigma_m = \dfrac{\sigma_{max} + \sigma_p}{2}$

$$= \frac{228 + 200}{2} = 214 \text{ MPa}$$




$$n = \frac{S_a}{\sigma_a} = \frac{S_m - \sigma_p}{\sigma_m - \sigma_p} \quad ; \quad m = \frac{\sigma_a}{\sigma_m - \sigma_p}$$

Goodman line: $S_m = S_{ut}\left(1 - \dfrac{S_a}{S_e}\right)$

Load line: $m = \dfrac{S_a}{S_m - \sigma_p}$

$$\text{So } S_a = m(S_m - \sigma_p) = mS_{ut}\left(1 - \frac{S_a}{S_e}\right) - m\sigma_p$$

$$= m(S_{ut} - \sigma_p) - \frac{mS_aS_{ut}}{S_e}$$

Then

$$S_a + \frac{mS_aS_{ut}}{S_e} = m(S_{ut} - \sigma_p)$$

$$S_a\left(1 + \frac{mS_{ut}}{S_e}\right) = m(S_{ut} - \sigma_p)$$

$$S_a = \frac{m(S_{ut} - \sigma_p)}{1 + \dfrac{mS_{ut}}{S_e}}$$

Here $m = \dfrac{\sigma_a}{\sigma_m - \sigma_p} = \dfrac{29.7}{214 - 200} = 2.12 = K_f$

Then $S_a = \dfrac{2.12(1400 - 200)}{1 + \dfrac{2.12(1400)}{122}} = 100$ MPa

Thus $n = S_a/\sigma_a = 100/29.7 = 3.37$ Ans.

---

7-24 Eq. (5-20):

$$S_{ut} = 0.45H_B = 0.45(380) = 171 \text{ kpsi}$$

$$S'_e = 0.504(171) = 86.2 \text{ kpsi}$$

Table 7-4: a = 14.4, b = −0.718

$$k_a = 14.4(171)^{-0.718} = 0.359$$

Eq. (7-18): $d_e = 0.370D = 0.370(0.375) = 0.139$

Eq. (7-23): $k_b = (0.139/0.3)^{-0.1133} = 1.09$

So $S_e = 0.359(1.09)(86.2) = 33.7$ kpsi

$$F_a = \frac{30 - 15}{2} = 7.5 \text{ lb}$$

$$F_m = \frac{30 + 15}{2} = 22.5 \text{ lb}$$

$$\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(22.5)(16)(10^{-3})}{\pi(0.375)^3} = 69.5 \text{ kpsi}$$

$$\sigma_a = \frac{7.5}{22.5}\sigma_m = \frac{7.5}{22.5}(69.5) = 23.2 \text{ kpsi}$$

Now, assuming a finite life, we rewrite

7-24 (Concluded)

Eq. (7-35) as

$$\frac{S_a}{S_f} + \frac{S_m}{S_{ut}} = 1; \text{ let } S_a = \sigma_a \text{ and } S_m = \sigma_m$$

Then

$$S_f = \frac{S_a}{1 - \dfrac{S_m}{S_{ut}}} = \frac{23.2}{1 - \dfrac{69.5}{171}} = 39.3 \text{ kpsi}$$

Eq. (7-6): $a = \dfrac{[0.9(171)]^2}{33.7} = 703$

$b = -\dfrac{1}{3} \log \dfrac{0.9(171)}{33.7} = -0.220$

Eq. (7-7): $N = (39.3/703)^{1/-0.220}$

$= 494$ kcycles  Ans.

---

7-25 (a) $I = \dfrac{bh^3}{12} = \dfrac{18(3)^3}{12} = 40.5 \text{ mm}^4$

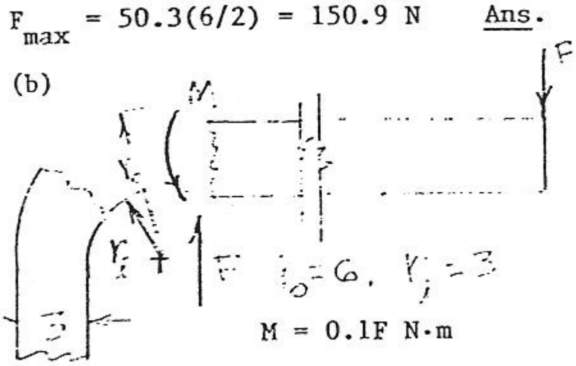
Since $y = \dfrac{F\ell^3}{3EI}$, $F = \dfrac{3EIy}{\ell^3}$

$$F_p = F_{min} = \frac{3(207)(40.5)(2)(10^3)}{(100)^3}$$

$= 50.3$ N  Ans.

$F_{max} = 50.3(6/2) = 150.9$ N  Ans.

(b)




M = 0.1F N·m

From curved-beam theory

$$r = \frac{h}{\ln \dfrac{r_o}{r_i}} = \frac{3}{\ln 2} = 4.328 \text{ mm}$$

$\bar{r} = 4.5$ mm, so $e = \bar{r} - r = 4.5 - 4.328$

$= 0.172$ mm

$c_o = 1.5 + 0.172 = 1.672$ mm

$c_i = 1.5 - 0.172 = 1.328$ mm

$$\sigma_i = -\frac{Mc_i}{Aer_i} = -\frac{0.1F(0.1328)}{0.3(1.8)(0.0172)(0.3)}$$

$= -4.766F$ MPa

Note that all dimensions have been transformed to centimeters in this computation. Similarly,

$$\sigma_o = \frac{Mc_o}{Aer_o} = \frac{0.1F(0.1672)}{0.3(1.8)(0.0172)(0.6)}$$

$= 3.000F$ MPa

$\sigma_{i,min} = -4.766(50.3) = -239.7$ MPa

$\sigma_{i,max} = -4.766(150.9) = -719.2$ MPa

$\sigma_{o,min} = 3(50.3) = 150.9$ MPa

$\sigma_{o,max} = 3(150.9) = 452.7$ MPa

Inner radius, yielding

Eq. (5-20): $S_u = 3.10H_B = 3.10(490)$

$= 1519$ MPa

Here we estimate that $S_y = 0.9S_u$, so

$S_y = 0.9(1519) = 1367$ MPa

$$n = \frac{S_{yc}}{\sigma_{max}} = \frac{-1367}{-719.2} = 1.90 \quad \text{Ans.}$$

The compressive mean stress has no effect on the endurance limit.

$$\sigma_a = \left| \frac{\sigma_{max} - \sigma_{min}}{2} \right| = \frac{-719.2 - (-239.7)}{2}$$

$= 239.8$ MPa

Eq. (7-4): $S'_e = 700$ MPa

Table 7-4: $a = 1.58$, $b = -0.085$

$k_a = 1.58(1519)^{-0.085} = 0.848$

Eq. (7-19): $d_e = 0.808[3(18)]^{\frac{1}{2}} = 5.94$ mm

Eq. (7-15): $k_b = \left(\dfrac{5.94}{7.62}\right)^{-0.1133} = 1.03$

$S_e = 0.848(1.03)(700) = 611.4$ MPa

So $n = \dfrac{611.4}{239.7} = 2.55$  Ans.

Outer radius

$$n_{static} = \frac{S_{yt}}{\sigma_{max}} = \frac{1367}{452.7} = 3.02 \quad \text{Ans.}$$

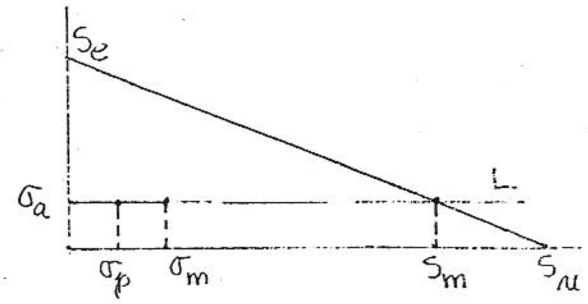
$S_e = 611.4$ MPa unchanged

$$\sigma_a = \frac{452.7 - 150.9}{2} = 150.9 \text{ MPa}$$

$$\sigma_m = \frac{452.7 + 150.9}{2} = 301.8 \text{ MPa}$$

$\sigma_p = 150.9$ MPa

7-25 (Concluded)




Failure occurs when $\sigma_m = S_m$ since $S_a = \sigma_a$.

Eq. (7-35): $S_m = S_{ut}\left(1 - \frac{S_a}{S_e}\right)$

$$= 1519\left(1 - \frac{150.9}{611.4}\right)$$

$$= 1144 \text{ MPa}$$

So $n = \frac{S_m}{\sigma_m} = \frac{1144}{301.8} = 3.79$ Ans.

---

7-26 Table A-20: $S_{ut} = 64$ kpsi

$S_{yt} = 54$ kpsi for cold drawn steel.

$S'_e = 0.504(64) = 32.3$ kpsi

Table 7-4: $a = 2.7$, $b = -0.265$

$k_a = 2.7(64)^{-0.265} = 0.897$

$k_b = 1$, $k_c = 0.923$ (axial)

$S_e = 0.897(1)(0.923)(32.3) = 26.7$ kpsi

At the fillet

$$\sigma_c = \frac{F_{max}}{A} = \frac{-16}{2.5(0.5)} = -12.8 \text{ kpsi}$$

$$n_{static} = \frac{-S_y}{\sigma_c} = \frac{-54}{-12.8} = 4.22 \quad \text{Ans.}$$

Fig. A-15-5: $D = 3.75$, $d = 2.5$,

$D/d = 3.75/2.5 = 1.5$, $r/d = 0.25/2.5$,

$r/d = 0.1$, $K_t = 2.1$

$q = 0.78$, $K_f = 1 + 0.78(2.1 - 1) = 1.86$

$$\sigma_{max} = \frac{4}{1.25} = 3.2 \text{ kpsi}$$

$$\sigma_{min} = \frac{-16}{1.25} = -12.8 \text{ kpsi}$$

$$\sigma_a = K_f\left|\frac{\sigma_{max} - \sigma_{min}}{2}\right|$$

$$\sigma_a = 1.86\left|\frac{3.2-(-12.8)}{2}\right| = 14.88 \text{ kpsi}$$

$$n = \frac{S_e}{\sigma_a} = \frac{26.7}{14.88} = 1.80 \quad \text{Ans. at fillet}$$

At hole $d/w = 0.75/3.75 = 0.20$

Fig. A-15-1: $K_t = 2.5$

$q = 0.78$, $K_f = 1 + 0.78(2.5 - 1) = 2.17$

$$\sigma_{max} = \frac{4}{0.5(3.75 - 0.75)} = 2.67 \text{ kpsi}$$

$$\sigma_{min} = \frac{-16}{0.5(3)} = -10.67 \text{ kpsi}$$

$$\sigma_a = 2.17\left|\frac{2.67-(-10.67)}{2}\right| = 14.47 \text{ kpsi}$$

$$n = \frac{26.7}{14.47} = 1.85 \quad \text{Ans.}$$

So failure would likely occur at the fillet.

---

7-27 (a) $\sigma'_a = 172$ MPa,

$\sigma'_m = [3(103)^2]^{\frac{1}{2}} = 178$ MPa

Eq. (7-45): $n = \frac{S_e S_{ut}}{\sigma_a S_{ut} + \sigma_m S_e}$

$$= \frac{276(551)}{172(551) + 178(276)}$$

$$= 1.06 \quad \text{Ans.}$$

(b) $S_{se} = 0.577S_e = 0.577(276) = 159$ MPa

Fatigue: $n = \frac{S_{se}}{\tau_a} = \frac{159}{69} = 2.30$ Ans.

Static: $n = \frac{S_{sy}}{\tau_{max}} = \frac{0.577(413)}{138 + 69} = 1.15$ Ans.

(c) $\sigma'_m = [3(103)^2]^{\frac{1}{2}} = 178$ MPa

$\sigma'_a = [(83)^2 + 3(69)^2]^{\frac{1}{2}} = 146$ MPa

Eq. (7-45): $n = \frac{276(551)}{146(551) + 178(276)}$

$$= 1.17 \quad \text{Ans.}$$

$\sigma'_{max} = [(83)^2 + 3(103 + 69)^2]^{\frac{1}{2}} = 309$ MPa

Static: $n = \frac{413}{309} = 1.34$ Ans.

7-27 (Concluded)

(d) $n = \dfrac{S_{se}}{\tau_a} = \dfrac{159}{207}$ No safety

Since $S_{su}$ is unknown we shall use the $S_e$-$S_{ut}$ instead of $S_{se}$-$S_{su}$ relations.

Thus $\sigma'_a = [3(207)^2]^{\frac{1}{2}} = 359$ MPa

Eq. (7-6): $a = \dfrac{[0.9(551)]^2}{276} = 891$

$b = -\dfrac{1}{3}\log\dfrac{0.9(551)}{276} = -0.0848$

Eq. (7-7): $N = (10^{-3})\left(\dfrac{359}{891}\right)^{1/-0.0848}$

$= 45.2$ kcycles  Ans.

(e) $\tau_a = 103$ MPa, $\sigma_m = 103$ MPa tension

$\sigma'_a = [3(103)^2]^{\frac{1}{2}} = 178$ MPa

$n = \dfrac{276(551)}{178(551) + 103(276)} = 1.20$  Ans.

---

7-28 OMITTED

---

7-29

$\sigma_{max} = \dfrac{4F_{max}}{\pi d^2} = \dfrac{4(15)}{\pi(1.25)^2} = 12.2$ kpsi

$\tau_{max} = \dfrac{16T_{max}}{\pi d^3} = \dfrac{16(3)}{\pi(1.25)^3} = 7.82$ kpsi

$\sigma' = [(12.2)^2 + 3(7.82)^2]^{\frac{1}{2}} = 18.2$ kpsi

Static $n = 112/18.2 = 6.15$  Ans.

$S'_e = 0.504(148) = 74.6$ kpsi

Table 7-4: $a = 1.34$, $b = -0.085$

$k_a = 1.34(148)^{-0.085} = 0.876$

Eq. (7-15): $k_b = (1.25/0.3)^{-0.1133}$

$= 0.851$

$S_e = 0.876(0.851)(74.6) = 55.6$ kpsi

$\sigma_{min} = \dfrac{4(2)}{\pi(1.25)^2} = 1.63$ kpsi

$\sigma_{a,0} = \dfrac{1}{k_{c,ax}}\dfrac{\sigma_{max} - \sigma_{min}}{2}$

$= 1.083\,\dfrac{12.2 - 1.63}{2} = 5.72$ kpsi

$\sigma_m = \dfrac{12.2 + 1.63}{2} = 6.92$ kpsi

$\tau_{min} = \dfrac{16(-0.3)}{\pi(1.25)^3} = -0.782$ kpsi

$\tau_{a,0} = \dfrac{\tau_{max} - \tau_{min}}{2} = \dfrac{7.82 - (-0.782)}{2}$

$= 4.30$ kpsi

Fig. A-15-8: $D/d = 1.75/1.25 = 1.4$,

$r/d = 0.125/1.25 = 0.10$, $K_{ts} = 1.42$

Fig. 5-17: $q = 0.99$

$K_{fs} = 1 + 0.99(1.42 - 1) = 1.42$

Fig. A-15-7: $K_t = 1.88$

Fig. 5-16: $q = 0.9$, $K_f = 1 + 0.9(1.88-1)$

$= 1.79$

So $\sigma_a = 1.79(5.72) = 10.2$ kpsi

$\tau_a = 1.42(4.30) = 6.11$ kpsi

$\tau_m = \dfrac{\tau_{max} + \tau_{min}}{2} = \dfrac{7.82 + (-0.782)}{2}$

$= 3.52$ kpsi

$\sigma'_a = [(10.2)^2 + 3(6.11)^2]^{\frac{1}{2}} = 14.7$ kpsi

$\sigma'_m = [(6.92)^2 + 3(3.52)^2]^{\frac{1}{2}} = 9.22$ kpsi

Eq. (7-45):

$n = \dfrac{S_e S_{ut}}{\sigma'_m S_e + \sigma'_a S_{ut}} = \dfrac{55.6(148)}{9.22(55.6)+14.7(148)}$

$= 3.06$  Ans.

---

7-30 (a) Tables 7-4 and 7-6:

$\underset{\sim}{k}_a = 2.70(1,\ 0.06)(80)^{-0.265}$

$= (0.845,\ 0.051)$

Eq. (7-16): $k_b = 1$

Eq. (7-29): $\underset{\sim}{k}_c = 0.923(1,\ 0.044)$

$= (0.923,\ 0.041)$

Fig. A-15-1: $d/w = 0.75/1.5 = 0.5$

$K_t = 2.17$

Fig. 5-16: $q = 0.82$ (end of chart)

$k_e = \dfrac{1}{1 + 0.82(2.17 - 1)} = 0.510$

Table 5-5: $C_{Kf} = 0.11 = C_{ke}$

$\hat{\sigma}_{ke} = 0.510(0.11) = 0.056$

7-30 (Concluded)

$\underset{\sim}{S}'_e = 0.504(1,\ 0.146)(80)$

$= (40.3,\ 5.89)$ kpsi

$\underset{\sim}{S}_e = (0.845,\ 0.051)(1)(0.923,\ 0.041) \times (0.510,\ 0.056)(40.3,\ 5.89)$

$\bar{S}_e = 0.845(0.923)(0.510)(40.3) = 16.0$ kpsi

$$C_{Se} = \left[\left(\frac{0.051}{0.845}\right)^2 + \left(\frac{0.041}{0.923}\right)^2 + \left(\frac{0.056}{0.510}\right)^2 + \left(\frac{5.89}{40.3}\right)^2\right]^{\frac{1}{2}} = 0.198$$

$\hat{\sigma}_{Se} = C_{Se}\bar{S}_e = 0.198(16.0) = 3.168$

$$\sigma_a = \frac{F_a}{(w - d)t} = \frac{2.1}{(1.5 - 0.75)(0.25)} = 11.2 \text{ kpsi}$$

Eq. (6-22): $z = -\dfrac{\mu_S - \mu_\sigma}{(\hat{\sigma}_S^2 + \hat{\sigma}_\sigma^2)^{\frac{1}{2}}}$

$$z = -\frac{16.0 - 11.2}{[(3.168)^2 + (0)^2]^{\frac{1}{2}}} = -1.52$$

Table A-10: R = (1 - 0.0643) = 0.936

Eq. (6-28): $z = -\dfrac{\mu_{\ln S} - \mu_{\ln \sigma}}{(\hat{\sigma}_{\ln S}^2 + \hat{\sigma}_{\ln\sigma}^2)^{\frac{1}{2}}}$

$$z = -\frac{\left[\ln 16.0 - \frac{(0.198)^2}{2}\right] - \left[\ln 11.2 - \frac{(0)^2}{2}\right]}{\left[(3.168)^2 + (0)^2\right]^{\frac{1}{2}}}$$

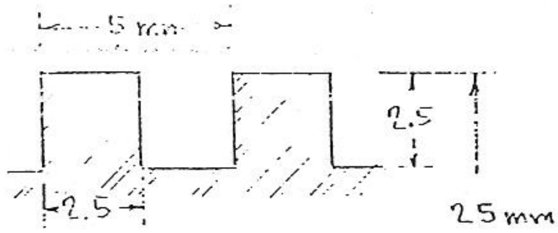
$= -1.70$

Table A-10: R = 1 - 0.0446 = 0.955

(b) Strength is the product of four random variables and so is robustly lognormal. Stress is deterministic, so stress margin is robustly lognormal, so Eq. (6-28) is a better estimate.

(c) Computer simulation with 10 000 trials gave $\underset{\sim}{S}_e = (16.06,\ 3.184)$ kpsi and R = 0.9501, confirming Eq. (6-28) as the better estimate.

8-1 (a)




Thread depth = 2.5 mm

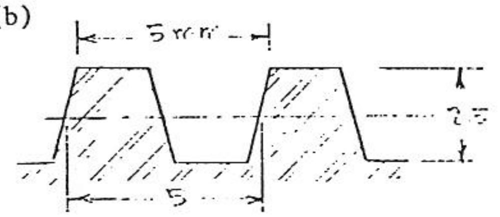
Width = 2.5 mm

$d_m = 25 - 1.25 - 1.25 = 22.5$ mm

$d_r = 25 - 5 = 20$ mm

$\ell = p = 5$ mm

(b)




Depth = 2.5 mm

Width at pitch line = 2.5 mm

$d_m = 22.5$ mm, $d_r = 20$ mm, $\ell = p = 5$ mm

---

8-2 and 8-3 OMITTED

---

8-4 Find torque to raise the load.

$F = 6$ kN, $\ell = 5$ mm, $d_m = 22.5$ mm

Eqs. (8-1) and (8-6):

$$T = \frac{6(22.5)}{2}\left[\frac{5 + \pi(0.08)(22.5)}{\pi(22.5) - 0.08(5)}\right] + \frac{6(0.05)(40)}{2}$$

$= 10.233 + 6 = 16.23$ N·m  Ans.

To lower the load, use Eqs. (8-2) and (8-6).

$$T = \frac{6(22.5)}{2}\left[\frac{\pi(0.08)(22.5) - 5}{\pi(22.5) + 0.08(5)}\right] + 6$$

$= 0.622 + 6 = 6.62$ N·m  Ans.

$$e = \frac{F\ell}{2\pi T} = \frac{6(5)}{2\pi(16.23)} = 0.294 \quad \text{Ans.}$$

---

8-5 Because there are no collar bearings at the top of the screws.

No, because they rotate in opposite directions.

---

8-6 (a) Screws rotate at

$n = 1720/75 = 22.93$ rev/min

The lead is ½ in and so the speed of the press head is

$V = 22.93(\frac{1}{2}) = 11.47$ in/min  Ans.

(b) $F = 2500$ lb/screw;

$d_m = 3 - 0.25 = 2.75$ in

$\sec\alpha = 1/\cos 29° = 1.143$

Eq. (8-5):

$$T = \frac{2500(2.75)}{2}\left[\frac{0.5 + \pi(0.05)(2.75)(1.143)}{\pi(2.75) - 0.05(0.5)(1.143)}\right]$$

$= 396.7$ lb·in

Eq. (8-6): $T_c = \frac{2500(0.06)(5)}{2} = 375$ lb·in

So $T_{tot} = 396.7 + 375 = 771.7$ lb·in per screw

Motor torque to include effect of gear efficiency

$$T_{motor} = \frac{771.7(2)}{75}\left(\frac{1}{0.95}\right) = 21.66 \text{ lb·in}$$

Motor horsepower required is

$$H = \frac{Tn}{63\ 000} = \frac{21.66(1720)}{63\ 000} = 0.591 \quad \text{Ans.}$$

---

8-7 OMITTED

---

8-8 $T = 6(2.75) = 16.5$ lb·in

$d_m = \frac{5}{8} - \frac{1}{12} = 0.5417$ in

$\ell = 1/6 = 0.1667$ in, $\alpha = 29°$

$\sec\alpha = 1.143$

Eq. (8-1): $T_{sc} = \frac{0.5417F}{2} \times$

$$\left[\frac{0.1667 + \pi(0.15)(0.5417)(1.143)}{\pi(0.5417) - 0.15(0.1667)(1.143)}\right]$$

$= 0.7421F$

8-8 (Concluded) Eq. (8-6):

$$T_c = \frac{0.15(7/16)F}{2} = 0.03281F$$

$$T_{tot} = 0.107F$$

$$F = \frac{16.5}{0.107} = 154 \text{ lb} \quad \underline{\text{Ans.}}$$

---

8-9 $d_m = 40 - 3 = 37$ mm

$\ell = 2(6) = 12$ mm

$$T = \frac{10(37)}{2}\left[\frac{12 + \pi(0.10)(37)}{\pi(37) - 0.10(12)}\right] + \frac{10(0.15)(60)}{2}$$

$$= 38.0 + 45 = 83.0 \text{ N·m}$$

Since $n = V/\ell = 48/12 = 4$ rev/s

$\omega = 2\pi n = 8\pi$ rad/s

So $H = T\omega = 83.0(8)\pi = 2086$ W  Ans.

---

8-10 (a) $d_m = 36 - 3 = 33$ mm,

$\ell = p = 6$ mm

$$T = \frac{33F}{2}\left[\frac{6 + \pi(0.14)(33)}{\pi(33) - 0.14(6)}\right] + \frac{0.09(90)F}{2}$$

$$= 3.2916 + 4.05 = 7.34 \text{ N·m}$$

$\omega = 2\pi n = 2\pi(1) = 2\pi$ rad/s

Since $H = T\omega$ we have

$$T = \frac{H}{\omega} = \frac{3000}{2\pi} = 477 \text{ N·m}$$

So $F = 477/7.34 = 65.0$ kN  Ans.

(b) $$e = \frac{F\ell}{2\pi T} = \frac{65.0(6)}{2\pi(477)} = 0.130 \quad \underline{\text{Ans.}}$$

---

QUIZ Two members, each 40 mm thick, are to be bolted together using an M10 × 1.5 bolt and a regular plain washer. Compute the bolt stiffness.

---

QUIZ Two members, each 1½ in thick, are to be bolted together using a ½"-13NC bolt and plain washer. Find the bolt stiffness and length.

---

QUIZ A ¼-in steel plate is to be bolted to an ASTM No. 30 cast iron member using 3/8"-16NC bolts. Estimate the stiffness of the members. (In using Table A-24 for the modulus of elasticity be sure to use E in compression corresponding to the top of the range given for tension.)

---

QUIZ A 6-mm steel plate is to be bolted to a 25-mm aluminum casting using an M5 × 0.8 bolt. Find the length of the bolt needed and the stiffness of the members.

---

QUIZ Two steel members, each ½ in thick, form a connection using 3/8"-UNF bolts and plain washers.

(a) Specify the bolt length.

(b) Find the bolt stiffness.

(c) Estimate the stiffness of the members.

---

QUIZ A No. 10-24 UNC fillister-head machine screw and nut are used to fasten a No. 16 Ga. sheet steel part to an aluminum casting 1/4 in thick.

(a) If the nut height is 3/8 in, what length of screw should be specified?

(b) Noting Fig. 8-10, what is the stiffness of this fastener?

---

TABLE 8-18 Basic Dimensions of American Standard Plain Washers. All dimensions in inches.

| Fastener size | Washer size | Diameters ID | Diameters OD | Thickness |
|---|---|---|---|---|
| #6 | 0.138 | 0.156 | 0.375 | 0.049 |
| #8 | 0.164 | 0.188 | 0.438 | 0.049 |
| #10 | 0.190 | 0.219 | 0.500 | 0.049 |
| 3/16 | 0.188 | 0.250 | 0.562 | 0.049 |
| #12 | 0.216 | 0.250 | 0.562 | 0.065 |
| 1/4 N | 0.250 | 0.281 | 0.625 | 0.065 |
| 1/4 W | 0.250 | 0.312 | 0.734 | 0.065 |
| 5/16 N | 0.312 | 0.344 | 0.688 | 0.065 |
| 5/16 W | 0.312 | 0.375 | 0.875 | 0.083 |
| 3/8 N | 0.375 | 0.406 | 0.812 | 0.065 |
| 3/8 W | 0.375 | 0.438 | 1.000 | 0.083 |
| 7/16 N | 0.438 | 0.469 | 0.922 | 0.065 |
| 7/16 W | 0.438 | 0.500 | 1.250 | 0.083 |
| 1/2 N | 0.500 | 0.531 | 1.062 | 0.095 |
| 1/2 W | 0.500 | 0.562 | 1.375 | 0.109 |
| 9/16 N | 0.562 | 0.594 | 1.156 | 0.095 |
| 9/16 W | 0.562 | 0.625 | 1.469 | 0.109 |
| 5/8 N | 0.625 | 0.656 | 1.312 | 0.095 |
| 5/8 W | 0.625 | 0.688 | 1.750 | 0.134 |
| 3/4 N | 0.750 | 0.812 | 1.469 | 0.134 |
| 3/4 W | 0.750 | 0.812 | 2.000 | 0.148 |
| 7/8 N | 0.875 | 0.938 | 1.750 | 0.134 |
| 7/8 W | 0.875 | 0.938 | 2.250 | 0.165 |
| 1 N | 1.000 | 1.062 | 2.000 | 0.134 |
| 1 W | 1.000 | 1.062 | 2.500 | 0.165 |
| 1 1/8 N | 1.125 | 1.250 | 2.250 | 0.134 |
| 1 1/8 W | 1.125 | 1.250 | 2.750 | 0.165 |
| 1 1/4 N | 1.250 | 1.375 | 2.500 | 0.165 |
| 1 1/4 W | 1.250 | 1.375 | 3.000 | 0.165 |
| 1 3/8 N | 1.375 | 1.500 | 2.750 | 0.165 |
| 1 3/8 W | 1.375 | 1.500 | 3.250 | 0.180 |
| 1 1/2 N | 1.500 | 1.625 | 3.000 | 0.165 |
| 1 1/2 W | 1.500 | 1.625 | 3.500 | 0.180 |
| 1 5/8 | 1.625 | 1.750 | 3.750 | 0.180 |
| 1 3/4 | 1.750 | 1.875 | 4.000 | 0.180 |
| 1 7/8 | 1.875 | 2.000 | 4.250 | 0.180 |
| 2 | 2.000 | 2.125 | 4.500 | 0.180 |
| 2 1/4 | 2.250 | 2.375 | 4.750 | 0.220 |
| 2 1/2 | 2.500 | 2.625 | 5.000 | 0.238 |
| 2 3/4 | 2.750 | 2.875 | 5.250 | 0.259 |
| 3 | 3.000 | 3.125 | 5.500 | 0.284 |

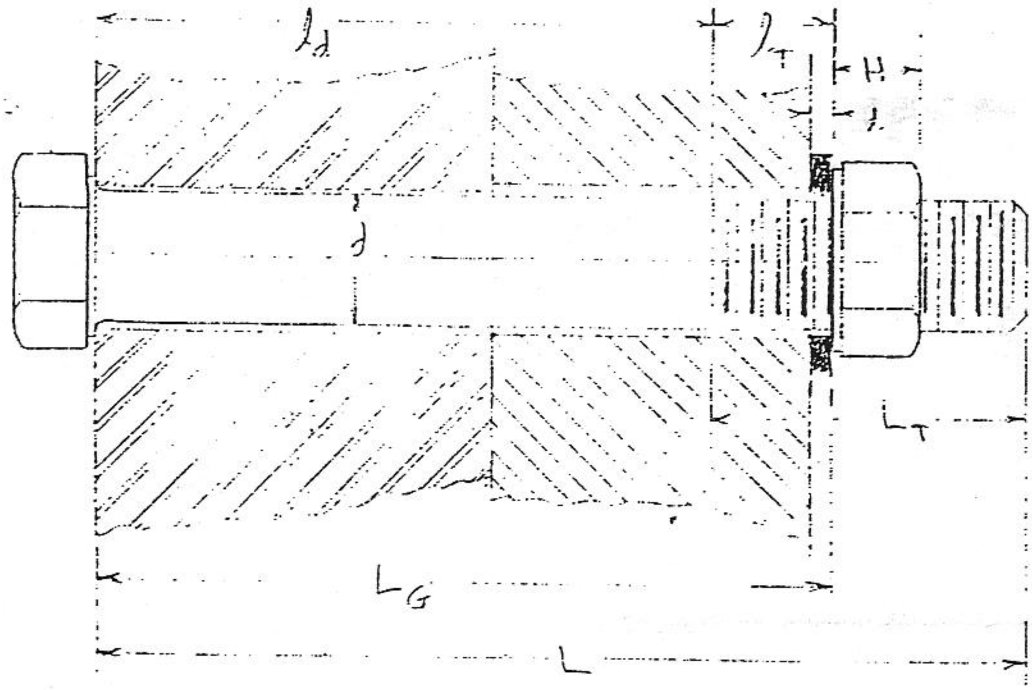
N = narrow; W = wide; use W when not specified.

TABLE 8-19 Dimensions of Metric Plain Washers. All dimensions in millimeters.

| Washer size[1] | Min. ID | Max. OD | Max. thick. |
|---|---|---|---|
| 1.6 N | 1.95 | 4.00 | 0.70 |
| 1.6 R | 1.95 | 5.00 | 0.70 |
| 1.6 W | 1.95 | 6.00 | 0.90 |
| 2 N | 2.50 | 5.00 | 0.90 |
| 2 R | 2.50 | 6.00 | 0.90 |
| 2 W | 2.50 | 8.00 | 0.90 |
| 2.5 N | 3.00 | 6.00 | 0.90 |
| 2.5 R | 3.00 | 8.00 | 0.90 |
| 2.5 W | 3.00 | 10.00 | 1.20 |
| 3 N | 3.50 | 7.00 | 0.90 |
| 3 R | 3.50 | 10.00 | 1.20 |
| 3 W | 3.50 | 12.00 | 1.40 |
| 3.5 N | 4.00 | 9.00 | 1.20 |
| 3.5 R | 4.00 | 10.00 | 1.40 |
| 3.5 W | 4.00 | 15.00 | 1.75 |
| 4 N | 4.70 | 10.00 | 1.20 |
| 4 R | 4.70 | 12.00 | 1.40 |
| 4 W | 4.70 | 16.00 | 2.30 |
| 5 N | 5.50 | 11.00 | 1.40 |
| 5 R | 5.50 | 15.00 | 1.75 |
| 5 W | 5.50 | 20.00 | 2.30 |
| 6 N | 6.65 | 13.00 | 1.75 |
| 6 R | 6.65 | 18.80 | 1.75 |
| 6 W | 6.65 | 25.40 | 2.30 |
| 8 N | 8.90 | 18.80 | 2.30 |
| 8 R | 8.90 | 25.40 | 2.30 |
| 8 W | 8.90 | 32.00 | 2.80 |
| 10 N | 10.85 | 20.00 | 2.30 |
| 10 R | 10.85 | 28.00 | 2.80 |
| 10 W | 10.85 | 39.00 | 3.50 |
| 12 N | 13.30 | 25.40 | 2.80 |
| 12 R | 13.30 | 34.00 | 3.50 |
| 12 W | 13.30 | 44.00 | 3.50 |
| 14 N | 15.25 | 28.00 | 2.80 |
| 14 R | 15.25 | 39.00 | 3.50 |
| 14 W | 15.25 | 50.00 | 4.00 |
| 16 N | 17.25 | 32.00 | 3.50 |
| 16 R | 17.25 | 44.00 | 4.00 |
| 16 W | 17.25 | 56.00 | 4.60 |
| 20 N | 21.80 | 39.00 | 4.00 |
| 20 R | 21.80 | 50.00 | 4.60 |
| 20 W | 21.80 | 66.00 | 5.10 |
| 24 N | 25.60 | 44.00 | 4.60 |
| 24 R | 25.60 | 56.00 | 5.10 |
| 24 W | 25.60 | 72.00 | 5.60 |
| 30 N | 32.40 | 56.00 | 5.10 |
| 30 R | 32.40 | 72.00 | 5.60 |
| 30 W | 32.40 | 90.00 | 6.40 |
| 36 N | 38.30 | 66.00 | 5.60 |
| 36 R | 38.30 | 90.00 | 6.40 |
| 36 W | 38.30 | 110.00 | 8.50 |

1 Same as screw or bolt size.
N = narrow; R = regular; W = wide

SUGGESTED PROCEDURE FOR DETERMINATION OF BOLT STIFFNESS




Given: 1. Grip or thickness of connected members, $L_G$.

2. Bolt diameter d and pitch p or number of threads per inch.

Find: 1. Nut height H from Table A-28.

2. Washer thickness t from Tables 8-18 or 8-19.

3. Thread length $L_T$ from Eqs. (8-7) or (8-8).

4. Bolt length L from the relation $L > L_G + H$

Use Table A-17 for preferred lengths.

5. Length of unthreaded portion from the relation $\ell_d = L - L_T$.

6. Length of threaded portion within the grip from the equation

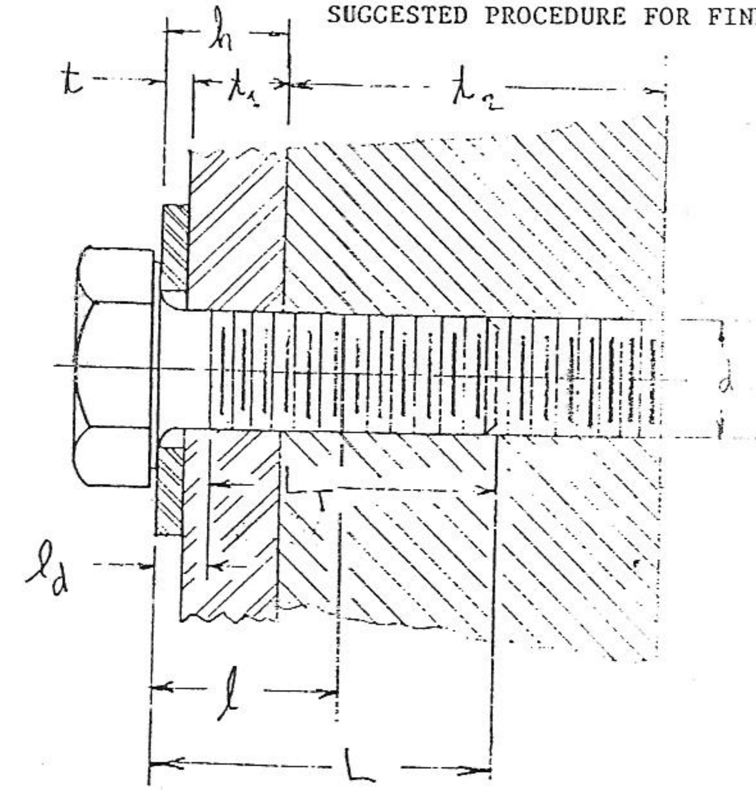
$\ell_T = L_G - \ell_d$

7. Area of unthreaded portion from the equation $A_d = \pi d^2/4$.

8. Area of threaded portion $A_T$ is the same as the tensile-stress area. Use Tables 8-1 or 8-2.

9. Bolt stiffness $k_b$ from Eq. (8-11).

SUGGESTED PROCEDURE FOR FINDING CAP-SCREW STIFFNESS




From Fig. 8-18

$$\ell = \begin{cases} h + \frac{t_2}{2} & t_2 < d \\ h + \frac{d}{2} & t_2 \geq d \end{cases}$$

Find:

1. Washer thickness t from Tables 8-18 or 8-19.
2. Thread length $L_T$; same as for bolts; use Eq. (8-7) or (8-8).
3. Cap screw length from $L > h + 1.5d$.
4. Length of unthreaded portion from $\ell_d = L - L_T$.
5. Length of useful threaded portion from $\ell_T = \ell - \ell_d$
6. Area of unthreaded portion from $A_d = \pi d^2/4$.
7. Area of threaded portion $A_T = A_t$ from Table 8-1 or 8-2.
8. Cap screw stiffness from Eq. (8-11).

8-11 The external tensile load per bolt is

$P = \frac{1}{10}\frac{\pi}{4}(150)^2(6)(10^{-3}) = 10.6$ kN

Also, $L_G = 45$ mm, $d = 12$ mm,

$H = 10.8$ mm, and no washer specified.

$L_T = 2D + 6 = 2(12) + 6 = 30$ mm

$L_G + H = 45 + 10.8 = 55.8$ mm

Table A-17: $L = 60$ mm

$\ell_d = 60 - 30 = 30$ mm

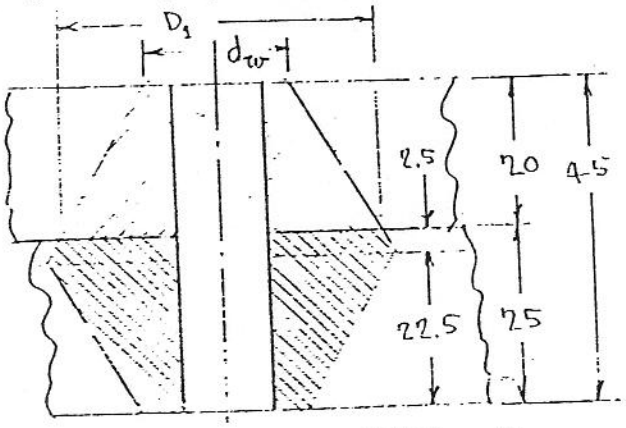
$\ell_T = 45 - 30 = 15$ mm

$A_d = \frac{\pi(12)^2}{4} = 113$ mm$^2$

Table 8-1: $A_T = A_t = 84.3$ mm$^2$

Eq. (8-11):

$$k_b = \frac{113(84.3)(207)}{113(15) + 84.3(30)} = 466.8 \text{ MN/m}$$




Three frusta: $d_w = 1.5(12) = 18$ mm

$D_1 = (20 \tan 30°)(2) + d_w$

$= (20 \tan 30°)(2) + 18 = 41.09$ mm

The following k's obtained by computer solution using $\alpha = 30°$:

1st frusta: $t = 20$ mm, $E = 207$ GPa

Eq. (8-13): $k_1 = 4470$ MN/m

2nd frusta: $t = 2.5$ mm,

Table A-24: $E = 16.4(6.89) = 113$ GPa

$k_2 = 59\ 040$ MN/m

3rd frusta: $t = 22.5$ mm, $E = 113$ GPa

$k_3 = 2343$ MN/m

$k_m = 1498$ MN/m

Eq. (8-21): $C = \frac{466.8}{466.8 + 1498} = 0.238$

Tables 8-1 and 8-6: $A_t = 84.3$ mm$^2$,

$S_p = 600$ MPa

Eq. (8-25): $F_i = 0.75(84.3)(600)(10^{-3})$

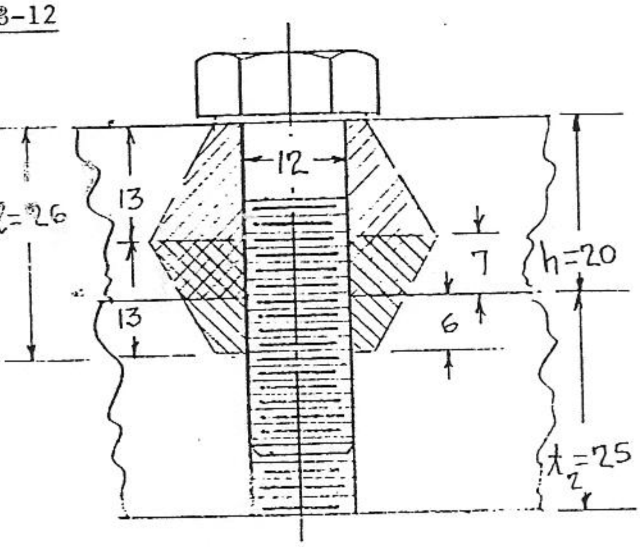
$= 37.9$ kN

Eq. (8-23):

$$n = \frac{600(10^{-3})(84.3) - 37.9}{0.238(10.6)} = 5.03 \text{ Ans.}$$

---

8-12




$P = \frac{1}{8}\frac{\pi}{4}(120)^2(6)(10^{-3}) = 8.48$ kN

Fig. 8-18: $t_1 = h = 20$, $t_2 = 25$,

$\ell = 20 + (12/2) = 26$ mm

$t = 0$ (no washer)

$L_T = 2(12) + 6) = 30$ mm

$L > h + 1.5d = 20 + 1.5(12) = 38$ mm

Use $L = 40$ mm

$\ell_d = 40 - 30 = 10$ mm

$\ell_T = \ell - \ell_d = 26 - 10 = 16$ mm

$A_d = 113$ mm$^2$, $A_T = 84.3$ mm$^2$

Eq. (8-11):

$$k_b = \frac{113(84.3)(207)}{113(16) + 84.3(10)} = 744 \text{ NM/m}$$

$D_1 = 1.5(12) + 0.577(26) = 33.0$ mm

8-12 (Concluded)

$D_2 = 1.5(12) = 18$ mm

Top frustum: $D = 18$, $t = 13$, $E = 207$ GPa

$k_1 = 5315.7$ MN/m (computer result)

Bottom frustum: $D = 18$, $t = 6$, $E = 113$ GPa

$k_3 = 4393$ MN/m

Mid frustum: $t = 7$, $E = 207$ GPa, $D = 24.9$

$k_2 = 15\ 660$ MN/m

$k_m = 2085$ MN/m

Eq. (8-21): $C = \frac{744}{744 + 2085} = 0.263$

Eq. (8-25) or Prob. 8-11: $F_i = 37.9$ kN

Eq. (8-23):

$$n = \frac{600(10^{-3})(83.4) - 37.9}{0.263(8.48)} = 5.69 \quad \text{Ans.}$$

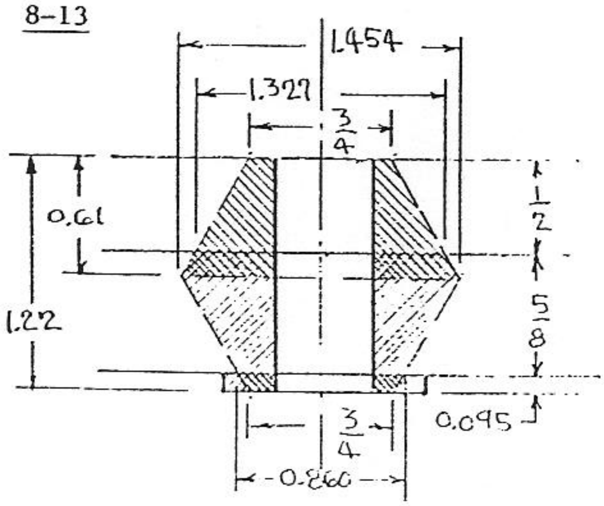
---

AN OBSERVATION

Slight differences in numerical results will often be obtained depending upon whether a hand-held calculator is used or a PC. The calculator seems to give more accurate results of the two. However both methods have been used in this manual, with some mixing even in individual problems.

---

8-13




For bolt stiffness, $H = 7/16$ in,

$L_T = 2(1/2) + 1/4 = 1\ 1/4$ in,

$L_G = 1/2 + 5/8 = 1\ 1/8$ in

$L > 1\frac{1}{8} + \frac{7}{16} + 0.095 = 1.66$ in

So use $L = 1\frac{3}{4}$ in

$\ell_d = L - L_T = 1.75 - 1.25 = 0.500$ in

$\ell_T = 1.125 + 0.095 - 0.500 = 0.72$ in

$A_d = \pi(0.25)^2/4 = 0.1963$ in$^2$

$A_t = A_T = 0.1419$ in$^2$

$$k_T = \frac{A_T E}{\ell_T} = \frac{0.1419(30)}{0.72} = 5.9125 \text{ Mlb/in}$$

$$k_d = \frac{A_d E}{\ell_d} = \frac{0.1963(30)}{0.500} = 11.778 \text{ Mlb/in}$$

$k_b = 3.94$ Mlb/in Ans.

Four frusta: By computer

Top frustum: $D = 0.75$, $t = 0.5$, $d = 0.5$, $E = 30$; $k = 33.2992$ mlb/in

2nd frustum: $D = 1.327$, $t = 0.11$, $d = 0.5$, $E = 18$; $k = 215.7433$ mlb/in

3rd frustum: $D = 0.860$, $t = 0.515$, $E = 18$; $k = 26.65056$ Mlb/in

4th frustum: $D = 0.75$, $t = 0.095$, $d = 0.5$, $E = 30$; $k = 97.27821$ Mlb/in

$k_m = 12.126$ Mlb/in Ans.

$$C = \frac{3.94}{3.94 + 12.126} = 0.245 \quad \text{Ans.}$$

---

8-14 and 8-15 OMITTED

---

8-16 Fig. 8-18: $t_1 = 0.25$ in,

$h = 0.25 + 0.065 = 0.315$ in

$\ell = h + (d/2) = 0.315 + (3/16)$

$= 0.5025$ in

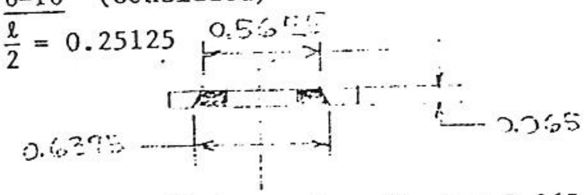
$D_1 = 1.5(0.375) + 0.577(0.5025)$

$= 0.8524$ in

$D_2 = 1.5(0.375) = 0.5625$ in

8-16 (Concluded)

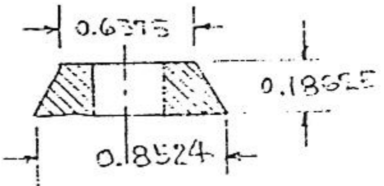
$\frac{\ell}{2} = 0.25125$




3 frusta Washer: $E = 30$, $t = 0.065$,

$D = 0.5625 + 2(0.065)(0.577) = 0.6375$;

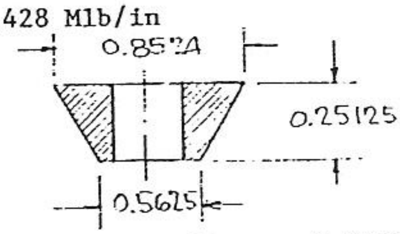
$k = 78.58$ Mlb/in (by computer)




Cap portion: $E = 14$, $t = 0.18625$,

$D = 0.6375 + 2(0.18625)(0.577) = 0.8524$;

$k = 23.428$ Mlb/in




Frame and cap: $E = 14$, $t = 0.25125$,

$k = 14.31$ Mlb/in

$k_m = 7.98$ Mlb/in Ans.

For bolt, $L_T = 2(3/8) + (1/4) = 1$ in

so it is threaded all the way. Since

$A_t = 0.0775\ \text{in}^2$,

$$k_b = \frac{0.0775(30)}{0.5025} = 4.63 \text{ Mlb/in} \quad \text{Ans.}$$

---

8-17 (a) $F'_b = RF'_{bmax} \sin\theta$

Half the external moment is contributed by the line load in the interval $0 \leq \theta \leq \pi$

$$\frac{M}{2} = \int_0^{\pi} F'_b R^2 \sin\theta\, d\theta$$

$$= \int_0^{\pi} F'_{bmax} R^2 \sin^2\theta\, d\theta$$

$$\frac{M}{2} = \frac{\pi}{2} F'_{bmax} R^2$$

from which $F'_{bmax} = \frac{M}{\pi R^2}$

$$F_{max} = \int_{\phi_1}^{\phi_2} F'_b R \sin\theta\, d\theta$$

$$= \frac{M}{\pi R^2} \int_{\phi_1}^{\phi_2} R \sin\theta\, d\theta$$

$$= \frac{M}{\pi R} (\cos\phi_1 - \cos\phi_2)$$

Noting $\phi_1 = 75°$, $\phi_2 = 105°$

$$F_{max} = \frac{12\ 000}{\pi(8/2)}(\cos 75° - \cos 105°)$$

$$= 494 \text{ lb} \quad \text{Ans.}$$

(b)

$$F_{max} = F'_{bmax} R\Delta\phi = \frac{M}{\pi R^2} \cdot R \cdot \frac{2\pi}{N} = \frac{2M}{RN}$$

$$F_{max} = \frac{2(12\ 000)}{(8/2)(12)} = 500 \text{ lb} \quad \text{Ans.}$$

(c) $F = F_{max} \sin\theta$

$$M = 2F_{max}R[(1)\sin^2 90° + 2\sin^2 60° + 2\sin^2 30° + (1)\sin^2(0)]$$

$$= 6F_{max}R$$

from which

$$F_{max} = \frac{M}{6R} = \frac{12\ 000}{6(8/2)} = 500 \text{ lb} \quad \text{Ans.}$$

The simple general equation resulted in part (b)

$$F_{max} = \frac{2M}{RN}$$

---

8-18 (a) Table 8-10: $K = 0.18$

Eq. (8-20): $\bar{F}_i = \frac{T}{Kd} = \frac{450}{0.18(0.375)}$

$= 6667$ lb Ans.

$\hat{\sigma}_{Fi} = 0.0843(6667) = 562$ lb Ans.

(b) $\bar{\sigma}_i = \frac{6667}{0.0878} = 75\ 900$ psi

8-18 (Concluded)

$$\hat{\sigma}_{\sigma i} = \frac{562}{0.0878} = 6400 \text{ psi}$$

Table 8-4 gives $S_p$ = 85 kpsi, but this is the minimum proof strength. The statistical parameters for this bolt material may be known only to the bolt manufacturers. And so the problem cannot be solved.

Suppose we use hypothetical numbers. Say, $\bar{S}_P$ = 90 kpsi and $\hat{\sigma}_{SP}$ = 2 kpsi. The coupling equation is Eq. (6-22) and is

$$z = -\frac{\bar{S}_P - \bar{\sigma}_i}{[\hat{\sigma}^2_{SP} + \hat{\sigma}^2{}_i]^{\frac{1}{2}}}$$

$$= -\frac{90 - 75.9}{[(2)^2 + (6.40)^2]^{\frac{1}{2}}} = -2.1028$$

Using Table A-10 we find

R = 1 - 0.01668 = 0.983

But see MORE on this page.

8-19 (a) To avoid confusion of terms, call preload $P_i$, reserving $F_i$ for CDF of ordered failure.

Table 8-6: $S_p$ = 600 MPa

Eq. (8-25): $P_i = 0.9A_tS_p = 0.9(245)(600)$

$= 132.3$ kN

Eq. (8-20) and Table 8-10:

T = 0.18(132.3)(20) = 476 N·m Ans.

(b) The medians of the smallest preload $\tilde{P}_1$ and largest $\tilde{P}_4$ can be found. Table 4-7: $\tilde{F}_1 = 0.1591$; $\tilde{F}_4 = 0.8409$. Alternately, from EQ. (4-33)

$$\tilde{F}_1 = \frac{1 - 0.3}{4 + 0.4} = 0.1591$$

$$\tilde{F}_4 = \frac{4 - 0.3}{4 + 0.4} = 0.8409$$

Table A-10: $\tilde{z}_1 = -0.9981$; $\tilde{z}_4 = 0.9981$

The standard deviation of $P_i$ is

$C_P\mu_P = 0.09(132.3) = 11.907$ kN

If $P_i \sim N(132.3, 11.907)$ then

$\tilde{P}_1 = \mu_P + z_1\hat{\sigma}_P = 132.3 + (-0.9981)11.907$

$= 120.4$ kN

$\tilde{P}_4 = \mu_P + z_4\hat{\sigma}_P = 132.3 + 0.9981(11.907)$

$= 144.2$ kN

This means that on many observations of the first of four preloads, half will exceed 120.4 kN and half will not. It doesn't mean that $P_1$ is 120.4 kN. It is possible to describe the mean value of the smallest preload $\bar{P}_1$ and the largest preload $\bar{P}_4$ by noting the mean CDF of the ith ordered failure is i/(n + 1) for any distribution, thus

$\bar{F}_1 = 1/(4 + 1) = 0.2$; $\bar{F}_4 = 4/(4 + 1) = 0.8$

From Table A-10, $\bar{z}_1 = -0.8415$ and $\bar{z}_4 = 0.8415$ and

$\bar{P}_1 = 132.3 + (-0.8415)11.907 = 122.3$ kN

$\bar{P}_4 = 132.3 + 0.8415(11.907) = 142.3$ kN

This means that on many observations of the first of four preloads, the mean observation will be 122.3 kN.

8-18 MORE: The worst case would be $\hat{\sigma}_{SP} = 0$ and $\bar{S}_P$ = 85 kpsi. Then the same approach gives a probability of failure of about 8%. So 8% is an upper bound and we know only that the actual probability of failure is less than 8%.

8-20 (a) $A_t = 245\ \text{mm}^2$, $S_p = 600$ MPa,

$A_d = \pi(20)^2/4 = 314.1\ \text{mm}^2$

$F_p = 245(0.600) = 147$ kN

$F_i = 0.90F_p = 0.90(147) = 132.3$ kN

$T = 0.18(132.3)(20) = 476$ N·m Ans.

(b) $L > 48 + 18 = 66$ mm

So use $L = 80$ mm

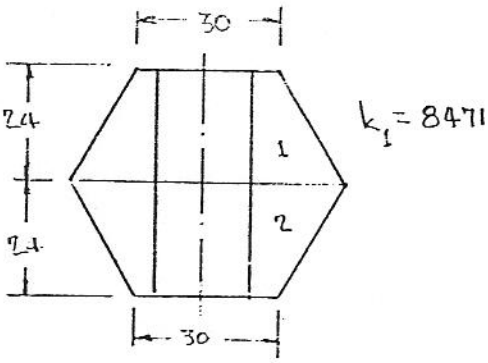
$L_T = 2D + 6 = 2(20) + 6 = 46$ mm

$\ell_T = 14$ mm, $\ell_d = 34$ mm

$$k_T = \frac{245(207)}{14} = 3622.5\ \text{MN/m}$$

$$k_d = \frac{314.1(207)}{34} = 1912.3\ \text{MN/m}$$

$k_b = 1252$ MN/m




$k_m = 4235$ MN/m; $C = 0.228$

$F_b = CP + F_i = 0.228(20) + 132.3$

$= 137$ kN Ans.

$F_m = P(1 - C) - F_i$

$= 20(1 - 0.228) - 132.3 = -116.9$ kN

Ans.

---

8-21 $$P = \frac{pA}{n} = \frac{\pi D^2 p}{4n} = \frac{\pi(0.9)^2(550)}{4(36)}$$

$= 9.72$ kN/bolt

$S_p = 830$ MPa, $S_u = 1040$ MPa,

$S_y = 940$ MPa, $A_t = 58.0\ \text{mm}^2$,

$A_d = \pi(10)^2/4 = 78.5\ \text{mm}^2$

$L_T = 2(10) + 6 = 26$ mm, $H = 8.4$ mm,

$L > 45 + 8.4 = 53.4$ mm

Use $L = 60$ mm

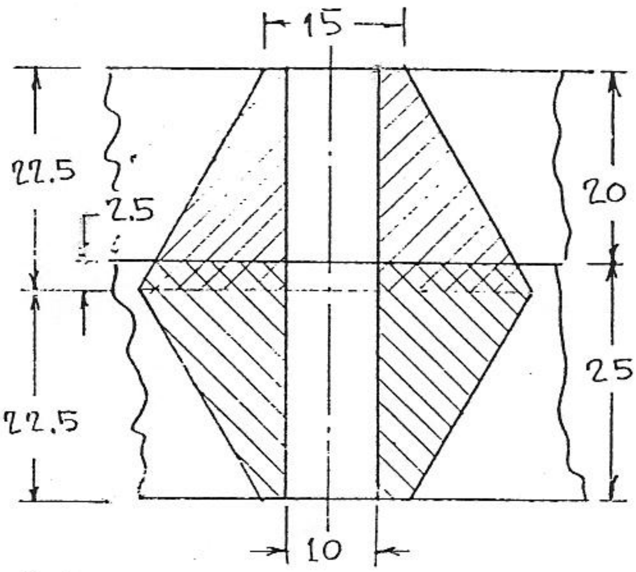
$\ell_d = 60 - 26 = 34$ mm

$\ell_T = 45 - 34 = 11$ mm

$$k_d = \frac{78.5(207)}{34} = 478\ \text{MN/m}$$

$$k_T = \frac{58.0(207)}{11} = 1091\ \text{MN/m}$$

$k_b = 332$ MN/m




3 frusta Top: $E = 207$, $t = 20$, $d_w = 15$

$k = 3503$ MN/m (computer solution)

Middle: $E = 120$, $t = 2.5$

$k = 55\ 076$ MN/m

Bottom: $E = 120$, $t = 22.5$

$k = 1958$ MN/m

$k_m = 1228$ MN/m

$$C = \frac{332}{332 + 1228} = 0.213$$

$F_i = 0.75A_t S_p = 0.75(58.0)(830)$

$= 36.1$ kN

Table 8-12: $S_e = 162$ MPa

$$\sigma_i = \frac{F_i}{A_t} = \frac{36.1}{58.0}(10^3) = 622\ \text{MPa}$$

8-21 (Concluded)

Eq. (8-37):

$$S_m = -\frac{(1040)^2}{2(162)} \pm \frac{1040}{2}\left[\left(\frac{1040}{162}\right)^2 + \frac{4(622+162)}{162}\right]^{\frac{1}{2}} = -3338.27 \pm 520(7.78275)$$

$$= -3338.27 \pm 4047 = 708.8 \text{ MPa}$$

$$S_a = S_m - \sigma_i = 708.8 - 622 = 86.8 \text{ MPa}$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.213(9.72)}{2(58.0)}(10^3) = 17.85 \text{ MPa}$$

$$n_{fatigue} = \frac{S_a}{\sigma_a} = \frac{86.8}{17.85} = 4.86 \qquad \underline{\text{Ans.}}$$

---

8-22 (a) Table A-22: $S_{ut} = 93.7$ kpsi

Using notation on p. 65, and, at Sect. A-A of the figure for Prob. 2-59

$r_i = 1$ in, $r_o = 2$ in, $\bar{r} = 1.5$ in,

$$r_n = \frac{(1)^2}{4[2(1.5) - \sqrt{4(1.5)^2 - (1)^2}\,]}$$

$$= 1.457\ 107 \text{ in}$$

$$e = \bar{r} - r_n = 1.5 - 1.457\ 107$$

$$= 0.042\ 893 \text{ in}$$

$$c_o = r_o - r_n = 2 - 1.457\ 107$$

$$= 0.542\ 893 \text{ in}$$

$$c_i = r_n - r_i = 1.457\ 107 - 1$$

$$= 0.457\ 107 \text{ in}$$

$$A = \pi(1)^2/4 = 0.7854 \text{ in}^2$$

$$M = P\bar{r} = 1.5P$$

$$\sigma_i = \frac{P}{A}\left(1 + \frac{\bar{r}c_i}{er_i}\right)$$

$$= \frac{P}{0.7854}\left[1 + \frac{1.5(0.457)}{0.0429(1)}\right] = 21.6P$$

$$\sigma_a = \frac{\sigma_i}{2} = \frac{21.6P}{2} = 10.81P$$

Eye Eq. (7-14) and Table 7-4:

$$k_a = 14.4(93.7)^{-0.718} = 0.553$$

Eq. (7-15): $k_b = (1/0.3)^{-0.1133} = 0.872$

Eq. (7-22): $k_c = 1$

Eq. (7-4):

$$S'_e = 0.504(93.7) = 47.2 \text{ kpsi}$$

Eq. (7-13):

$$S_e = 0.553(0.872)(47.2) = 22.8 \text{ kpsi}$$

From Eq. (7-39) recognizing that $\sigma_a = \sigma_m$ we have

$$n_f = \frac{1}{\sigma_a\left(\frac{1}{S_e} + \frac{1}{S_{ut}}\right)}$$

$$= \frac{1}{10.81P\left(\frac{1}{22\ 800} + \frac{1}{93\ 700}\right)} = \frac{1696}{P}$$

Thread

$k_a = 0.553$, $k_b = 1$, $k_c = 0.923$

Table 8-11: $k_e = \frac{1}{K_f} = \frac{1}{3.8} = 0.263$

$$S'_e = 47.2 \text{ kpsi}$$

$$S_e = 0.553(1)(0.923)(0.263)(47.2)$$

$$= 6.34 \text{ kpsi}$$

$$\sigma = \frac{P}{A_t} = \frac{P}{0.663} = 1.51P$$

Using same equation as before,

$$n_f = \frac{1}{0.754P\left(\frac{1}{6340} + \frac{1}{93\ 700}\right)}$$

$$= \frac{7876}{P}$$

Therefore the eye is weakest.

(b) Cold form the eye.

(c) $$P = \frac{1696}{n_f} = \frac{1696}{2} = 848 \text{ lb}$$

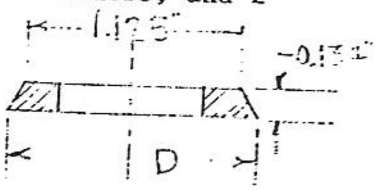
---

8-23 (a) $L > 1.5 + 2(0.134) + \frac{41}{64} = 2.41"$

Use $L = 2\frac{1}{2}$ in Ans.

(b) 4 frusta, 2 washers, and 2 members.

Washer




$E = 30$,

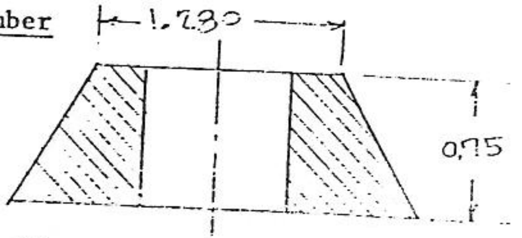
$t = 0.134$,

$D = 1.125$, $k = 153.3379$ mlb/in

Next $D = 1.125 + 2(0.134)\tan 30°$

$= 1.280$ in

Member




$E = 16$, $t = 0.75$, $D = 1.280$

$k = 35.49476$ Mlb/in

$$k_m = \frac{1}{\frac{2}{153} + \frac{2}{35.5}} = 14.41 \text{ Mlb/in} \quad \text{Ans.}$$

Bolt $L_T = 2(3/4) + 1/4 = 1\ 3/4$ in

$\ell = 2(0.134) + 2(0.75) = 1.768$ in

$\ell_d = 0.75$ in

$\ell_T = 1.768 - 0.75 = 1.018$ in

$$k_d = \frac{A_d E}{\ell_d} = \frac{\pi(0.75)^2(30)}{4(0.75)} = 17.67 \text{ Mlb/in}$$

$A_t = 0.373$ in²

$$k_T = \frac{A_t E}{\ell_T} = \frac{0.373(30)}{1.018} = 11.00 \text{ Mlb/in}$$

$k_b = 6.78$ Mlb/in Ans.

$$C = \frac{6.78}{6.78 + 14.41} = 0.320 \quad \text{Ans.}$$

(c)

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.320(6)}{2(0.373)} = 2.573 \text{ kpsi}$$

$$\sigma_m = \sigma_a + \frac{F_i}{A_t} = 2.573 + \frac{25}{0.373} = 69.60 \text{ kpsi}$$

By Goodman Here Table 8-12 gives

$S_e = 18.6$ kpsi

$$S_a = \frac{S_{ut} - (F_i/A_t)}{1 + \frac{S_{ut}}{S_e}}$$

$$= \frac{120 - 67.02}{1 + \frac{120}{18.6}} = 7.11 \text{ kpsi}$$

$$n = \frac{S_a}{\sigma_a} = \frac{7.11}{2.573} = 2.76 \quad \text{Ans.}$$

(d)

$$S_a = \frac{S_p - \sigma_i}{2} = \frac{85 - 67.02}{2} = 8.99 \text{ kpsi}$$

$$n = \frac{S_a}{\sigma_a} = \frac{8.99}{2.573} = 3.49 \quad \text{Ans.}$$

---

8-24 (a) Table 8-2: $A_t = 0.1419$ in²

Table 8-4: $S_p = 85$ kpsi, $S_{ut} = 120$ kpsi

Table 8-12: $S_e = 18.6$ kpsi

Eq. (8-25) $F_i = 0.75 A_t S_p$

$= 0.75(0.1419)(85)$

$= 9.046$ kip

$$C = \frac{4.94}{4.94 + 15.97} = 0.236$$

Eq. (8-31): $\sigma_a = \frac{CP}{2A_t} = \frac{0.236P}{2(0.1419)}$

$= 0.832P$ kpsi

Eq. (8-32): $\sigma_m = 0.832P + \frac{9.046}{0.1419}$

$= 0.832P + 63.75$ kpsi

Eq. (8-35):

$$S_a = \frac{S_{ut} - \sigma_i}{1 + \frac{S_{ut}}{S_e}} = \frac{120 - \frac{9.046}{0.1419}}{1 + \frac{120}{18.6}} = 7.55 \text{ kpsi}$$

Using $\sigma_a = S_a/n$ gives $0.832P_a = \frac{7.55}{2}$

and so $P_a = 4.537$ kip Ans.

(b) $\sigma_a = 0.832(4.537) = 3.775$ kpsi

$\sigma_m = 3.775 + 63.75 = 67.52$ kpsi

8-24 (Concluded)

Eq. (8-230: $n = \dfrac{85(0.1419) - 9.046}{0.236(4.537)}$

$= 2.82$ Ans.

Eq. (8-24): $n = \dfrac{9.046}{4.537(1 - 0.236)}$

$= 2.61$ Ans.

---

8-25 OMITTED

---

8-26 Most designers will use coarse threads unless there is a particular reason for another pitch. So here coarse threads are assumed.

$A_t = 561 \text{ mm}^2$, $S_p = 600$ MPa, $S_u = 830$ MPa, $S_e = 129$ MPa

$F_i = 0.75(561)(600)(10^{-3}) = 252.45$ kN

$\sigma_i = \dfrac{252.45}{561}(10^3) = 450$ MPa

$\sigma_a = \dfrac{CP}{2A_t} = \dfrac{0.33(80)(10^3)}{2(561)} = 23.53$ MPa

$S_a = \dfrac{S_{ut} - \sigma_i}{1 + \dfrac{S_{ut}}{S_e}} = \dfrac{830 - 450}{1 + \dfrac{830}{129}} = 51.12$ MPa

$n_f = \dfrac{S_a}{\sigma_a} = \dfrac{51.12}{23.53} = 2.17$ Ans.

Eq. (8-23): $n = \dfrac{A_t S_p - F_i}{CP}$

$= \dfrac{561(600)(10^{-3}) - 252.45}{0.33(80)}$

$= 3.19$ based on bolt stress less than proof strength. Ans.

Eq. (8-24): $n = \dfrac{F_i}{P(1 - C)}$

$= \dfrac{252.45}{80(1 - 0.33)} = 4.71$

based on joint separation.

---

8-27 (a) $A_t = 0.0775 \text{ in}^2$, $S_p = 85$ kpsi, $S_{ut} = 120$ kpsi, $S_e = 18.6$ kpsi

Assume entire bolt within the grip is unthreaded. Then

$k_b = \dfrac{A_d E}{\ell} = \dfrac{\pi(0.375)^2(30)}{4(13.5)} = 0.245$ Mlb/in Ans.

Cross sectional cylinder area is

$A = \dfrac{\pi}{4}[(D + 2t)^2 - D^2]$

$= \dfrac{\pi}{4}[(4.75)^2 - (4)^2] = 5.154 \text{ in}^2$

$k_m = \dfrac{AE}{\ell} = \dfrac{5.154(30)}{12} \cdot \dfrac{1}{6}$

$= 2.148$ Mlb/in/bolt Ans.

So $C = \dfrac{k_b}{k_b + k_m} = \dfrac{0.245}{0.245 + 2.148} = 0.102$

(b) $F_i = 0.75(0.0775)(85) = 4.94$ kip

$\sigma_i = 0.75(85) = 63.75$ kpsi

$P = pA = 2000 \dfrac{\pi(4)^2}{4} \cdot \dfrac{1}{6}$

$= 4189$ lb/bolt

$\sigma_a = \dfrac{CP}{2A_t} = \dfrac{0.102(4.189)}{2(0.0775)} = 2.76$ kpsi

Based on Goodman

$S_a = \dfrac{S_{ut} - \sigma_i}{1 + \dfrac{S_{ut}}{S_e}} = \dfrac{120 - 63.75}{1 + \dfrac{120}{18.6}}$

$= 7.55$ kpsi

$n = \dfrac{S_a}{\sigma_a} = \dfrac{7.55}{2.76} = 2.74$ Ans.

(c) Since $n = \dfrac{F_i}{P(1 - C)}$ we have

$P_{cr} = \dfrac{F_i}{1 - C} = \dfrac{4.94}{1 - 0.102} = 5.50$ kip

So joint separation would occur at

$p = \dfrac{P_{cr}}{A} = \dfrac{5500(6)}{4\pi} = 2630$ psi Ans.

---

8-28 Members, $S_y = 71$ kpsi,

$S_{sy} = 0.577(71) = 41.0$ kpsi

Grade 5 bolts, $S_y = 130$ kpsi,

$S_{sy} = 0.577(130) = 75.01$ kpsi

1) Shear of bolts

$$A_s = 2 \cdot \frac{\pi(0.375)^2}{4} = 0.221 \text{ in}^2$$

$$F_s = \frac{A_s S_{sy}}{n} = \frac{0.221(75.01)}{3} = 5.53 \text{ kip}$$

2) Bearing on bolts $A_b = 0.188 \text{ in}^2$

$$F_b = \frac{A_b S_{yc}}{n} = \frac{0.188(130)}{2} = 12.22 \text{ kip}$$

3) Bearing on member

$$F_b = \frac{0.188(71)}{2.5} = 5.34 \text{ kip}$$

4) Tension of members $A_t = 0.219 \text{ in}^2$

$$F_t = \frac{0.219(71)}{3} = 5.18 \text{ kip}$$

F = 5.18 kip based on tension of members. Ans.

---

8-29 Members, $S_y = 32$ kpsi

Bolts, $S_y = 92$ kpsi, $S_{sy} = 53.08$ kpsi

1) Shear of bolts

$$A_s = \frac{2\pi(0.375)^2}{4} = 0.221 \text{ in}^2$$

$$\tau = \frac{F}{A_s} = \frac{4}{0.221} = 18.1 \text{ kpsi}$$

$$n = \frac{53.08}{18.1} = 2.93 \quad \text{Ans.}$$

2) Bearing on bolts

$$A_b = 2(0.25)(0.375) = 0.188 \text{ in}^2$$

$$\sigma_b = \frac{-4}{0.188} = -21.3 \text{ kpsi}$$

$$n = \frac{92}{|-21.3|} = 4.32 \quad \text{Ans.}$$

3) Bearing on members

$$n = \frac{S_{yc}}{\sigma_b} = \frac{32}{|-21.3|} = 1.50 \quad \text{Ans.}$$

4) Tension on members

$$A_t = (2.375 - 0.75)(1/4) = 0.406 \text{ in}^2$$

$$\sigma_t = \frac{4}{0.406} = 9.85 \text{ kpsi}$$

$$n = \frac{32}{9.85} = 3.25 \quad \text{Ans.}$$

---

8-30 Members, $S_y = 71$ kpsi

Bolts, $S_y = 92$ kpsi, $S_{sy} = 53.08$ kpsi

1) Shear of bolts

$$F_s = \frac{1.20(53.08)}{1.8} = 35.39 \text{ kip}$$

2) Bearing on bolts

$$F_b = \frac{1.31(92)}{2.2} = 54.78 \text{ kip}$$

3) Bearing on members

$$F_b = \frac{1.31(71)}{2.4} = 38.75 \text{ kip}$$

4) Tension of members

$$F_t = \frac{1.59(71)}{2.6} = 43.42 \text{ kip}$$

F = 35.4 kip based on shear of bolt Ans.

---

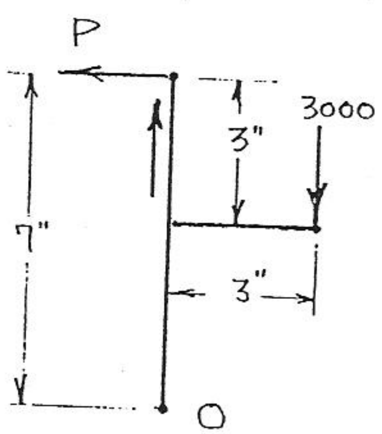
8-31, 8-32, and 8-33 OMITTED

---

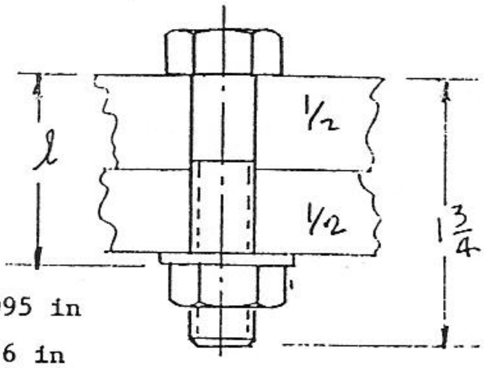
8-34

$F_s = 3000$ lb

$$P = \frac{3000(3)}{7} = 1286 \text{ lb}$$



8-34 (Continued)




$\ell = 1.095$ in

$H = 7/16$ in

$L > 1.095 + (7/16) = 1.532$ in

Use 1 3/4-in bolts; $L_T = 1\ 1/4$ in

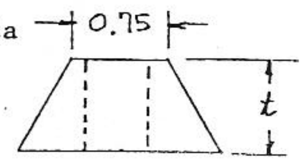
$A_d = \dfrac{\pi(0.5)^2}{4} = 0.1963\ \text{in}^2$

$A_t = 0.1419\ \text{in}^2$

$k_d = \dfrac{0.1963(30)}{0.5} = 11.78$ Mlb/in

$k_T = \dfrac{0.1419(30)}{0.595} = 7.155$ Mlb/in

2 identical frusta




$t = 1.095/2$

$= 0.5475$ in

$k = 31.94576$ Mlb/in

So $k_m = 15.97$ Mlb/in,

$k_b = 4.449$ Mlb/in

$C = \dfrac{4.449}{4.449 + 15.97} = 0.218$

$S_p = 85$ kpsi

$F_i = 0.75(0.1419)(85) = 9.046$ kip

$\sigma_i = 0.75(85) = 63.75$ kpsi

Due to the tensile load

$\sigma_b = \dfrac{CP}{A_t} + \dfrac{F_i}{A_t} = \dfrac{1}{0.1419}[0.218(1.286)$

$+ 9.046] = 65.72$ kpsi

$\tau = \dfrac{F_s}{A_s} = \dfrac{3}{0.1963} = 15.28$ kpsi

$\sigma' = [(65.72)^2 + 3(15.28)^2] = 70.8$ kpsi

So the stress margin is

$m = 85 - 70.8 = 14.2$ kpsi    Ans.

---

8-35 OMITTED

---

8-36 Using a result of Prob. 6-23

$F_x = \dfrac{2\pi\mu T}{0.18d}$ with a design factor of n

gives

$T = \dfrac{0.18nF_x d}{2\pi\mu} = \dfrac{0.18(3)(1000)d}{2\pi(0.12)} = 716d$

or $T/d = 716$

Also $T/d = K(0.75\ S_p A_t)$

$= 0.18(0.75)(85\ 000)A_t$

$= 11\ 475\ A_t$

Now, prepare a table

| Size | $A_t$ | $T/d = 11\ 475A_t$ |
|---|---|---|
| 1/4-28 | 0.0364 | 417.7 |
| 5/16-24 | 0.058 | 665.55 |
| 3/8-24 | 0.0878 | 1007.5 |

$T = (11\ 475A_t)d = 1007(0.375) = 377.8$ lb·in

So specify a wrench torque of 400 lb.in

Check factor of safety:

$n = \dfrac{2\pi\mu T}{0.18F_x d} = \dfrac{2\pi(0.12)(400)}{0.18(1000)(0.375)} = 4.47$

---

8-37 $8(1250) = 2P(5)$

$P = \dfrac{8(1250)}{10} = 1000$ lb/bolt

(b) $A_t = 0.0775\ \text{in}^2$, $S_p = 85$ kpsi

$F_i = 0.90(0.0775)(85) = 5.93$ kip

$\sigma_i = 0.90(85) = 76.5$ kpsi

Eq. (8-23):

$n = \dfrac{85(0.0775) - 5.93}{0.173(1.000)} = 3.80$   Ans.

8-37 (Concluded)

(c) Total clamping load is

$N = 4(5.93) = 23.72$ kip

$n = \frac{V_{crit}}{V} = \frac{\mu N}{V} = \frac{0.25(23.72)}{1.25} = 4.74$ Ans.

(d) $A_s = 0.221\ in^2$ for both bolts

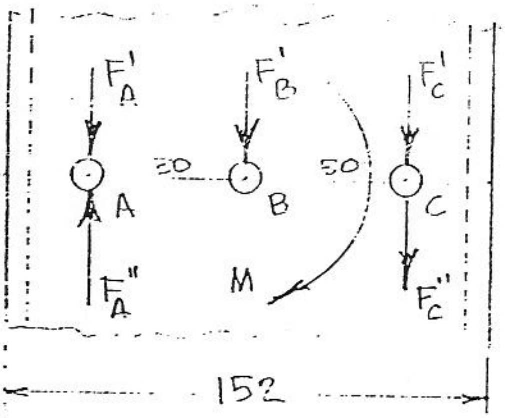
$\tau = \frac{F}{A} = \frac{1250}{0.221} = 5660$ psi; $S_y = 92$ kpsi

$S_{sy} = 0.577(92) = 53.08$ kpsi

$n = \frac{53.08}{5.66} = 9.38$ Ans.

---

8-38




Bolts: $S_p = 380$ MPa, $S_y = 420$ MPa

Channel: $t = 6.4$ mm, $S_y = 170$ MPa

Cantilever: $S_y = 190$ MPa

Nut: $H = 10.8$ mm

$L_T = 30$ mm

$L > 12 + 6.4 + 10.8 = 29.2$ mm

Use $L = 30$ mm

All threads, so $A_t = 84.3\ mm^2$

$M = (50 + 26 + 125)F = 201F$

$F''_A = F''_C = \frac{201F(50)}{2(50)^2} = 2.01F$

$F_C = F'_C + F''_C = (2.01 + \frac{1}{3})F = 2.343F$

Bolts: $S_{sy} = 0.577(420) = 242.3$ MPa

Shear of bolt $A_s = A_t = 84.3\ mm^2$

$F = \frac{S_{sy}}{n}\frac{A_s}{2.343} = \frac{242.3(84.3)(10^{-3})}{2.8(2.343)}$

$= 3.11$ kN

Bearing on bolt: $A_b = td_m$

For a 12-mm bolt, $d_m = 12 - 0.649\,519(1.75)$

$= 10.86$ mm

$A_b = 6.4(10.86) = 69.5\ mm^2$

$F = \frac{S_y}{n}\frac{A_b}{2.343} = \frac{420}{2.8}\frac{69.5(10^{-3})}{2.343} = 4.45$ kN

Bearing on member:

$F = \frac{170}{2.8}\frac{69.5(10^{-3})}{2.343} = 1.80$ kN

Strength of cantilever: $M = 151F$

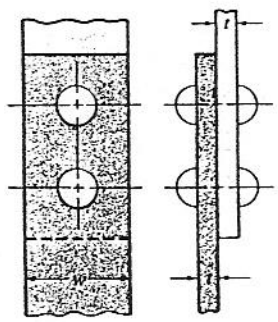
$I/c = 4930\ mm^3$

$F = \frac{4930(190)}{2.8(151)(10^3)} = 2.22$ kN

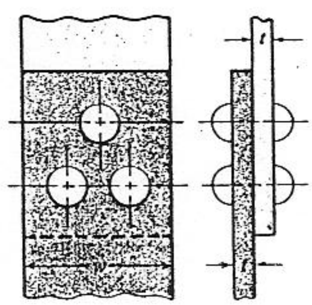
So $F = 1.80$ kN based on bearing on member. Ans.

---

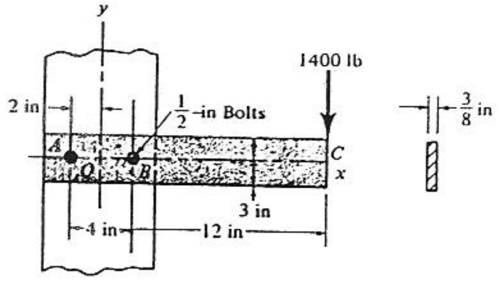
SUGGESTIONS FOR QUIZ PROBLEMS

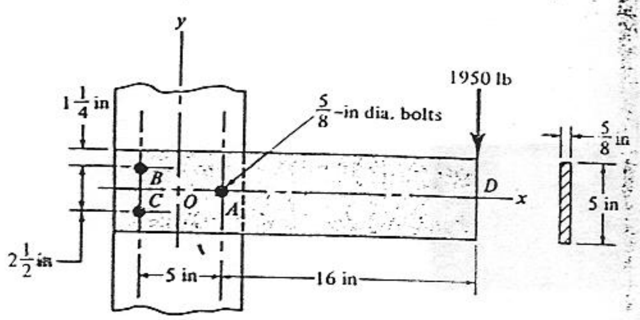


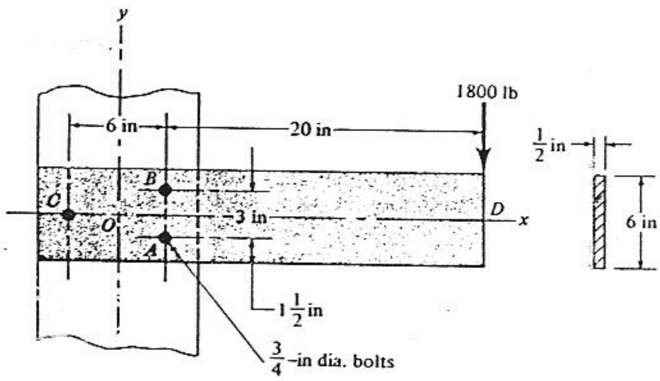
(a)

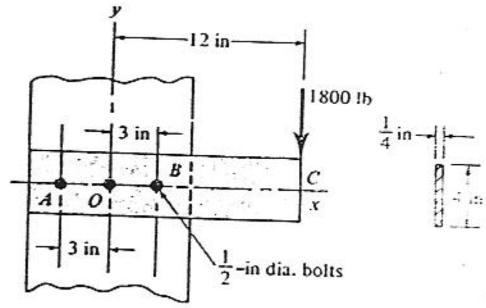


(c)



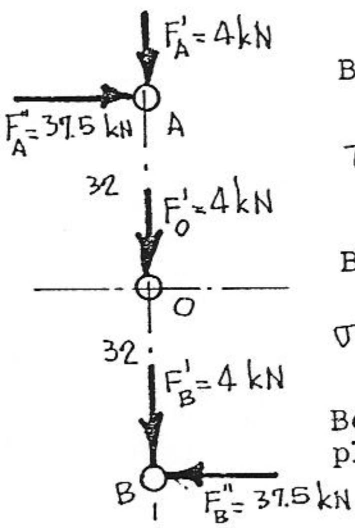




8-39 $F' = 4$ kN; $M = 12(200) = 2400$ N·m,

$$F''_A = F''_B = \frac{2400}{64} = 37.5 \text{ kN};\ F_A = F_B = \sqrt{(4)^2 + (37.5)^2}$$
$$= 37.7 \text{ kN} \quad \text{Ans.}$$

$F_O = 4$ kN Ans.



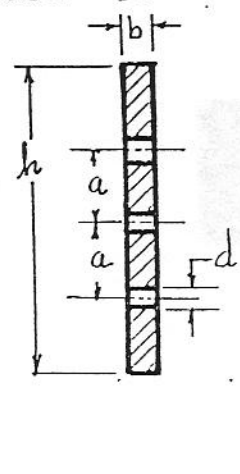

Bolt shear: $A_s = \dfrac{\pi(12)^2}{4} = 113$ mm$^2$,

$$\tau = \frac{37.7(10)^3}{113} = 334 \text{ MPa} \quad \text{Ans.}$$

Bearing on member: $A_b = 12(8) = 96$ mm$^2$,

$$\sigma = -\frac{37.7(10)^3}{96} = -393 \text{ MPa} \quad \text{Ans.}$$

Bending stress in plate:



8-39 (Continued)

$$I = \frac{bh^3}{12} - \frac{bd^3}{12} - 2\left(\frac{bd^3}{12} + a^2bd\right)$$

$$= \frac{8(136)^3}{12} - \frac{8(12)^3}{12}$$

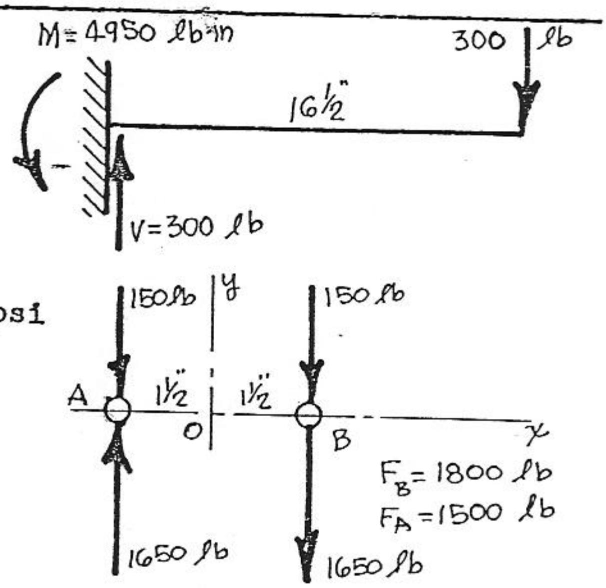
$$- 2\left[\frac{8(12)^3}{12} + (32)^2(8)(12)\right] = 1.48(10)^6 \text{ mm}^4 \quad \text{Ans.}$$

$$M = 12(200) = 2400 \text{ N·m}; \; \sigma = \frac{Mc}{I} = \frac{2400(10)^3(68)}{1.48(10)^6} = 110 \text{ MPa}$$

Ans.

---

8-40 Shear of bolt:




$$A_s = \frac{\pi}{4}(0.5)^2 = 0.196 \text{ in}^2$$

$$\tau = \frac{F}{A} = \frac{1800}{0.196} = 9180 \text{ psi}$$

$$S_{sy} = 0.577(85) = 49.0 \text{ kpsi}$$

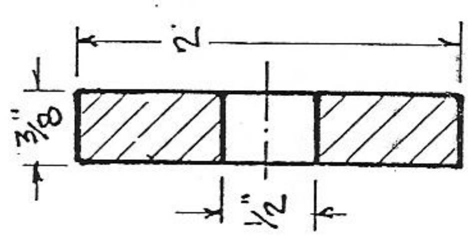
$$n = \frac{49.0}{9.18} = 5.35 \quad \text{Ans.}$$

Bearing on bolt:

$$A_b = \frac{1}{2}\left(\frac{3}{8}\right) = 0.1875 \text{ in}^2$$

$$\sigma = -\frac{F}{A} = -\frac{1800}{0.1875} = -9600 \text{ psi}; \; n = \frac{85}{9.6} = 8.85 \quad \text{Ans.}$$

Bearing on members: $S_y = 54$ kpsi, $n = \frac{54}{9.6} = 5.63$ Ans.



8-40 (Concluded)

Strength of members: The moment at the RH bolt is

$$M = 300(15) = 4500 \text{ lb·in}$$

$$I = \frac{0.375(2)^3}{12} - \frac{0.375(0.5)^3}{12} = 0.246 \text{ in}^4$$

$$\sigma = \frac{Mc}{I} = \frac{4500(1)}{0.246} = 18\ 300 \text{ psi}$$

$$n = \frac{54(10)^3}{18\ 300} = 2.95 \quad \underline{\text{Ans.}}$$

8-41 The direct shear load per bolt is $F' = 2500/6 = 417$ lb. The two inside bolts are loafing and so the moment is taken only by the four outside bolts. This moment is $M = 2500(5) = 12\ 500$ lb.in

Thus

$$F'' = \frac{(12\ 500)(2.5)}{2(2.5)^2} = 1250 \text{ lb};$$ thus the resultant bolt load is

$$F = \sqrt{(417)^2 + (1250)^2} = 1320 \text{ lb}$$

Bolt strength, $S_y = 52$ kpsi; channel strength, $S_y = 46$ kpsi; plate strength, $S_y = 45.5$ kpsi

Shear of bolt: $A_s = \pi(0.625)^2/4 = 0.307 \text{ in}^2$

$$n = \frac{S_{sy}}{\tau} = \frac{(0.577)(52\ 000)}{1320/0.307} = 6.98 \quad \underline{\text{Ans.}}$$

Bearing on bolt: Channel thickness is $t = 3/16$ in;

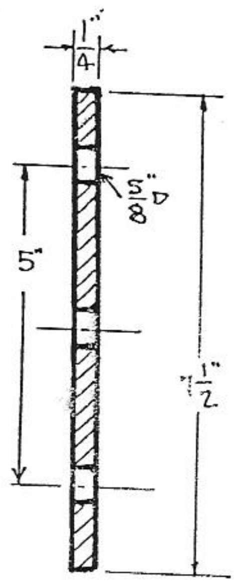
$$A_b = (0.625)(3/16) = 0.117 \text{ in}^2; \quad n = \frac{52\ 000}{1320/0.117} = 4.61 \quad \underline{\text{Ans.}}$$

Bearing on channel: $n = \dfrac{46\ 000}{1320/0.117} = 4.08$ Ans.

Bearing on plate: $A_b = (0.625)(1/4) = 0.156 \text{ in}^2$

$$n = \frac{45\ 500}{1320/0.156} = 5.38 \quad \underline{\text{Ans.}}$$

8-41 (Concluded)




Strength of plate:

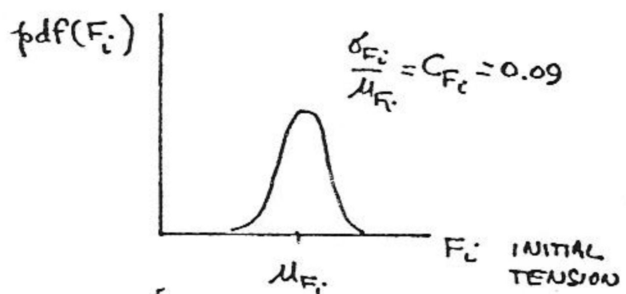
$$I = \frac{0.25(7.5)^3}{12} - \frac{0.25(0.625)^3}{12} - 2\left[\frac{0.25(0.625)^3}{12} + \left(\frac{1}{4}\right)\left(\frac{5}{8}\right)(2.5)^2\right] = 6.82 \text{ in}^4$$

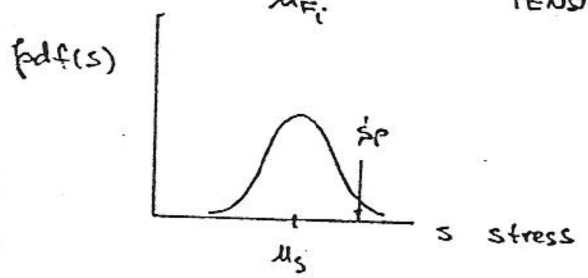
$M = 6250$ lb·in per plate

$$\sigma = \frac{Mc}{I} = \frac{6250(3.75)}{6.82} = 3440 \text{ psi}$$

$$n = \frac{45\ 500}{3440} = 13.2 \quad \text{Ans.}$$

8-42




$$\mathbf{F}_b = \mathbf{S}A_t = \mathbf{F}_i; \quad \mathbf{S} = \frac{\mathbf{F}_i}{A_t}; \quad \mu_S = \frac{\mu_{Fi}}{A_t}$$

$$C_S = C_{Fi}$$

$$\mu_S + z\hat{\sigma}_S = S_p = \mu_S + zC_{Fi}\mu_S$$

$$= \mu_S(1 + zC_{Fi})$$

$$\mu_S = \frac{S_p}{1 + zC_{Fi}} = \xi' S_p$$

from which $\xi' = 1/(1 + zC_{Fi})$

From Table A-10 $z = 2.32$

$$\xi' = \frac{1}{1 + 2.32(0.09)} = 0.827 \quad \text{Ans.}$$

$$T = KF_i d = K(\xi' S_p A_t)d$$

$$= 0.2(0.827)(85\ 000)(0.226)(0.625)$$

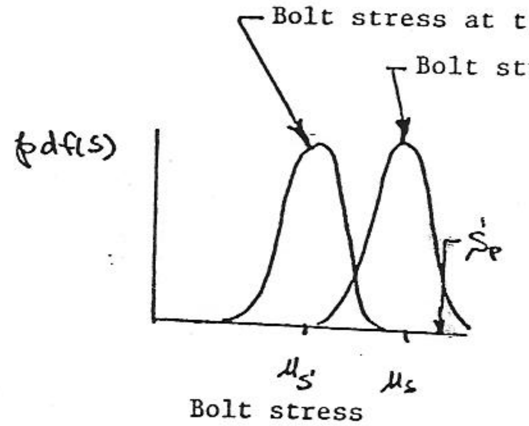
$$= 1986 \text{ lb·in} \quad \text{Ans.}$$

8-43 Since the external load on the joint increases the tension beyond the initial preload, define $\xi$ as the fraction of proof load existing under joint load.

Eq. (8-17): $F_b = CnP + F_i$, $\xi S_p A_t = CnP + \xi' S_p A_t$

This equation tells us that both distributions are the same and merely displaced a horizontal distance of $CnP/A_t$.

8-43 (Concluded)




$$S_p = \mu_{S'} + \frac{CnP}{A_t} + z\hat{\sigma}_{S'}$$

$$S'_p = \mu_{S'} + \frac{CnP}{A_t} + zC_{Fi}\mu_{S'}$$

$$S_p = \mu_{S'}(1 + zC_{Fi}) + \frac{CnP}{A_t}$$

$$\mu_{S'} = \frac{S_p - \frac{CnP}{A_t}}{1 + zC_{Fi}} = \xi' S_p$$

$$\xi' = \frac{1 - \frac{CnP}{S_p A_t}}{1 + zC_{Fi}}$$

This is the proper preload fraction $\xi'$ to run the risk indicated by z under joint load. Solve for z

$$z = \frac{1}{C_{Fi}}\left[\frac{1 - \frac{CnP}{S_p A_t}}{\xi'} - 1\right] = \frac{1}{0.09}\left[\frac{1 - \frac{0.444(2)(36\ 000)/7}{85\ 000(0.226)}}{0.60}\right] = 3.00$$

Table A-10: Chance of overproofloading a single bolt: 0.00135

Chance of not over-proofloading a single bolt: (1 - 0.00135) = 0.99865

Chance of not over-proofloading any of seven bolts: $0.99865^7 = 0.9906$

Chance of over-proofloading one or more bolts: 1 - 0.9906 = 0.0094≅0.01

---

8-44 What should be the torque-wrench setting for the bolts of Example 8-2 if the chance of getting too much proof load is set at 1 in 100? The lubricant is machine oil.

Solution: The reliability of joint against one or more bolts over-proofloaded is $R_j = 0.99$. So the reliability of an individual bolt must be

$R_i = R_j^{1/n} = 0.99^{1/7} = 0.9986$; From Table A-10, z = 2.98 Using equation from Prob. 8-43 gives

$$\xi' = \frac{1 - \frac{0.444(2)(36\ 000)/7}{85\ 000(0.226)}}{1 + 2.98(0.09)} = 0.601$$

$T = KF_i d = K\xi' S_p A_t d = 0.2(0.601)(85\ 000)(0.226)(0.625) = 1443$ lb·in Ans.

---

QUIZ For a given bolt diameter the spring rate of the members decreases as the grip length increases. Each successive increase in grip results in a smaller decrease in spring rate of the members. Show that this effect is asymptotic, with the asymptote being

$$\frac{k_m}{Ed} = \frac{\pi \tan\alpha}{2\ \ell n\ [(1 + d/D)/(1 - d/D)]}$$

QUIZ Not only is it important to provide wrench clearance in a bolting pattern, but it is important that the largest diameter of a pair of frusta not interfere with that of an adjacent bolt. Show that the minimum spacing in bolt diameters is

$$D/d = 1.5 + (\ell/d)\tan\alpha$$

9-1 Using $\tau = \dfrac{F}{2(0.707)h\ell}$ gives

$$F = 2(0.707)h\ell = 2(0.707)\left(\frac{5}{16}\right)(2)(20)$$

$$= 17.7 \text{ kip} \quad \underline{\text{Ans.}}$$

---

9-2 This problem requires access to a two-sided t table.

| N | x |
|---|---|
| 1 | 17.1 |
| 2 | 20.4 |
| 3 | 21.2 |
| 4 | 19.5 |
| 5 | 20.1 |

$\bar{F} = 19.66 \quad s = 1.557$

$\nu = 5 - 1 = 4$

$\alpha = 1 - 0.9 = 0.10$

$t_{0.10;\nu} = t_{0.10;4} = 2.132$

$$\mu \leqq 19.66 + 1.557(2.132)/\sqrt{5} = 21.14 \text{ kip}$$

$$\mu \geqq 19.66 - 1.577(2.132)/\sqrt{5} = 18.18 \text{ kip}$$

---

9-3 Eq. (4-33):

$$CDF_1 = \frac{1 - 0.3}{20 + 0.4} = 0.0343$$

Table A-10: $\tilde{z}_1 = -1.82$

$$\tilde{F}_1 = \bar{F} + \tilde{z}_1 s_F = 18.0 - 1.82$$

$$= 15.09 \text{ kip} \quad \underline{\text{Ans.}}$$

Note this is the median value.

---

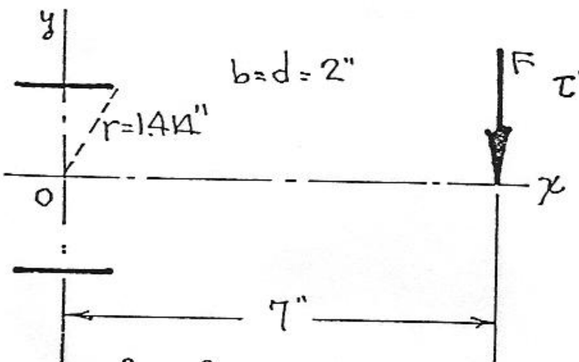
9-4

$$\tau = \frac{F}{4(0.707)h\ell} = \frac{32}{4(0.707)\left(\frac{5}{16}\right)(2)}$$

$$= 18.1 \text{ kpsi} \quad \underline{\text{Ans.}}$$

---

9-5




$$\tau' = \frac{V}{A} = \frac{F}{2(0.707)h\ell} = \frac{F}{1.414\left(\frac{5}{16}\right)(2)}$$

$$= 1.13F \text{ kpsi}$$

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{2[3(4) + 4]}{6} = 5.33 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707\left(\frac{5}{16}\right)(5.33) = 1.18 \text{ in}^4$$

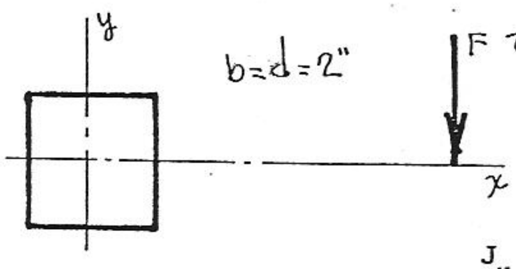
$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{7F(1)}{1.18} = 5.93F \text{ kpsi}$$

$$\tau_{max} = \sqrt{\tau_x^2 + \tau_y^2} = F\sqrt{(5.93)^2 + (1.13 + 5.93)^2} = 9.22F \text{ kpsi, then}$$

$$F = \frac{20}{9.22} = 2.17 \text{ kip} \quad \underline{\text{Ans.}}$$

---

**9-6**




$$\tau' = \frac{V}{A} = \frac{F}{4(0.707)h\ell} = \frac{F}{4(0.707)(\frac{5}{16})(2)} = 0.566F \text{ kpsi}$$

$$J_u = \frac{(b+d)^3}{6} = \frac{4^3}{6} = 10.67 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(\tfrac{5}{16})(10.67) = 2.36 \text{ in}^4$$

$$\tau_x'' = \tau_y'' = \frac{Mr_y}{J} = \frac{7F(1)}{2.36} = 2.97F \text{ kpsi}$$

$$\tau_{max} = \sqrt{\tau_x^2 + \tau_y^2} = F\sqrt{(2.97)^2 + (2.97 + 0.566)^2} = 4.62F \text{ kpsi}$$

$$F = \frac{20}{4.62} = 4.33 \text{ kip} \quad \underline{\text{Ans.}}$$

---

**9-7** Weld area: $A = 721 \text{ mm}^2$

$S_{ut} = 320$ MPa; $S'_e = 0.504(320) = 161$ MPa

Table 7-6: $a = 272$, $b = -0.995$

$k_a = 272(320)^{-0.995} = 0.875$

$k_b = 1$ for axial loading

$k_e = 1/K_f = 1/2.7 = 0.370$

$S_e = 0.875(0.370)(161) = 52.1$ MPa

$S_{se} = 0.577(52.1) = 30.1$ MPa

$$\tau_a = \frac{S_{se}}{n} = \frac{30.1}{3} = 10.0 \text{ MPa}$$

$F_a = \tau_a A = 10.0(721)(10^{-3}) = 7.21$ kN Ans.

---

**9-8** $A = 255 \text{ mm}^2$ ; $S_{ut} = 320$ MPa

$$\tau_a = \tau_m = \frac{F}{2A} = \frac{F}{510} \text{ MPa (F in newtons)}$$

$$\sigma'_a = \sigma'_m = [3\frac{F^2}{(510)^2}]^{\frac{1}{2}} = \frac{F}{294.4}$$

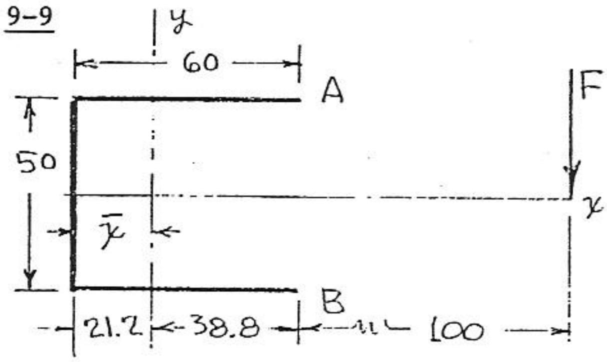
$$\text{Eq(7-55): } n = \frac{S_e S_{ut}}{\sigma_a S_{ut} + \sigma_m S_e} \quad \text{or}$$

$$2.8 = \frac{52(320)}{\frac{F}{294.4}(320 + 520)}$$

$F = 4703$ N or 4.70 kN Ans.

---

**9-9**




The electrode is unspecified. E60XX is superior to base metal, so use the base metal properties for the weld. The heel is the worst place with puddling of base metal and weld metal.

Table A-20: $S_{ut} = 320$ MPa, $S_y = 180$ MPa

$S_{sy} = 0.577(180) = 104$ MPa

$$A = 0.707h(2b + d) = 0.707(6)(120 + 50) = 721 \text{ mm}^2$$

$$\bar{x} = \frac{b^2}{2b + d} = \frac{(60)^2}{120 + 50} = 21.2 \text{ mm}$$

$$J_u = \frac{8b^3 + 6bd^2 + d^3}{12} - \frac{b^4}{2b + d}$$

$$\tau' = \frac{F}{A} = \frac{2000}{721} = 2.77 \text{ MPa}$$

9-9 (Continued)

$$J_u = \frac{8(6)^3 + 6(6)(5)^2 + (5)^3}{12} - \frac{(6)^4}{2(6) + 5)} = 153 \text{ cm}^3$$

$$J = 0.707hJ_u = 0.707(0.6)(153) = 64.9 \text{ cm}^4$$

$$M = (100 + 38.8)(2) = 277.6 \text{ N·m}$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{277.6(2.5)}{64.9} = 10.7 \text{ MPa}$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{277.6(3.88)}{64.9} = 16.6 \text{ MPa}$$

$$\tau_{max} = (\tau_x^2 + \tau_y^2)^{\frac{1}{2}}$$

$$= [(10.7)^2 + (16.6 + 2.77)^2]^{\frac{1}{2}}$$

$$= 22.1 \text{ MPa}$$

For static failure (first cycle yielding):

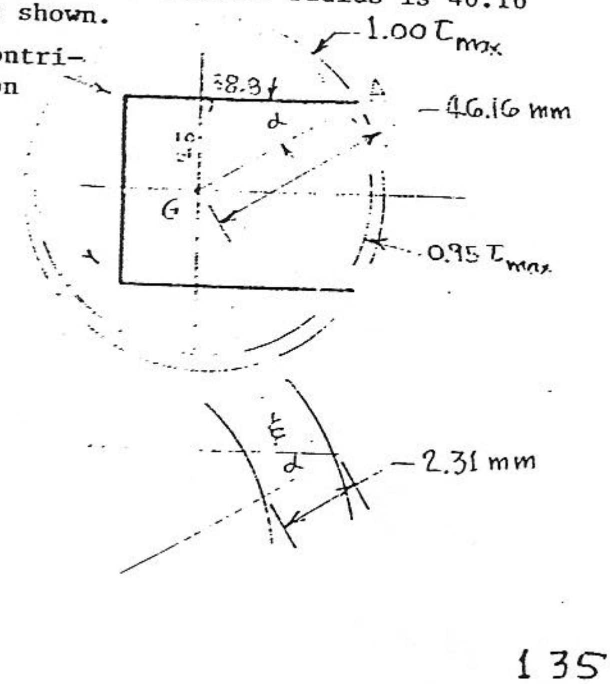
$$n = \frac{S_{sy}}{\tau_{max}} = \frac{104}{22.1} = 4.71$$

Fatigue: $S'_e = 0.504(320) = 161$ MPa

Table 7-6: $k_a = 272(320)^{-0.995} = 0.875$

The determination of $k_b$ is more complicated. To find an equivalent diameter, identify throat area subjected to 0.95 maximum stress or more. If we ignore the primary shear, the stress is proportional to the distance from the centroid. The extreme radius is 46.16 mm as shown.

No contribution from the corners




$$\cos\alpha = \frac{38.8}{46.16}$$

$$46.16 - 0.95(46.16) = 2.31 \text{ mm}$$

$$\frac{2.31}{\xi} = \cos\alpha = \frac{38.8}{46.16}$$

$$\xi = 2.75 \text{ mm}$$

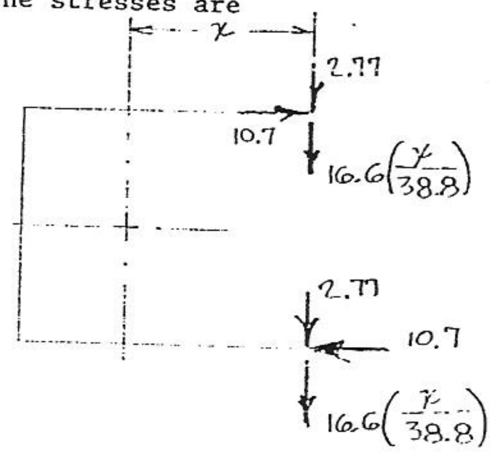
$$A_{0.95} = 2(0.707)\xi h$$

$$= 2(0.707)(2.75)(6) = 23.33 \text{ mm}^2$$

$$d_e = \left(\frac{A_{0.95}}{0.0766}\right)^{\frac{1}{2}} = \left(\frac{23.33}{0.0766}\right)^{\frac{1}{2}} = 17.5 \text{ mm}$$

$$k_b = \left(\frac{17.5}{7.62}\right)^{-0.1133} = 0.910$$

Actually, the primary shear destroys the strict proportionality of stress to radius from centroid. An algebraic approach works if primary shear is significant. At position x from the centroid the stresses are




$$\tau_{max} = \left[(10.7)^2 + \left(2.77 + 16.6\cdot\frac{x}{38.8}\right)^2\right]^{\frac{1}{2}}$$

$$\left(\tau_{max}\right)_{x = 38.8} = 22.128\ 870$$

What value of x results in

$$\tau = 0.95(22.128\ 870) = 21.022\ 427 \text{ MPa?}$$

$$x = 35.821\ 391$$

$$\xi = 38.8 - 35.821\ 391 = 2.98 \text{ mm}$$

$$A_{0.95} = 2(0.707)\xi h = 2(0.707)(2.98)(6) = 25.28 \text{ mm}^2$$

$$d_e = \left(\frac{25.28}{0.0766}\right)^{\frac{1}{2}} = 18.2 \text{ mm}$$

Eq. (7-23): $k_b = \left(\frac{18.2}{7.62}\right)^{-0.1133} = 0.906$

9-9 (Concluded) $k_c = 0.577$

$k_e = 1/K_f = 1/2.7 = 0.370$

$$S_{se} = 0.875(0.906)(0.577)(0.370)(161)$$
$$= 27.25 \text{ MPa}$$

$$n = \frac{S_{se}}{\tau_a} = \frac{27.25}{22.1} = 1.23$$

So the risk of fatigue failure is greater than that of first-cycle yielding in the weldment.

Parent metal. The peak moment occurs near the beginning of the weld.

$$\sigma_a = \frac{Mc}{I} = \frac{6M}{bd^2} = \frac{6(2000)(0.1)}{\frac{10}{10}\left(\frac{50}{10}\right)^2} = 48 \text{ MPa}$$

$$k_a = 272(320)^{-0.995} = 0.875$$

$$d_e = 0.808[10(50)]^{\frac{1}{2}} = 18.1 \text{ mm}$$

$$k_b = \left(\frac{18.1}{7.62}\right)^{-0.1133} = 0.906$$

$$k_e = 1/K_f = 1/2.7 = 0.370$$

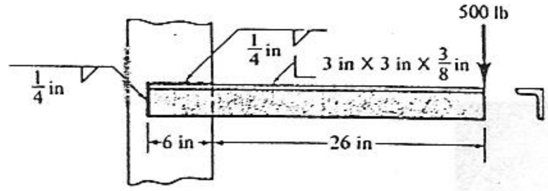
$$S'_e = 0.504(320) = 161 \text{ MPa}$$

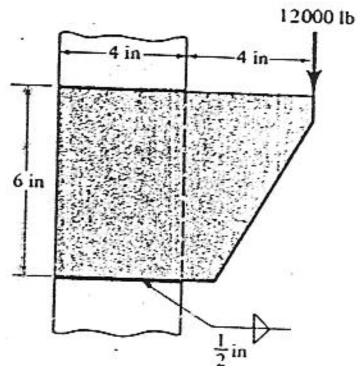
$$S_e = 0.875(0.906)(0.370)(161) = 47.3 \text{ MPa}$$

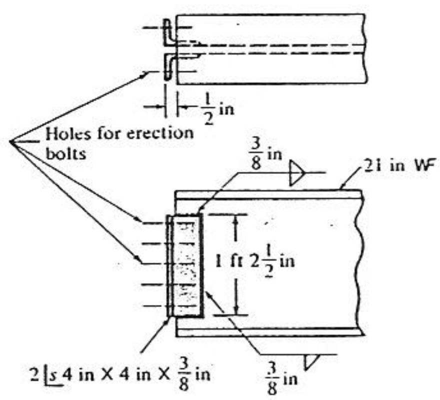
$$n = \frac{S_e}{\sigma_a} = \frac{47.3}{48} = 0.985$$

So the parent metal, that is, the cantilever, will fail in fatigue.

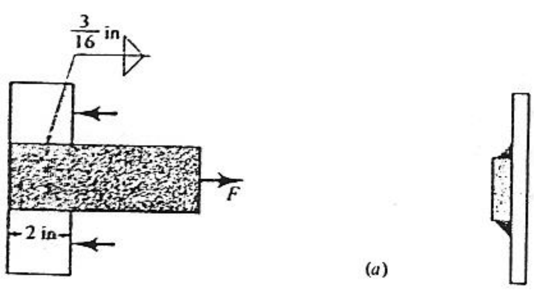
---



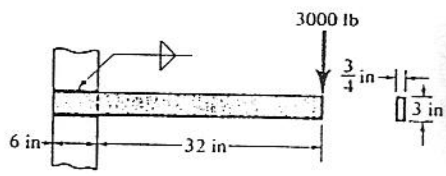




PROBLEM IDEAS




(a)



**9-10** $\tau' = 0$, $J_u = 2\pi r^3 = 2\pi(4)^3$

$= 402\ \text{cm}^3$

$J = 0.707hJ_u = 0.707(0.5)(402) = 142\ \text{cm}^4$

$M = (200 + 40)F = 240\ F\ \text{N·m}$ (F in kN)

$$\tau'' = \frac{Mr}{2J} = \frac{240F(4)}{2(142)} = 3.38F\ \text{MPa}$$

Since $\tau_{all} = 140$ MPa, $F = \dfrac{140}{3.38}$

$= 41.4$ kN  Ans.

---

**9-11** $J_u = 2\pi r^3 = 2\pi(1)^3 = 6.28\ \text{in}^3$

$J = 0.707hJ_u = 0.707(0.25)(6.28)$

$= 1.11\ \text{in}^4$

$$\tau = \frac{Mr}{J} = \frac{Tr}{J} = \frac{20(1)}{1.11} = 18.0\ \text{kpsi}$$  Ans.

---

**9-12** Table 9-3, item 2: $h = 0.375$ in,

$d = 8$ in, $b = 1$ in,

$A = 1.414(0.375)(8) = 4.24\ \text{in}^2$

$I_u = d^3/6 = (8)^3/6 = 85.3\ \text{in}^3$

$I = 0.707hI_u = 0.707(0.375)(85.3)$

$= 22.6\ \text{in}^4$

$$\tau' = \frac{F}{A} = \frac{5}{4.24} = 1.18\ \text{kpsi}$$

$M = 5(6) = 30$ kip·in

$c = (1 + 8 + 1 - 2)/2 = 4$ in

$$\tau'' = \frac{Mc}{I} = \frac{30(4)}{22.6} = 5.31\ \text{kpsi}$$

$\tau_{max} = [(5.31)^2 + (1.18)^2]^{\frac{1}{2}}$

$= 5.44$ kpsi  Ans.

---

**9-13**

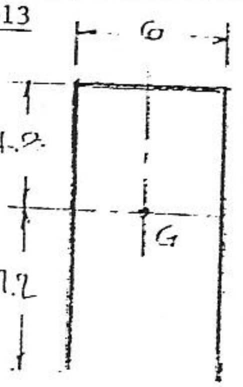



Table 9-3, item 5:

$h = 0.6$ cm, $b = 6$ cm,

$d = 12$ cm,

$A = 0.707h(b + 2d)$

$= 0.707(0.6)(30)$

$= 12.7\ \text{cm}^2$

$$\bar{y} = \frac{d^2}{b + 2d}$$

$$\bar{y} = \frac{(12)^2}{6 + 2(12)} = 4.80\ \text{cm}$$

$$I_u = \frac{2d^3}{3} - 2d^2\bar{y} + (b + 2d)\bar{y}^2$$

$$= \frac{2(12)^3}{3} - 2(12)^2(4.80)$$

$+ [6 + 2(12)](4.80)^2 = 461\ \text{cm}^3$

$I = 0.707hI_u = 0.707(0.6)(461)$

$= 196\ \text{cm}^4$

$M = 7.5(120) = 900$ N·m, $c = 7.2$ cm

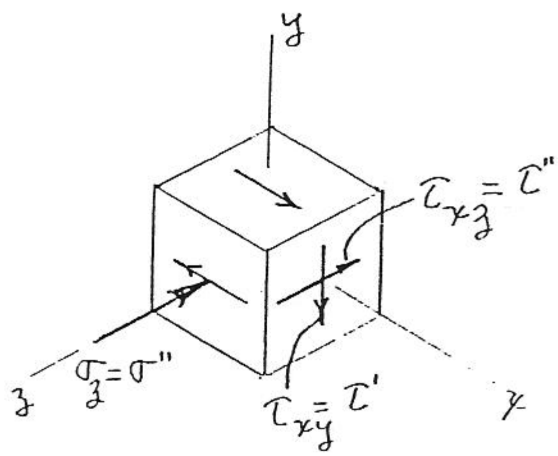
Primary shear, $\tau' = \dfrac{V}{A} = \dfrac{7.5(10^3)}{12.7(10^2)}$

$= 5.91$ MPa

Secondary stresses,

$$\tau'' = \sigma'' = \frac{Mc}{I} = -\frac{900(7.2)}{196} = -33.1\ \text{MPa}$$

These stresses occur at the bottom of the leg and are oriented as follows:




Since the shear stresses are at right angles

$\tau_{max} = (\tau'^2 + \tau''^2)^{\frac{1}{2}}$

$= [(33.1)^2 + (5.91)^2]^{\frac{1}{2}} = 33.6$ MPa

$$n = \frac{S_{sy}}{\tau_{max}} = \frac{120}{33.6} = 3.57$$  Ans.

9-14 Largest possible weld size is 1/16.

Table 9-3: $A = 1.414h(b + d)$

$$= 1.414(\frac{1}{16})(1 + 7.5)$$
$$= 0.751\ in^2$$

$\bar{x} = 0.5$ in, $\bar{y} = 3.75$ in

$$I_u = \frac{d^2}{6}(3b + d) = \frac{(7.5)^2}{6}[3(1) + 7.5]$$
$$= 98.4\ in^3$$

$$I = 0.707hI_u = 0.707(\frac{1}{16})(98.4) = 4.35\ in^4$$

$M = 4W$

Secondary stress:

$$\tau'' = \sigma'' = \frac{Mc}{I} = \frac{4W(3.75)}{4.35} = 3.45W$$

Primary shear:

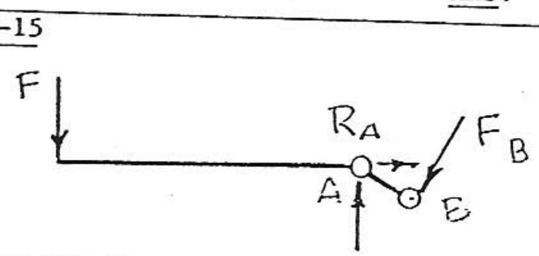
$$\tau' = \frac{V}{A} = \frac{W}{0.751} = 1.33W$$

The primary and secondary shear are at right angles to each other, and so the von Mises stress is

$$\sigma' = [\sigma''^2 + 3(\tau''^2 + \tau'^2)]^{\frac{1}{2}}$$
$$= W\{(3.45)^2 + 3[(3.45)^2 + (1.33)^2]\}^{\frac{1}{2}}$$
$$= 7.27W$$

Thus $W = 900/7.27 = 124$ lb Ans.

9-15




$F = 100$ lb

$F_B = 100\ (16/3) = 533$ lb

At A: $\underset{\sim}{R}_A = 533 \sin 60°\ \underset{\sim}{i}$
$+ (100 + 533 \cos 60°)\underset{\sim}{j}$

$$R_A = [(462)^2 + (367)^2]^{\frac{1}{2}} = 590\ lb$$

$$A = 2(1.414)\pi hr = 2(1.414)\pi h(\frac{1}{2})$$
$$= 4.44h\ in^2$$

$$J_u = 2\pi r^3 = 2\pi(\tfrac{1}{2})^3 = 0.785\ in^3$$

$$J = 2(0.707)hJ_u = 1.414(0.785h)$$
$$= 1.11h\ in^4$$

Primary shear:

$$\tau' = \frac{V}{A} = \frac{590}{4.44h} = \frac{133}{h}$$

Secondary shear:

$$\tau'' = \frac{Tc}{J} = \frac{100(16)(0.5)}{1.11h} = \frac{720.7}{h}$$

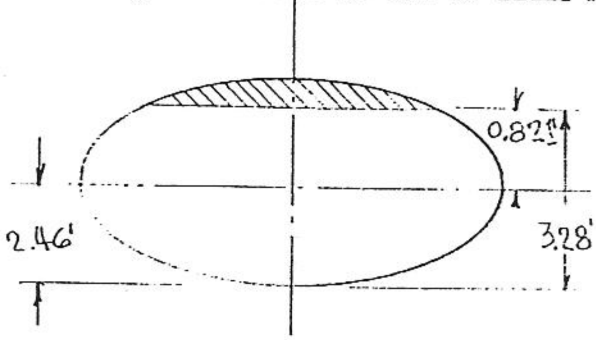
$$\tau = \tau' + \tau'' = \frac{1}{h}(133 + 720.7) = \frac{854}{h}$$

Since the allowable stress is 3000 psi

$h = 854/3000 = 0.285$ in

So use a 5/16-in leg. Ans.

9-16 Solved using U. S. customary units.
Tank weight estimate is 10% of fluid wt.




Length of tank = 6 m = 19.7 ft

60 km/h = 37.3 mi/h = 54.7 ft/s

200 m = 656.2 ft

The equation in feet is

$$\left(\frac{x}{4.92}\right)^2 + \left(\frac{y}{2.46}\right)^2 = 1$$

Tank volume = $\pi ab\ell = \pi(4.92)(2.46)(19.7)$
$= 749\ ft^3$

Solving ellipse for x gives

$$x = \left[1 - \left(\frac{y}{2.46}\right)^2\right]^{\frac{1}{2}}(4.92)$$

Simpson's rule for the right half of the void follows

9-16 (Concluded)

| y | x | | |
|---|---|---|---|
| 0.821 | 4.638 | 1 | 4.638 |
| 1.231 | 4.260 | 4 | 17.039 |
| 1.641 | 3.665 | 2 | 7.331 |
| 2.051 | 2.717 | 4 | 10.866 |
| 2.46 | 0 | 1 | 0 |
| | | | 39.874 |

$A = \frac{0.410}{3}(39.874) = 5.449\ \text{ft}^3$

$2A = 2(5.449) = 10.899\ \text{ft}^3$

Void volume $= 10.899(19.7) = 214.7\ \text{ft}^3$

Liquid volume $= 749 - 214.7 = 534.3\ \text{ft}^3$

Liquid weight, $W = 534.3(62.4)(0.9)$

$= 30\ 000\ \text{lb}$

Mass $m = \frac{30\ 000}{32.174} = 932.6$ slugs

$a = v^2/2s = (54.7)^2/[2(656.2)]$

$= 2.28\ \text{ft/s}^2$

$F = ma = 1.10(932.6)(2.28) = 2339\ \text{lb}$

or $2339/5 = 468$ lb/support, which is trivial.

$F = \frac{mv^2}{r} = \frac{1026(54.7)^2}{656/2} = 4679\ \text{lb}$

or $4679/5 = 935$ lb per support

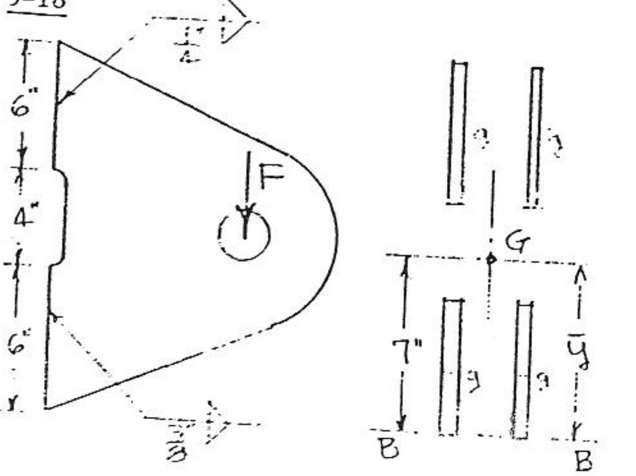
In terms of the 150° saddles the weld stresses are not significant for the loading conditions stated.

---

9-17 OMITTED

---

9-18




E6010 electrode, $S_y = 50$ kpsi

The centrode of the weld beads is

$$\bar{y} = \frac{6(0.707)(3/8)(3) + 6(0.707)(\frac{1}{4})(13)}{6(0.707)(3/8) + 6(0.707)(1/4)}$$

$= 7$ in

Estimate the second moment of area about G:

$$I_{1/4} = 2(I_G + Ay^2)$$

$$= 2\left[\frac{0.707(\frac{1}{4})(6)^3}{12} + 0.707(\frac{1}{4})(6)(6)^2\right]$$

$$= 82.7\ \text{in}^4$$

$$I_{3/8} = 2(I_G + Ay^2)$$

$$= 2\left[\frac{0.707(3/8)(6)^3}{12} + 0.707(3/8)(6)(4)^2\right] = 60.4\ \text{in}^4$$

$$I = I_{1/4} + I_{3/8} = 143.1\ \text{in}^4$$

Estimate the primary and secondary stresses:

Eq. (9-3): $\tau' = \frac{F}{2[6(0.707)(3/8 + 1/4)]}$

$= 0.189F$

$\tau'' = \frac{Mc}{I} = \frac{8F(9)}{143.1} = 0.503F$

The companion normal stress is

$\sigma = \tau'' = 0.503F$

The capacity using conventional weld analysis (load carried by shear) is

$\tau = (\tau'^2 + \tau''^2)^{\frac{1}{2}}$

$= [(0.189)^2 + (0.503)^2]^{\frac{1}{2}}F = 0.537F$

$F = \frac{\frac{1}{2}S_y}{0.537n} = \frac{\frac{1}{2}(50\ 000)}{0.537(2)} = 23\ 300\ \text{lb}$ Ans.

The capacity including normal stress using distortion-energy theory is

$\sigma' = \{(0.503)^2 + 3[(0.189)^2 + (0.503)^2]\}^{\frac{1}{2}}F = 1.058F$

$F = \frac{S_y}{1.058n} = \frac{50\ 000}{1.058(2)} = 23\ 600\ \text{lb}$

---

9-19 Endurance strength of parent metal:

Table A-20: $S_{ut} = 58$ kpsi, $S_y = 32$ kpsi

Eq. (7-14) and Table 7-4:

$k_a = 39.9(58)^{-0.995} = 0.702$ (as forged)

Eq. (7-16): $k_b = 1$

Eq. (7-22): $k_c = 0.923$

Table 9-6: $k_e = \frac{1}{K_f} = \frac{1}{2} = 0.5$

Eq. (7-13):

$S_e = 0.702(1)(0.923)(0.5)(0.504)(58)$

$= 9.5$ kpsi

Strength of weld metal:

Table 9-4: $S_y = 50$ kpsi, $S_{ut} = 62$ kpsi

Eq. (7-14) and Table 7-4:

$k_a = 39.9(62)^{-0.995} = 0.657$

Tensile loading of member places the weld throat in uniform though varying shear, $k_b = 1$. Bending would place the weld throat in stress condition analogous to bending. Use Eq. (7-17) and (7-15) in that case.

Eq. (7-22): $k_c = 0.577$

[illegible]

Eq. (7-4): $S'_e = 0.504(62) = 31.2$ kpsi

Eq. (7-13):

$S_{se} = 0.657(1)(0.577)(0.5)(31.2)$

$= 5.91$ kpsi

Permissible load in parent metal:

$$\sigma_a = \sigma_m = \frac{F}{2A} = \frac{F}{2(3/8)(2)(2)} = 0.333F$$

Eq. (7-39): $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$

$$\frac{0.333F}{9.5} + \frac{0.333F}{58} = \frac{1}{2};\ F = 12.3 \text{ kip}$$

Permissible load in weld metal:

$$\tau_a = \tau_m = \frac{\tau_{max}}{2} = \frac{F}{2(0.707)(3/8)(2)(2)}$$

$= 0.471F$

Sec. 7-14 and Eq. (7-39):

$$\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{sut}} = \frac{1}{n}$$

$$\frac{0.471F}{5.91} + \frac{0.471F}{0.67(62)} = \frac{1}{2};\ F = 5.49 \text{ kip}$$

The weld governs and so F = 5.49 kip <u>Ans.</u>

---

9-20 OMITTED

---

9-21 Compute A and I of weld:

$A = 2(0.707)(h\ell) = 2(0.707)(\frac{1}{4})(12)$

$= 4.24\ \text{in}^2$

$I_u = d^3/6 = (12)^3/6 = 288\ \text{in}^3$

$I = 0.707(\frac{1}{4})(288) = 50.9\ \text{in}^4$

Estimate the endurance limits:

Table A-20: AISI 1010, $S_{ut} = 47$ kpsi

Eq. (7-4): $S'_e = 0.504(47) = 23.7$ kpsi

AISI 1015, $S_{ut} = 50$ kpsi

$S'_e = 0.504(50) = 25.3$ kpsi

Table 9-4: $S_{ut} = 62$ kpsi,

[illegible]

The weld will control because it carries the load as a shear stress on the weld throat.

Eq. (7-14): $k_a = 39.9(62)^{-0.995} = 0.657$

$d_e = 0.808[\frac{1}{4}(0.707)(12)]^{\frac{1}{2}} = 1.177$ in

Eq. (7-15): $k_b = (1.177/0.3)^{-0.1133}$

$= 0.857$

Eq. (7-22): $k_c = 0.577$

Table 9-6: $k_e = 1/K_f = 1/2 = 0.5$

Eq. (7-13):

9-21 (Concluded)

$$S_{se} = 0.657(0.857)(0.577)(0.5)(31.2) = 5.1 \text{ kpsi}$$

Express shear stresses in terms of the load amplitude $P_a$:

$$\tau'_a = \frac{P_a}{2(0.707)h\ell} = \frac{P_a}{2(0.707)(\frac{1}{4})(12)} = 0.236P_a$$

$$\tau''_a = \frac{Mr}{I} = \frac{3P_a}{I/r} = \frac{3P_a}{50.9/6} = 0.354P_a$$

$$\tau_{a,max} = \tau'_a \quad \tau''_a = (0.236 + 0.354)P_a = 0.590P_a$$

The Goodman criterion for shear adapts Eq. (7-39) to

$$\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}} = \frac{1}{n}$$

$$\frac{0.590P_a}{5.1} + \frac{0.590P_a}{0.67(62)} = \frac{1}{2}$$

Therefore $P_a = 3.85$ kip and

$P_{max} = 2P_a = 2(3.85) = 7.70$ kip Ans.

---

9-22 Neglect primary shear

$$\frac{S_y}{n} = \tau = \frac{My}{I} = \frac{Md/2}{0.707hI_u}$$

$$\text{or } h = \frac{nMd}{2(0.707)S_yI_u}$$

Then the leg size for vertical weld beads is

$$h_1 = \frac{nMd}{2(0.707)S_y(d^3/6)} = K\frac{6}{d^2}$$

The leg size for the horizontal beads is

$$h_2 = \frac{nMd}{2(0.707)S_y(bd^2/2)} = K\frac{2}{bd^2}$$

The cost of the vertical beads is

$$\$_1 = 9h_1\ell_1 = 9K\frac{6}{d^2}(2d) = 108K/d$$

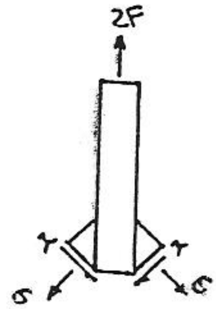
The cost of the horizontal beads is

$$\$_2 = 9h_2\ell_2 = 9K\frac{2}{bd^2}(2b) = 36K/d$$

And so the horizontal beads are cheaper by a factor of about 3.

---

9-23




$$A = \frac{2h\ell}{\cos\theta + \sin\theta}$$

$$\tau = \frac{F}{h\ell}\sin\theta(\cos\theta + \sin\theta)$$

$$\sigma = \frac{F}{h\ell}\cos\theta(\cos\theta + \sin\theta)$$

$$\tau_{max} = \frac{1.21F}{h\ell} \text{ at } \theta = 67\tfrac{1}{2}°$$

$$= \frac{F}{0.826h\ell} \qquad \text{Conservative}$$

---

9-24

$$\tau = \frac{P}{A} = \frac{2F}{2h\ell(\cos\theta + \sin\theta)}$$

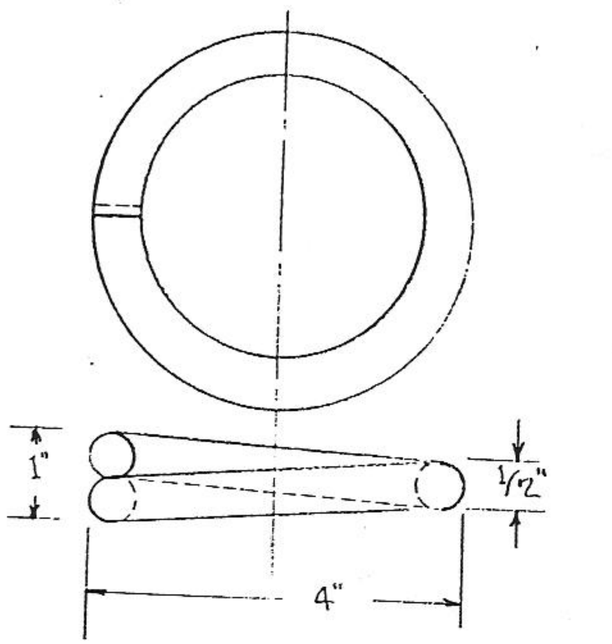
$$= \frac{F}{h\ell}(\sin\theta + \cos\theta)$$

$$\tau_{max} = \frac{\sqrt{2}F}{h\ell} \text{ at } = 45°$$

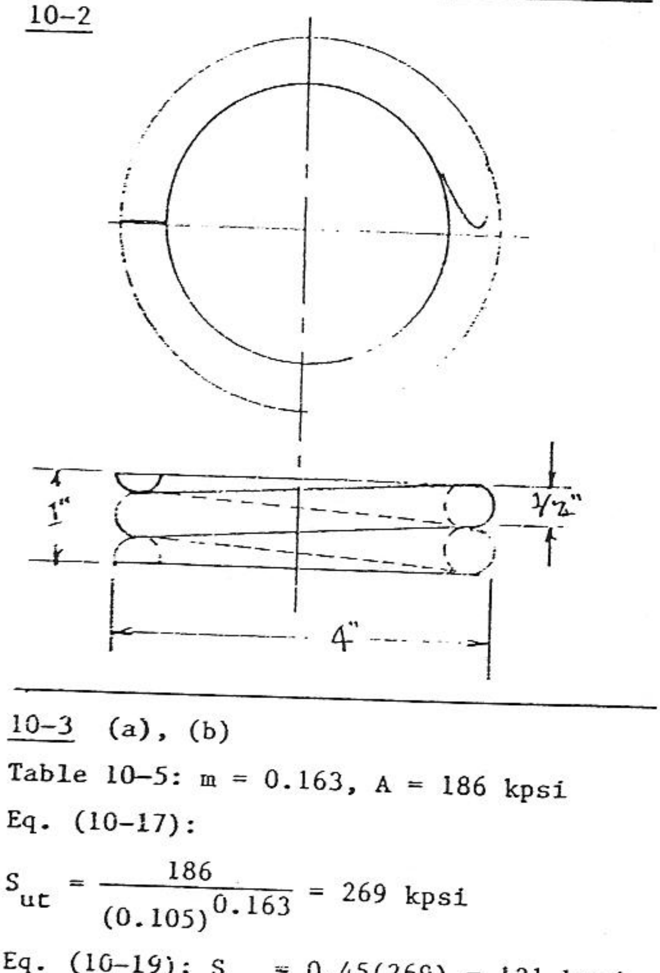
$$= F/(0.707h\ell) \qquad \text{The same}$$

---

10-1




10-2




10-3 (a), (b)

Table 10-5: $m = 0.163$, $A = 186$ kpsi

Eq. (10-17):

$$S_{ut} = \frac{186}{(0.105)^{0.163}} = 269 \text{ kpsi}$$

Eq. (10-19): $S_{sy} = 0.45(269) = 121$ kpsi

$D = 1.225 - 0.105 = 1.120$ in

$C = D/d = 1.120/0.105 = 10.67$

Eq. (10-4): $K_s = \dfrac{2(10.67) + 1}{2(10.67)} = 1.05$

Table 10-2: $N_a = 12 - 1 = 11$ coils

$L_s = 0.105(12) = 1.26$ in

Eq. (10-3): $F_s = \dfrac{\pi d^3 S_{sy}}{8K_s D}$

$$= \frac{\pi(0.105)^3(121)(10^3)}{8(1.05)(1.120)}$$

$= 46.77$ lb  Ans.

(c) Eq. (10-9):

$$k = \frac{d^4 G}{8D^3 N} = \frac{(0.105)^4(11.5)(10^6)}{8(1.12)^3(11)}$$

$= 11.31$ lb/in  Ans.

$L_0 = F_s/k + L_s = (46.77/11.31) + 1.26$

$= 5.40$ in  Ans.

(d) If $\alpha = 0.5$, then

$$\frac{2.63D}{\alpha} = \frac{2.63(1.12)}{0.5} = 5.89 \text{ in}$$

So spring will not buckle if both ends are fixed.

10-4 Table 10-5: $m = 0.201$, $A = 1510$ MPa

Eq. (10-17): $S_{ut} = \dfrac{1510}{(2)^{0.201}} = 1314$ MPa

Eq. (10-19): $S_{sy} = 0.45(1314) = 591$ MPa

$D = 20$ mm; $C = 20/2 = 10$

Eq. (10-4): $K_s = 21/20 = 1.05$

Table 10-2: $N_a = 8\frac{1}{2} - 1 = 7.5$ coils

$L_s = 2(8.5) = 17$ mm

Eq. (10-3):

$$F_s = \frac{\pi d^3 S_{sy}}{8K_s D} = \frac{\pi(2)^3(591)}{8(1.05)(20)}\left[\frac{(10^{-3})^3(10^6)}{10^{-3}}\right]$$

$= 88.4$ N  Ans.

Eq. (10-9):

$$k = \frac{d^4 G}{8D^3 N} = \frac{(2)^4(79.3)}{8(20)^3(7.5)}\left[\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}\right]$$

$= 2640$ N/m  Ans.

10-4 (Concluded)

$y = \frac{F_s}{k'} = \frac{88.4(10^3)}{2640} = 33.5$ mm

$L_0 = y + L_s = 33.5 + 17 = 50.5$ mm   Ans.

Table 10-2: $p = L_0/N_t = 50.5/8.5$

$= 5.94$ mm   Ans.

Table 10-3 and Eq. (10-16):

$\frac{2.63D}{\alpha} = \frac{2.63(20)}{0.5} = 105.2$ mm

Spring will not buckle.

---

10-5 (a) $N_t = 12\frac{1}{2}$, $D = 46.6$ mm,

$C = 46.6/3.4 = 13.7$

Table 10-2: $N_a = 11\frac{1}{2}$, $p = 120/12.5$

$= 9.6$ mm

$L_s = 3.4(12.5) = 42.5$ mm   Ans.

(b) $k = \frac{d^4G}{8D^3N} = 1140$ N/m

(c) $F = 1140(120 - 42.5)(10^{-3})$

$= 88.4$ N   Ans.

(d) $K_s = \frac{2(13.7) + 1}{2(13.7)} = 1.036$

$\tau = \frac{8FD}{\pi d^3} = 276$ MPa   Ans.

---

10-6 OMITTED

---

10-7 Table 10-2:

$N_t = L_s/d = 14.35/1.40 = 10.25$

$D = 12.19 - 1.40 = 10.79$ mm

$C = 10.79/1.40 = 7.707$

$N_a = 10.25 - 2 = 8.25$ coils

$K_s = \frac{2(7.707) + 1}{2(7.707)} = 1.065$

Table 10-5: $m = 0.163$, $A = 2060$ MPa

Eq. (10-17): $S_{ut} = \frac{2060}{(1.40)^{0.163}}$

$= 1950$ MPa

Eq. (10-19): $S_{sy} = 0.45(1950) = 878$ MPa

$\tau_{max} = 0.9(878) = 790$ MPa

Eq. (10-3):

$F = \frac{\pi(1.40)^3(790)}{8(1.065)(10.79)} \quad \frac{(10^{-3})^3(10^6)}{10^{-3}}$

$= 74.1$ N   Ans.

Eq. (10-9):

$k = \frac{(1.40)^4(79.3)}{8(10.79)^3(8.25)} \quad \frac{(10^{-3})^4(10^9)}{(10^{-3})^3}$

$= 3674$ N/m

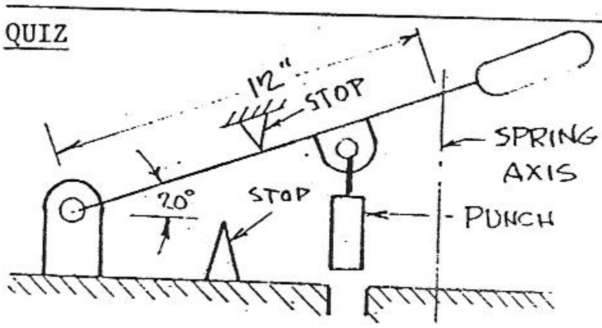
$y_s = \frac{F}{k} = \frac{74.1(10^3)}{3674} = 20.17$ mm

$L_0 = L_s + y_s = 14.35 + 20.17$

$= 34.5$ mm   Ans.

---

QUIZ




Design a compression coil spring to hold the punch lever against the upper stop. The lever weighs 3 lb.

---

QUIZ A helical compression coil spring is made of music wire with squared and ground ends. Data include: $d = 3$ mm, $N_t = 13$ coils, $D = 63.4$ mm, and $L_0 = 257.5$ mm. Find the shear strength, the stress at the solid height, and the spring rate. Will the spring buckle when compressed solid?

---

10-8

COMPUTER SOLUTION

```
WIRE DIA. =  2
WIRE DIA. =  2.5
NA =  3 NT =  5 LS =  12.5 LO =  231.0315 LCR =  403.4636
D =  76.70411 TAU =  794.0347 SSY =  798.3908
NA =  4 NT =  6 LS =  15 LO =  233.5315 LCR =  366.5711
D =  69.69031 TAU =  722.5926 SSY =  798.3908
NA =  5 NT =  7 LS =  17.5 LO =  236.0315 LCR =  340.2944
D =  64.69476 TAU =  671.7083 SSY =  798.3908
NA =  6 NT =  8 LS =  20 LO =  238.5315 LCR =  320.2293
D =  60.88009 TAU =  632.8524 SSY =  798.3908
NA =  7 NT =  9 LS =  22.5 LO =  241.0315 LCR =  304.1904
D =  57.83087 TAU =  601.7933 SSY =  798.3908
NA =  8 NT =  10 LS =  25 LO =  243.5315 LCR =  290.9476
D =  55.31323 TAU =  576.1489 SSY =  798.3908
NA =  9 NT =  11 LS =  27.5 LO =  246.0315 LCR =  279.7461
D =  53.18366 TAU =  554.4571 SSY =  798.3908
NA =  10 NT =  12 LS =  30 LO =  248.5315 LCR =  270.0919
D =  51.34827 TAU =  535.762 SSY =  798.3908
NA =  11 NT =  13 LS =  32.5 LO =  251.0315 LCR =  261.6459
D =  49.74256 TAU =  519.4064 SSY =  798.3908
NA =  12 NT =  14 LS =  35 LO =  253.5315 LCR =  254.1662
D =  48.32056 TAU =  504.922 SSY =  798.3908
```

In the solution above LCR = $2.63D/\alpha$. This is Eq. (10-16).

$k = (50-10)/140 = 0.286$ kN/m; $A = 2060$, $m = 0.163$.

Use 2.5 mm wire; the most highly stressed spring has $N_t = 5$, $L_0 = 231$ mm, $S_{sy} = 798$ MPa and $\tau = 794$ MPa. This is also the shortest spring. $D = 76.7$ mm.

The smallest diameter spring has $D = 48.3$ mm, $N_t = 14$ coils, $L_0 = 253.5$ mm, $S_{sy} = 798$ MPa, and $\tau = 505$ MPa

A spring with more than 14 coils will buckle. A spring with less than 3 coils will be too highly stressed. See next page for solutions using d = 3 mm.

10-8 (Concluded)

```
WIRE DIA. =  3
NA =   5 NT =   7 LS =   21 LO =   239.5315 LCR =   433.9401
D =  82.49811 TAU =  495.138 SSY =  775.013
NA =   6 NT =   8 LS =   24 LO =   242.5315 LCR =   408.3533
D =  77.63371 TAU =  466.4642 SSY =  775.013
NA =   7 NT =   9 LS =   27 LO =   245.5315 LCR =   387.9006
D =  73.74536 TAU =  443.5437 SSY =  775.013
NA =   8 NT =   10 LS =   30 LO =   248.5315 LCR =   371.0136
D =  70.53491 TAU =  424.6193 SSY =  775.013
NA =   9 NT =   11 LS =   33 LO =   251.5315 LCR =   356.7294
D =  67.81923 TAU =  408.6117 SSY =  775.013
NA =   10 NT =   12 LS =   36 LO =   254.5315 LCR =   344.4185
D =  65.47881 TAU =  394.8154 SSY =  775.013
NA =   11 NT =   13 LS =   39 LO =   257.5315 LCR =   333.6483
D =  63.43123 TAU =  382.7457 SSY =  775.013
NA =   12 NT =   14 LS =   42 LO =   260.5315 LCR =   324.1102
D =  61.61791 TAU =  372.0569 SSY =  775.013
NA =   13 NT =   15 LS =   45 LO =   263.5315 LCR =   315.577
D =  59.99563 TAU =  362.4941 SSY =  775.013
NA =   14 NT =   16 LS =   48 LO =   266.5315 LCR =   307.8769
D =  58.53173 TAU =  353.865 SSY =  775.013
NA =   15 NT =   17 LS =   51 LO =   269.5315 LCR =   300.8773
D =  57.20101 TAU =  346.0208 SSY =  775.013
NA =   16 NT =   18 LS =   54 LO =   272.5315 LCR =   294.4737
D =  55.98359 TAU =  338.8446 SSY =  775.013
NA =   17 NT =   19 LS =   57 LO =   275.5315 LCR =   288.5826
D =  54.86361 TAU =  332.2428 SSY =  775.013
NA =   18 NT =   20 LS =   60 LO =   278.5315 LCR =   283.1364
D =  53.82821 TAU =  326.1394 SSY =  775.013
```

---

10-9 and 10-10 OMITTED

---

10-11 (a) Table 10-5: A = 1510, m = 0.201

Eq. (10-17): $S_{ut} = \dfrac{1510}{(2)^{0.201}} = 1314$ MPa Ans.

Estimate $S_{yt} = 0.75S_{ut} = 0.75(1314) = 985$ MPa Ans.

(b) Eq. (10-19): $S_{sy} = 0.45(1314) = 591$ MPa

Eq. (10-2): $C = D/d = 12/2 = 6$

Eq. (10-4):

$$K_s = \frac{2(6)+1}{2(6)} = 1.083$$

Eq. (10-3):

$$\tau = \frac{1.083(8)(30)(12)}{\pi(2)^3} = 124 \text{ MPa} \quad \text{Ans.}$$

(c) Eq. (10-9):

$$k = \frac{(2)^4(79.3)(10^6)}{8(12)^3(120)} = 765 \text{ N/m} \quad \text{Ans.}$$

(d) Eq. (10-3):

$$F = \frac{\pi d^3 S_{sy}}{8K_s D} = \frac{\pi(2)^3(591)}{8(1.083)(12)} = 143 \text{ N} \quad \text{Ans.}$$

(e) Fig. 10-3b: $r_m = 3$ mm, $r_i = 3-1 = 2$ mm

Eq. (10-10): $K = 3/2 = 1.5$

Eq. (10-3):

$$F_{max} = \frac{\pi(2)^3(591)}{8(1.5)(12)} = 103 \text{ N} \quad \text{Ans.}$$

10-11 (Continued)

(f) The normal stress in the hook is

$$\sigma = S_{yt} = \frac{M}{I/c} + \frac{F}{A}$$

$$= K\left(\frac{32Fr_m}{\pi d^3}\right) + \frac{4F}{\pi d^2} \qquad (1)$$

Here $r_i = 6 - (d/2) = 5$ mm,

$K = r_m/r_i = 6/5 = 1.2$

$$985 = 1.2\left(\frac{32F_{max}(6)}{\pi(2)^3}\right) + \frac{4F_{max}}{\pi(2)^2}$$

$F_{max} = 104$ N  Ans.

(g) The spring preload F = 30 N must be overcome before the spring will stretch. Thus

$$y = \frac{F}{k} = \frac{103 - 30}{765}(10^3) = 95.4 \text{ mm}$$

The distance between hook ends will then be

$L = 95.4 + 264 = 359.4$ mm  Ans.

---

10-12 $r_i = 6 - (2/2) = 5$ mm,

$r_m = \bar{r} = 6$ mm, $r_o = 6 + (2/2) = 7$ mm

$$r = \frac{d^2}{4[2r_m - (4r_m^2 - d^2)^{\frac{1}{2}}]}$$

$$= \frac{(2)^2}{4\{2(6) - [4(6)^2 - (2)^2]^{\frac{1}{2}}\}}$$

$= 5.958\ 04$ mm

$e = r_m - r = 6 - 5.958\ 04 = 0.041\ 96$ mm

$c_i = (d/2) - e = 1 - 0.041\ 96$

$= 0.958\ 04$ mm

$$\sigma_i = S_{yt} = \frac{Fr_m c_i}{Aer_i} + \frac{F}{A}$$

$$= \frac{4F}{\pi d^2}\left(\frac{r_m c_i}{er_i} + 1\right)$$

$$985 = \frac{4F}{\pi(2)^2}\left[\frac{6(0.958\ 04)}{0.041\ 96(5)} + 1\right]$$

$= 9.039\ 6F$

So $F = \dfrac{985}{9.039\ 6} = 109.0$ N  Ans.

---

10-13 OMITTED

---

10-14 (a) $D = (7/16) - 0.042 = 0.3955"$

$L_s = dN_t = 0.042(14) = 0.588$ in  Ans.

Table 10-2: $N_a = 14 - 2 = 12$ coils

Eq. (10-9):

$$k = \frac{(0.042)^4(11.5)(10^6)}{8(0.3955)^3(12)}$$

$= 6.025$ lb/in  Ans.

$y_s = 1.25 - 0.588 = 0.662$ in

$F_s = ky_s = 6.025(0.662) = 3.99$ lb

Eq. (10-2): $C = 0.3955/0.042 = 9.42$

Eq. (10-3):

$$\tau_s = 1.053\frac{8(3.99)(0.3955)}{\pi(0.042)^3}(10^3)$$

$= 57.1$ kpsi  Ans.

(b) Eq. (10-6):

$$K_B = \frac{4(9.42) + 2}{4(9.42) - 3} = 1.144$$

$$F_m = \frac{3.5 + 1.5}{2} = 2.5 \text{ lb}$$

$$F_a = \frac{3.5 - 1.5}{2} = 1 \text{ lb}$$

Eq. (10-3):

$$\tau_a = K_B\frac{8F_aD}{\pi d^3}$$

$$= 1.144\frac{8(1)(0.3955)(10^{-3})}{\pi(0.042)^3}$$

$= 15.55$ kpsi

$$\tau_m = K_s\frac{8F_mD}{\pi d^3}$$

10-14 (Concluded)

$$\tau_m = 1.053\,\frac{8(2.5)(0.3955)}{\pi(0.042)^3} = 35.79 \text{ kpsi}$$

Table 10-5: A = 137 kpsi, m = 0.201

$$S_{ut} = \frac{137}{(0.042)^{0.201}} = 259 \text{ kpsi}$$

$$S_{su} = 0.67(259) = 174 \text{ kpsi}$$

Eq. (10-31):

$$n = \frac{45(174)}{15.56(174) + 35.79(45)} = 1.813 \quad \underline{\text{Ans.}}$$

---

10-15 D = 5 − 0.60 = 4.40 mm

C = 4.4/0.60 = 7.33

Eq. (10-4): $K_s = \frac{2(7.33)+1}{2(7.33)} = 1.068$

Eq. (10-6): $K_B = \frac{4(7.33)+2}{4(7.33)-3} = 1.190$

Eqs. (10-26, 27): $F_a = 2.5$ N, $F_m = 3.5$N

Eq. (10-3):

$$\tau_a = 1.190\,\frac{8(2.5)(4.40)}{\pi(0.60)^3} = 154.3 \text{ MPa}$$

$$\tau_m = 1.068\,\frac{8(3.5)(4.40)}{\pi(0.60)^3} = 193.9 \text{ MPa}$$

Table 10-5: A = 2060, m = 0.163

Eq. (10-17): $S_{ut} = \frac{2060}{(0.60)^{0.163}}$

$= 2239$ MPa

Eq. (10-30): $S_{su} = 0.67(2239) = 1500$ MPa

Page 436: $S_{se} = 310$ MPa

Eq. (10-31):

$$n = \frac{310(1500)}{154.3(1500) + 193.9(310)} = 1.59 \quad \underline{\text{Ans.}}$$

---

10-16 (a) D = 28 − 3 = 25 mm,

C = 25/3 = 8.33, $N_a = 9 - 2 = 7$ coils

Eq. (10-9):

$$k = \frac{(3)^4(79.3)}{8(25)^3(7)}\;\frac{(10^{-3})^4(10^9)}{(10^{-3})^3}$$

$= 7.34$ N/mm $\underline{\text{Ans.}}$

Eq. (10-4): $K_s = \frac{2(8.33)+1}{2(8.33)} = 1.06$

Eq. (10-6): $K_B = \frac{4(8.33)+2}{4(8.33)-3} = 1.165$

Assume $F_{min} = 0$; then

$F_a = F_m = 60/2 = 30$ N

Eq.(10-28):

$$\tau_a = 1.165\,\frac{8(30)(25)}{\pi(3)^3} = 82.4 \text{ MPa}$$

Eqs. (10-28) and (10-29):

$\tau_m = \tau_a(K_s/K_B) = 82.4(1.06/1.165)$

$= 75.0$ MPa

Table 10-5): A = 2060 MPa, m = 0.163

Eq. (10-17):

$$S_{ut} = \frac{2060}{(3)^{0.163)}} = 1722 \text{ MPa}$$

Eq. (10-30): $S_{su} = 0.67(1722) = 1154$ MPa

Page 436: $S_{se} = 310$ MPa

Eq. (10-31):

$$n = \frac{310(1154)}{82.4(1154) + 75.0(310)}$$

$= 3.02$ $\underline{\text{Ans.}}$

---

10-17 From 10-16 solution, D = 25 mm, C = 8.33, $N_a = 7$, d = 3 mm, k = 7.34N/mm

Table 10-6: $C_d = 0.0022$

$\underset{\sim}{d} = 3(1, 0.0022) = (3, 0.0066)$ mm

By linear interpolation from Table 10-9:

$\hat{\sigma}_{Do} = 0.1165$

Then $\hat{\sigma}_D = (\hat{\sigma}^2_{Do} + \hat{\sigma}^2_d)^{1/2}$

$= [(0.1165)^2 + (0.0066)^2)^{1/2}$

$= 0.1167$ mm

So $\underset{\sim}{D} = (25, 0.1167) = 25(1, 0.00467)$ mm

$$\underset{\sim}{k} = \frac{\underset{\sim}{d}^4\underset{\sim}{G}}{8\underset{\sim}{D}^3 N_a}$$ (See next page)

10-17 (Concluded)

$$\underset{\sim}{k} = \frac{[3(1,\ 0.0022)]^4[79.3(1,\ 0.012)]}{8[25(1,\ 0.00467)]^3(7)}(10^3)$$

Table 4-4:

$$C_k = \{(4(0.0022)]^2 + (0.012)^2 + [3(0.00467)]^2\}^{\frac{1}{2}} = 0.0204$$

So $\underset{\sim}{k} = 7.34(1,\ 0.0204)$

$= (7.34,\ 0.150)$ N/mm

$3\ \hat{\sigma}_k = 3(0.150) = 0.450$

So the spread is

$k_{min} = 7.34 - 0.45 = 6.89$ N/mm

to $k_{max} = 7.34 + 0.45 = 7.79$ N/mm

or, about $6C_k = 6(0.0204) = 0.122$, which is about 12 percent. Ans.

---

10-18 Since $\bar{L}_0 = 60$ mm (given), enter Table 10-7 with $N_a/L_0 = 7/60 = 0.1167$ and $C = 8.33$ and use linear interpolation to get

$$\frac{T}{L_0} = 0.018\ 55 \text{ mm/mm}$$

Then $T = \pm\ 0.018\ 55(60) = \pm\ 1.113$ mm

Ans.

---

10-19 (a) $N_a = N_t - 2 = 12.5 - 2$

$= 10.5$ turns

$D = 1.46 - 0.162 = 1.298$ in

$C = 1.298/0.162 = 8.01 \cong 8$

$N_a/L_0 = 10.5/4.5 = 2.333$

Table 10-7: $T/L_0 = 0.017\ 66$

So $T = \pm 0.017\ 66(4.5) = \pm\ 0.079$ in

$\hat{\sigma}_{L0} = 0.079/3 = 0.0263$ in

Therefore

$\underset{\sim}{L}_0 = (4.5,\ 0.0263) = 4.5(1,\ 0.0059)$ in

Ans.

(b) Table 10-6: $C_d = 0.0025$

So $\underset{\sim}{d} = 0.162(1,\ 0.0025)$

$= (0.162,\ 0.000\ 405)$ in

Table 10-10: $\hat{\sigma}_{Do} = 0.005\ 47$ in

So $\hat{\sigma}_D = (\hat{\sigma}^2_{Do} + \hat{\sigma}^2_d)^{\frac{1}{2}}$

$= [(0.005\ 47)^2 + (0.000\ 405)^2]^{\frac{1}{2}}$

$= 0.005\ 48$ in

So $\underset{\sim}{D} = (1.298,\ 0.005\ 48)$

$= 1.298(1,\ 0.004\ 23)$ in

Now $k = \dfrac{\underset{\sim}{d}^4\underset{\sim}{G}}{8\underset{\sim}{D}^3N_a}$

$$k = \frac{[0.162(1,\ 0.0025)]^4[11.5(10^6)(1,0}{8[1.298(1,\ 0.004\ 23)]^3(10.5)}$$

Table 4-4:

$$C_k = \{[4(0.0025))]^2 + (0.01)^2 + [3(0.004\ 23)]^2\}^{\frac{1}{2}} = 0.019$$

$$\bar{k} = \frac{(0.162)^4(11.5)(10^6)}{8(1.298)^3(10.5)} = 43.12 \text{ lb/in}$$

Then

$\underset{\sim}{k} = 43.12(1,\ 0.019)$

$= (43.12,\ 0.819)$ lb/in Ans.

(c)

$$\underset{\sim}{y} = \frac{F}{\underset{\sim}{k}} = \frac{50}{43.12(1,\ 0.019)}$$

$= (1.160,\ 0.022)$

$= 1.160(1,\ 0.019)$ in Ans.

(d) $K_s = 17/16 = 1.0625$

Table 10-10: $\hat{\sigma}_{Do} = 0.005\ 47$ in

$\underset{\sim}{D}_o = (1.46,\ 0.005\ 47)$

$= 1.46(1,\ 0.003\ 75)$ in

$\underset{\sim}{D} = \underset{\sim}{D}_o - \underset{\sim}{d} = (1.46,\ 0.005\ 47) - 0.162(1,\ 0.0025)$

$\underset{\sim}{D} = (1.298,\ 0.005\ 48)$

$= 1.298(1,\ 0.004\ 22)$ in

10-19 (Concluded)

$$\tilde{\tau} = K_s \frac{8F\tilde{D}}{\pi \tilde{d}^3}$$

$$= 1.0625 \frac{8(50)[1.298(1,\ 0.004\ 22)]}{\pi[0.162(1,\ 0.0025)]^3} (10^{-3})$$

Table 4-4:

$$C_\tau = \{[3(0.0025)]^2 + (0.0042)^2\}^{\frac{1}{2}}$$

$$= 0.008\ 60$$

$$\bar{\tau} = 41.3 \text{ kpsi}$$

$$\tilde{\tau} = 41.3(1,\ 0.008\ 60)$$

$$= (41.3,\ 0.355) \text{ kpsi} \quad \text{Ans.}$$

---

10-20 Table 10-6: $\tilde{d} = 2(1,\ 0.0033)$

$$= (2,\ 0.0066) \text{ mm}$$

$D = 12.5 - 2 = 10.5$ mm,

$C = 10.5/2 = 5.25$

Table 10-9: $\hat{\sigma}_{Do} = 0.0513$

$$\tilde{D}_o = (12.5,\ 0.0513) = 12.5(1,\ 0.0041) \text{mm}$$

$$\tilde{D} = \tilde{D}_o - \tilde{d} = (12.5 - 0.0513) - (2,\ 0.0066)$$

$$= (10.5,\ 0.0517)$$

$$= 10.5(1,\ 0.004\ 93) \text{ mm}$$

$$K_s = \frac{2(5.25) + 1}{2(5.25)} = 1.095$$

$$K_B = \frac{4(5.25) + 2}{4(5.25) - 3} = 1.278$$

$$F_m = \frac{130 + 10}{2} = 70 \text{ N}$$

$$F_a = \frac{130 - 10}{2} = 60 \text{ N}$$

$$\tilde{\tau}_a = K_B \frac{8F_a\tilde{D}}{\pi \tilde{d}^3}$$

$$= 1.278 \frac{8(60)[10.5(1,\ 0.004\ 93]}{\pi[2(1,\ 0.0033)]^3}$$

$$C_{\tau a} = \{(0.004\ 93)^2 + [3(0.0033)]^2\}^{\frac{1}{2}}$$

$$\tilde{\tau}_a = 256(1,\ 0.011) = (256,\ 2.816) \text{ MPa}$$

$$\tilde{\tau}_m = K_s \frac{8F_m\tilde{D}}{\pi \tilde{d}^3}$$

$$= 1.095 \frac{8(70)[10.5(1,\ 0.004\ 93)]}{\pi[2(1,\ 0.0033)]^3}$$

$$= 256(1,\ 0.011) = (256,\ 2.816) \text{ MPa}$$

The 99 percentile involves $|z| = 2.33$

$$\bar{S}_{se} = (S_{se})_{.99} + z\hat{\sigma}_{Sse} = (S_{se})_{.99}(1 + zC_{Sse})$$

$$= 310[1 + 2.33(0.05)] = 346 \text{ MPa}$$

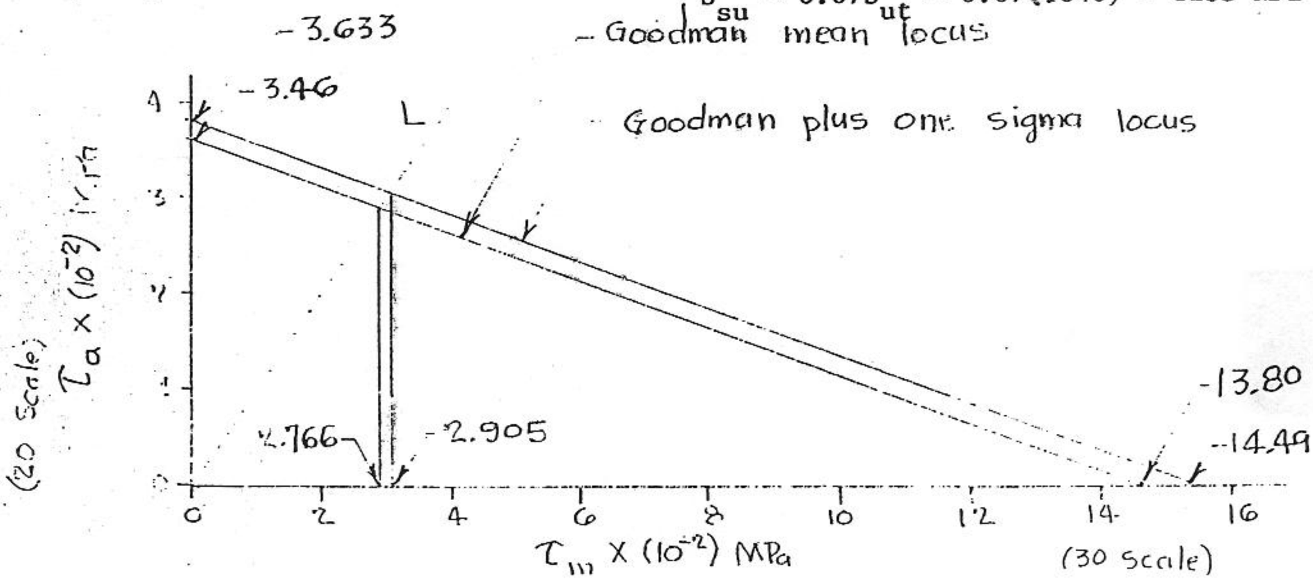
Therefore

$$\tilde{S}_{se} = 346(1,\ 0.05) = (346,\ 17.3) \text{ MPa}$$

Table 10-5: $A = 2060$ MPa, $m = 0.163$

$$S_{ut} = \frac{2060}{(2)^{0.163}} = 1840 \text{ MPa}$$

$$S_{su} = 0.67S_{ut} = 0.67(1840) = 1233 \text{ MPa}$$



10-20 (Concluded) As before

$$\bar{S}_{su} = (S_{su})_{.99}[1 + 2.33(0.05)]$$

$$= 1233[1 + 2.33(0.05)] = 1380 \text{ MPa}$$

On the previous page, the mean Goodman line is

$$S_a = S_{se} - \frac{S_{se}}{S_{su}} S_m$$

Load line L is $\tau_a = r\tau_m$ or $S_a = rS_m$

Solving simultaneously yields

$$S_m = \frac{S_{se}}{r + \frac{S_{se}}{S_{su}}} = \frac{1}{\frac{r}{S_{se}} + \frac{1}{S_{su}}}$$

The mean lines intersect at

$$\bar{S}_m = \frac{1}{\frac{1}{346} + \frac{1}{1380}} = 276.6 \text{ MPa}$$

The plus one sigma Goodman locus has an equation of

$$\bar{S}_m + \hat{\sigma}_{Sm} = \frac{1}{\frac{r}{\bar{S}_{se} + \hat{\sigma}_{Sse}} + \frac{1}{\bar{S}_{su} + \hat{\sigma}_{Ssu}}}$$

$$= \frac{1}{\frac{1}{346 + 17.3} + \frac{1}{1380 + 69}}$$

$$= 290.5 \text{ MPa}$$

Then

$$\hat{\sigma}_{Sm} = 290.5 - 276.6 = 13.9 \text{ MPa}$$

Finally, use the coupling equation to interfere $\tau_m$ with $S_m$.

$$z = -\frac{276.6 - 256}{[(13.9)^2 + (2.816)^2]^{1/2}} = -1.45$$

Table A-10: $R = 1 - \Phi = 1 - 0.0735$

$$= 0.9265 \quad \underline{\text{Ans.}}$$

---

10-21 OMITTED

---

10-22 (a) $D = 0.654 - 0.055 = 0.5$

Table 10-5: $A = 186$ kpsi, $m = 0.16$

Eq. (10-17):

$$S_{ut} = \frac{186}{(0.055)^{0.163}} = 298 \text{ k}$$

Eq. (10-39): $S_y = 0.78(298) = 232$

$C = D/d = 0.599/0.055 = 10.89$

Eq. (10-32):

$$K_i = \frac{4(10.89)^2 - 10.89 -}{4(10.89)(9.89)}$$

$$= 1.0735$$

Eq. (10-33):

$$Fr = \frac{\pi d^3 \sigma}{32K_i} = \frac{\pi(0.055)^3(232)(10}{32(1.0735)}$$

$$= 3.53 \text{ lb·in}$$

(b) Eq. (10-36):

$$k' = \frac{(0.055)^4(30)(10^6)}{10.8(0.599)(6)}$$

$$= 7.07 \text{ lb·in/revolution}$$

At $Fr = 3.53$ lb·in

$n = Fr/k' = 3.53/7.07 = 0.499$ rev.

$D_i = D_o - 2d = 0.654 - 2(0.055)$

$= 0.544$ in

$N' = 6 + 0.499 = 6.499$ turns, so

$$D'_i = \frac{N}{N'} D_i = \frac{6}{6.499}(0.544) = 0.502 \text{ i}$$

(c) $\theta = 0.499(360°) = 179.6°$ An

---

10-23 Table 10-6: $C_d = 0.0036$, so

$d = 0.055(1, 0.0036) = (0.055, 0.00$

$C = 10.89$ so, by linear interpolati

for a compression spring,

Table 10-10: $\hat{\sigma}_{Do} = 0.003\ 75$ or, for compression spring

$$C_{Do} = \hat{\sigma}_{Do}/D_o = 0.003\ 75/0.654$$

$$= 0.005\ 73$$

So, for this torsion spring

$$C_{Do} = 0.8(0.005\ 73) = 0.004\ 59$$

s the OD is specified as

$= 0.654(1,\ 0.004\ 59) = (0.654,\ 0.003)$

$\underset{\sim}{D}_o - \underset{\sim}{d}$

$(0.654,\ 0.003) - (0.055,\ 0.000\ 198)$

$(0.599,\ 0.003) = 0.599(1,\ 0.005)$ in

now have

$$= \frac{\underset{\sim}{d}^4 \underset{\sim}{E}}{10.8\underset{\sim}{D}N}$$

$$= \frac{[0.055(1,\ 0.0036)]^4[30(1,\ 0.02)](10^6)}{10.8[0.599(1,\ 0.005)](6)}$$

.e 4-4:

$$= \{[4(0.0036)]^2 + (0.02)^2 + (0.005)^2\}^{\frac{1}{2}} = 0.025$$

) $k' = 7.072$ lb·in/ rev

$k' = 7.072(1,\ 0.025)$

$= (7.072,\ 0.177)$

; $T = \pm 3(0.177) = \pm 0.53$ lb·in/ rev

---

24 Let $T = Fr$; then $T_{min} = 0.2T_{max}$

$$= \frac{T_{max} - T_{min}}{2} = \frac{T_{max} - 0.2T_{max}}{2} = 0.4T_{max}$$

$$= \frac{T_{max} + T_{min}}{2} = 0.6T_{max}$$

b. 10-22: $K_i = 1.0735$

(10-33): $\sigma_m = K_i \dfrac{32T_m}{\pi d^3} = \dfrac{1.0735(32)(0.6T_{max})}{\pi(0.055)^3}$

$= 39.4(10^3)T_{max}$ psi

b. 10-22: $S_{ut} = 298$ kpsi

(10-40): $S_e = 78.0$ kpsi

ng the modified Goodman line

$$+ \frac{\sigma_m}{S_{ut}} = 1$$

$$\left(\frac{26.3}{78.0} + \frac{39.4}{298}\right) = 1$$

$= 2.13$ lb·in <u>Ans.</u>

---

10-25 OMITTED

---

10-26 Table 10-6: The half range on d is $\Delta d = 0.0006$ in

$$C = \frac{D}{d} = \frac{D_o - d}{d} = \frac{0.600 - 0.060}{0.060} = 9$$

$N_a = N_t - 2 = 5 - 2 = 3$

$$\frac{N_a}{L_0} = \frac{3}{5} = 0.6$$

Now enter Table 10-7 and use 2-way interpolation. $T/L_0 = 0.0131$

$T = 0.0131L_0 = 0.0131(5) = 0.0655$

$\cong 0.066$ in

Similarly, from Table 10-10

$\hat{\sigma}_{Do} = 0.003\ 184$ in

$3\hat{\sigma}_{Do} = 3(0.003\ 184) = 0.009\ 55$

$\cong 0.010$ in

Summary: $d = 0.060 \pm 0.0006$ in

$L_0 = 5.000 \pm 0.066$ in

$D_o = 0.600 \pm 0.010$ in

---

PROBLEM A manufacturer has on hand a supply of springs in which the spring rate is $k \sim N(4.0,\ 0.10)$ lb/in. There is an application for a spring in which the rate must exceed 4 lb/in. It is decided to use a fixture which compresses the spring a definite amount and shows the corresponding force. Using the fixture the springs are separated into two groups, one group larger than 4 lb/in and the other smaller. If springs are selected at random from the bin containing the higher spring rate, estimate the mean and the standard deviation of the spring rate.

---

11-1 1800 h

---

11-2 OMITTED

---

11-3 Eq. (11-1):

$$\frac{L}{L_{10}} = \left(\frac{2.89}{3.8}\right)^{1/3} = 0.913$$

Eq. (11-6):

$$R = \exp\left[-\left(\frac{0.913 - 0.02}{4.439}\right)^{1.483}\right]$$

$$= 0.9115 \quad \text{Ans.}$$

---

11-4 Eq. (11-6): Rewrite as

$$R = \exp\left[-\left(\frac{L - 0.02L_{10}}{4.439L_{10}}\right)^{1.483}\right]$$

Substituting, gives

$$0.96 = \exp\left[-\left(\frac{1800 - 0.02L_{10}}{4.439L_{10}}\right)^{1.483}\right]$$

Now take the natural log of both sides and solve. Thus

$$L_{10} = \frac{1800}{0.4936} = 3373 \text{ h} \quad \text{Ans.}$$

---

11-5 The average median life is between 4 and 5 times the $L_{10}$ life.

(a) Assume $L_{av} = 4L_{10}$

Eq. (11-3: $F_C = 1570(\frac{1}{4})^{1/3} = 989$ lb Ans.

(b) Assume $L_{av} = 5L_{10}$, then

$F_C = 1570(\frac{1}{5})^{1/3} = 918$ lb Ans.

So $918 \le F_C \le 989$ lb Ans.

---

11-6 Try $F_e = 9$ kN 02-series deep groove

$L_{10} = (5000)(900)(60)(10^{-6})$

$= 270$ millions

Eq. (11-3): $C = 9(270)^{1/3} = 58$ kN

Table 11-3: Tentative selection is 70-mm bore with $C_0 = 37.5$ kN.

Table 11-2: $\frac{F_a}{C_0} = \frac{4}{37.5} = 0.107$

$X_2 = 0.56$, $Y_2 = 1.45$

Eq. (11-11):

$$F_e = 0.56(8) + 1.45(4) = 10.28$$

Eq. (11-3): $C = 10.28(270)^{1/3} = 66.4$ kN

Use 80-mm bore with $C = 70.2$ kN and $C_0 = 45$ kN

Check: $\frac{F_a}{C_0} = \frac{4}{45} = 0.089$

Table 11-2: $X_2 = 0.56$, $Y_2 = 1.53$

Eq. (11-11): $F_e = 0.56(8) + 1.53(4)$

$= 10.60$ kN

Eq. (11-3): $C = 10.60(270)^{1/3} = 68.5$ kN

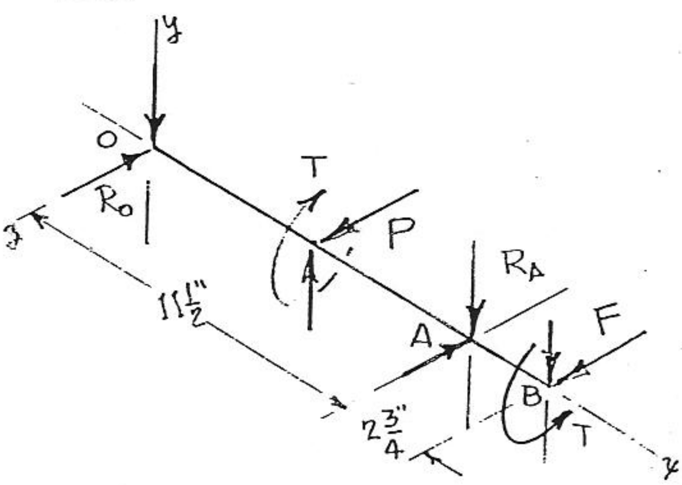
Ok. Ans.

---

11-7 and 11-8 OMITTED

---

11-9




Assume concentrated forces, as shown.

$P_z = 8(24) = 192$ lb

$P_y = 8(30) = 240$ lb

$T = 192(2) = 384$ lb·in

$\Sigma T^x = -384 + 1.5F \cos 20° = 0$

$$F = \frac{384}{1.5(0.940)} = 272 \text{ lb}$$

11-9 (Continued)

$\Sigma M_O^{\ z} = 5.75P_y + 11.5R_A^{\ y}$

$\qquad - 14.25F \sin 20° = 0$; thus

$5.75(240) + 11.5R_A^y - 14.25(272)(0.342) = 0$

$R_A^{\ y} = -4.73$ lb

$\Sigma M_O^{\ y} = -5.75P_z - 11.5R_A^{\ z}$

$\qquad -14.25F \cos 20° = 0$; thus

$-5.75(192) + 11.5R_A^z - 14.25(272)(0.940) = 0$

$R_A^{\ z} = -413$ lb; $R_A = [(-413)^2 + (-4.73)^2]^{\frac{1}{2}}$

$\qquad = 413$ lb

$\Sigma F^z = R_O^{\ z} + P_z + R_A^{\ z} + F \cos 20° = 0$

$R_O^{\ z} + 192 - 413 + 272(0.940) = 0$

$R_O^{\ z} = -34.6$ lb

$\Sigma F^y = R_O^{\ y} + P_y + R_A^{\ y} - F \sin 20° = 0$

$R_O^{\ y} + 240 - 4.73 - 272(0.342) = 0$

$R_O^{\ y} = -142$ lb

$R_O = [(-34.6)^2 + (-142)^2]^{\frac{1}{2}} = 146$ lb

So the reaction at A governs.

$L_{10} = 30(10^3)(60)(300)(10^{-6})$

$\qquad = 540$ millions of revolutions

Since the applications factor is 1.2 the bearing must be selected for a force of

$F = 1.2(413) = 496$ lb

Eq. (11-3): $C = 496(540)^{1/3} = 4039$ lb

Converting to kN gives

$C = 4039(4.45)(10^{-3}) = 18.0$ kN

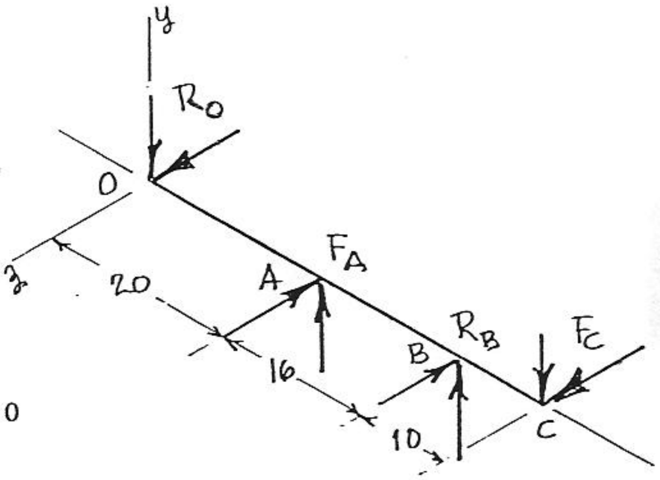
Table 11-3: Use 30-mm bore deep-groove bearings rated at 19.5 kN.

---

11-10 Static analysis omitted.

$R_O = 610$ lb, $R_B = 1650$ lb

The equivalent load for the bearing




at O is $F_e = 1.4(610) = 854$ lb

$$C = 854\left[\frac{50(10)^3(60)(480)}{10^6}\right]^{1/3} = 9640 \text{ lb}$$

or, in kN, $C = 9.64(4.45) = 42.9$ kN

So use a 55-mm bore, 02 series, ball bearing at O rated at 43.6 kN.

For the roller bearing

$F_e = 1.4(1650) = 2310$ lb

$$C = 2310\left[\frac{50(10)^3(60)(480)}{10^6}\right]^{3/10}$$

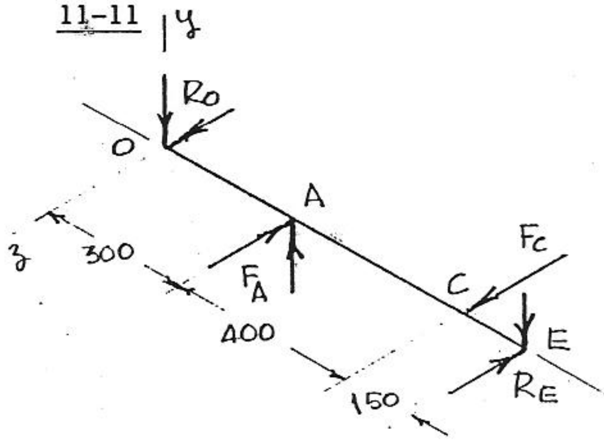
$\qquad = 20.5(10^3)$ lb

In kN, $C = 20.5(4.45) = 91.2$ kN

So use a 55-mm bore, 03 series, straight roller bearing at B rated at 102 kN.

---

11-11



11-11 (Concluded)

$R_O^y = 164$ N, $R_O^z = 107$ N, $R_O = 196$ N

$R_E^y = 89.3$ N, $R_E^z = 174$ N, $R_E = 196$ N

Eq. (11-9):

$$F_R = 0.196\left[\frac{60(10)^3(1800)(60)(10^{-6})}{0.02 + 4.439\left(\ln \frac{1}{0.999}\right)^{1/1.483}}\right]^{\frac{1}{3}}$$

$= 9.226$ kN

A 25-mm 02 series bearing has a rating of 14.0 kN which is ample. An extra light bearing might do.

---

11-12 There is no external thrust, and so Eq. (11-13) is

$$F_{eA} = 0.4F_{rA} + K_A\left(\frac{0.47F_{rB}}{K_B}\right)$$

$$= 0.4(1120) + 1.5\left[\frac{0.47(2190)}{1.5}\right]$$

$= 1477$ lb

$F_{eB} = 0.4(2190) + 0.47(1120) = 1402$ lb

$F_{eA} = 1.4(1477) = 2068$ lb

$F_{eB} = 1.4(1402) = 1963$ lb

$$F_{RA} = 2068\left[\frac{40(10^3)(400)}{3000(500)}\right]^{3/10}$$

$= 4207$ lb Ans.

$$F_{RB} = 1963\left[\frac{40(10^3)(400)}{3000(500)}\right]^{3/10}$$

$= 3993$ lb Ans.

---

11-13 OMITTED

---

11-14 Bearing A:

$F_r = [(35)^2 + (212)^2]^{\frac{1}{2}} = 215$ lb

$F_a = 555$ lb, $\frac{F_a}{F_r} = \frac{555}{215} = 2.58$

Table 11-3: Assume 85-mm deep-groove bearing. $C_0 = 63.0$ kN

$$\frac{F_a}{C_0} = \frac{0.555(4.45)}{63.0} = 0.0392$$

Table 11-2: Try $X_2 = 0.56$, $Y_2 = 1.85$

Eq. (11-11):

$$F_e = 0.56(215)\left(\frac{4.45}{1000}\right) + 1.85(555)\left(\frac{4.45}{1000}\right) = 5.10 \text{ kN}$$

With application factor

$F_D = 1.3(5.10) = 6.63$ kN

$$\frac{L}{L_{10}} = \frac{25\ 000}{10^6}(60)(600) = 900$$

Eq.(11-9):

$$F_R = 6.64\left[\frac{900}{0.02 + 4.439\left(\ln \frac{1}{.99}\right)^{1/1.483}}\right]^{1/3}$$

$= 106.3$ kN

But an 85-mm bearing is rated at 90.4kN

Try 95 mm; $C = 121$ kN, $C_0 = 85.0$ kN

$$\frac{F_a}{C_0} = \frac{0.555(4.45)}{85.0} = 0.029$$

Table 11-2: $Y_2 = 1.98$

$$F_e = \frac{4.45}{1000}[0.56(215) + 1.98(555)]$$

$= 5.43$ kN

$F_D = 1.3(5.43) = 7.05$ kN

$F_R = 7.05[\quad]^{1/3} = 112.9$ kN

So use a 95-mm angular contact ball bearing at point A.

Bearing at B:

$F_r = [(35)^2 + (66)^2]^{\frac{1}{2}} = 74.7$ lb

$$F_D = 74.7(1.3)\left(\frac{4.45}{1000}\right) = 0.432 \text{ kN}$$

$$C = 0.432\left[\frac{900}{0.02 + 4.439\left(\ln \frac{1}{.99}\right)^{1/1.483}}\right]^{3/10}$$

$= 5.24$ kN

Table 11-4: Use 02-series cylindrical roller bearing having a 25 mm bore and $C = 16.8$ kN

---

11-15 Eq. (4-28):

$$\tilde{x} = 4.48[\ln (1/0.5)]^{2/3} = 3.51$$

$$x_{10} = 4.48[\ln(1/0.9)]^{2/3} = 0.999$$

$$\frac{\tilde{x}}{x_{10}} = \frac{3.51}{0.999} = 3.51$$

Eq. (4-25):

$$\bar{x} = 4.48\Gamma(1 + \frac{1}{3/2}) = 4.48\Gamma(1.67) = 4.05$$

$$\frac{\bar{x}}{x_{10}} = \frac{4.05}{0.999} = 4.05$$

---

11-16 Eq. (4-28):

$$\tilde{x} = 0.0403 + 6.58[\ln (1/0.5)]^{1/1.165}$$
$$= 4.844$$

$$x_{10} = 0.0403 + 6.58[\ln (1/0.9)]^{1/1.165}$$
$$= 0.994$$

$$\frac{\tilde{x}}{x_{10}} = \frac{4.844}{0.994} = 4.87$$

---

11-17 OMITTED

---

11-18

$$x_1 = \frac{60(115)(2000)}{10^6} = 13.8$$

$$x_2 = \frac{60(600)(2000)}{10^6} = 72$$

From $R = \exp\left[-\left(\frac{x}{\theta}\right)^b\right]$

$$[\ln (1/R_1)]^{1/b} = x_1/\theta$$

$$[\ln (1/R_2)]^{1/b} = x_2/\theta$$

Then

$$b = \frac{\ln\left[\frac{\ln (1/R_1)}{\ln (1/R_2)}\right]}{\ln (x_1/x_2)} = \frac{\ln\left[\frac{\ln (1/0.9)}{\ln (1/0.2)}\right]}{\ln (115/600)}$$
$$= 1.65$$

Eq. (11-10):

$$\left(\frac{F_R}{F_D}\right)^3 = \frac{x}{\theta[\ln (1/R)]^{1/b}} = \left(\frac{39\ 600}{18\ 000}\right)^3$$
$$= 10.65$$

$$\theta = \frac{x_1}{10.65[\ln (1/R_1)]^{1/b}}$$

$$= \frac{13.8}{10.65[\ln (1/0.9)]^{1/1.65}} = 5.07$$

and

$$R = \exp\left[-\left(\frac{x}{5.07}\right)^{1.65}\right]$$

---

11-19 First, estimate b and θ.

$$x_1 = [60(360)(2000)]/10^6 = 43.2$$

$$x_2 = [60(2000)(2000)]/10^6 = 240$$

From Prob. 11-18 solution

$$b = \frac{\ln\left[\frac{\ln (1/R_1)}{\ln (1/R_2)}\right]}{\ln (x_1/x_2)} = \frac{\ln\left[\frac{\ln (1/0.9)}{\ln (1/0.2)}\right]}{\ln (43.2/240)}$$
$$= 1.59$$

$$\theta = \frac{x_1}{10.65[\ln (1/R_1)]^{1/b}}$$

$$= \frac{43.2}{10.65[\ln (1/0.9)]^{1/1.59}} = 16.70$$

At any reliability level

$$R = \exp\left[-\left(\frac{x_1}{\theta_1}\right)^{b_1}\right] = \exp\left[-\left(\frac{x_2}{\theta_2}\right)^{b_2}\right]$$

$$\left(\frac{x_1}{\theta_1}\right)^{b_1} = \left(\frac{x_2}{\theta}\right)^{b_2}$$

For comparable exponents $b_1 \cong b_2$

$$\frac{x_2}{x_1} = \frac{\theta_2}{\theta_1} = \frac{16.7}{5.07} = 3.3$$

So the increase is at least threefold everywhere.

For different exponents $b_1 \neq b_2$, let $x_2/x_1 = r$. Then

$$\left(\frac{x_1}{\theta_1}\right)^{b_1} = \left(\frac{rx_1}{\theta_2}\right)^{b_2}$$

**11-19** (Concluded)

$$\left(\frac{43.2}{5.07}\right)^{1.65} = \left(\frac{43.2r}{16.7}\right)^{1.59}$$

$$r = \left(\frac{34.3}{4.532}\right)^{1/1.59} = 3.57$$

So the increase in life is at least threefold at the $B_{10}$ level.

---

**11-20** Eq. (11-1):

$$F_a{}^a L_1 = F_2{}^a L_2 = K = \text{constant}$$

K can be found from the basic load rating of 20.3 kN and rating life of $10^6$ revolutions.

$$K = (20.3)^3(10^6) = 8.365(10^9)$$

At a load of 18 kN life $L_1$ is

$$L_1 = \frac{K}{F_a{}^a} = \frac{8.365(10^9)}{(18)^3} = 1.434(10^6) \text{ rev}$$

At a load of 30 kN life $L_2$ is

$$L_2 = \frac{K}{F_2{}^a} = \frac{8.365(10^9)}{(30)^3} = 0.310(10^6) \text{ rev}$$

Eq. (7-46): (Miner's rule)

$$\frac{\ell_1}{L_1} + \frac{\ell_2}{L_2} = 1$$

$$\frac{200\ 000}{1.434(10^6)} + \frac{\ell_2}{0.310(10^6)} = 1$$

$$\ell_2 = 0.267(10^6) \text{ rev} \qquad \underline{\text{Ans.}}$$

Check:

$$\frac{200\ 000}{1.434(10^6)} = \frac{0.267(10^6)}{0.310(10^6)} = 0.14 + 0.86 = 1$$

---

**11-21** This problem was devised after the page proof had been received. Since there may not have been space for it, here is the problem statement.

The same 02-30 angular-contact ball bearing is to be subjected to a two-step loading cycle of 4 min with a loading of 18 kN and 6 min with a loading of 30 kN. This cycle is to be repeated until failure. Estimate the total life in revolutions.

Define: $\ell$ = total turns

$f_1$ = fraction of turns at load $F_1$

$f_2$ = fraction of turns at load $F_2$

From Prob. 11-20:

$$L_1 = 1.434(10^6) \text{ rev}$$
$$L_2 = 0.310(10^6) \text{ rev}$$

From Miner's rule [Eq. (7-46)]:

$$\frac{\ell_1}{L_1} + \frac{\ell_2}{L_2} = 1, \quad \frac{f_1\ell}{L_1} + \frac{f_2\ell}{L_2} = 1$$

$$\ell = \frac{1}{\dfrac{f_1}{L_1} + \dfrac{f_2}{L_2}} = \frac{1}{\dfrac{0.40}{1.434(10^6)} + \dfrac{0.60}{0.310(10^6)}}$$

$$= 452\ 000 \text{ rev} \qquad \underline{\text{Ans.}}$$

---

PROBLEM When the misalignment in a rolling contact bearing exceeds some threshold amount (0.001 rad in the case of cylindrical roller bearings) there is a life reduction penalty fraction $f_r$ imposed. This factor would appear in Eq. (11-9) as

$$F_R = (AF)F_D\left[\frac{x_D/f_r}{x_0 + (\theta - x_0)[\ln(1/R)]^{1/b}}\right]^{1/a}$$

where $x_D$ is the design life expressed in multiples of the rating life, and AF is the application factor. When a bearing is selected the extant reliability is higher than the goal. Show that the realized reliability $R'$ is given by

$$R' = \exp\left[-\left(\frac{x_D/f_r - x_0[C_R/(AF\cdot F_D)]^a}{(\theta - x_0)[C_R/(AF\cdot F_D)]^a}\right)^b\right]$$

---

12-1 $r = 0.5$ in, $r/c = 0.5/0.000\,75 = 667$

$N = 1100/60 = 18.3$ rev/s

$P = 250/1 = 250$ psi

$$S = (667)^2 \frac{8(10^{-6})(18.3)}{250} = 0.261$$

$$\frac{h_0}{c} = 0.595; \ \frac{r}{c}f = 5.8; \ \frac{Q_s}{Q} = 0.5$$

$$\frac{Q}{rcN\ell} = 3.98$$

$h_0 = 0.595().000\,75) = 0.000\,446$ in Ans.

$$f = \frac{5.8}{r/c} = \frac{5.8}{667} = 0.0087$$

$$H = \frac{2\pi fWrN}{778(12)} = \frac{2\pi(0.0087)(250)(0.5)(18.3)}{778(12)}$$

$= 0.0134$ Btu/s Ans.

$Q = 3.98(0.5)(0.000\,75)(18.3)(1)$

$= 0.0273$ in$^3$/s

$Q_s = 0.5(0.0273) = 0.0137$ in$^3$/s Ans.

---

12-2 OMITTED

---

12-3 $r = 0.625$ in, $r/c = 0.625/0.001 = 625$

$$N = \frac{1150}{60} = 19.2 \text{ rev/s}$$

$$P = \frac{400}{1.25(2.5)} = 128 \text{ psi}$$

$\ell/d = 2$

$$S = (625)^2 \frac{10(10^{-6})(19.2)}{128} = 0.586$$

Use $\ell/d = \infty$; $h_0/c = 0.96$; $P/p_{max} = 0.83$

$Q/rcN\ell = 3.11$

$h_0 = 0.96(0.001) = 0.000\,96$ in Ans.

$Q = 3.11(0.625)(0.001)(19.2)(2.5)$

$= 0.0933$ in$^3$/s Ans.

$$p_{max} = \frac{128}{0.83} = 154 \text{ psi}$$ Ans.

---

12-4 $r = 1.5$ in, $\ell/d = 0.5$

$r/c = 1.5/0.0025 = 600$

$N = 600/60 = 10$ rev/s

$P = 800/[1.5(3)] = 178$ psi

SAE 10 $\mu = 1.75$ μreyn

$$S = (600)^2 \frac{1.75(10^{-6})(10)}{178} = 0.0354$$

$h_0/c = 0.11$; $P/p_{max} = 0.21$

$h_0 = 0.11(0.0025) = 0.000\,275$ in Ans.

$$p_{max} = \frac{178}{0.21} = 848 \text{ psi}$$ Ans.

SAE 40 $\mu = 4.75$ μreyn

$$S = 0.0354 \frac{4.75}{1.75} = 0.0961$$

$h_0/c = 0.21$; $P/p_{max} = 0.275$

$h_0 = 0.21(0.0025) = 0.000\,525$ in Ans.

$$p_{max} = \frac{178}{0.275} = 647 \text{ psi}$$ Ans.

---

12-5 and 12-6 OMITTED

---

12-7 $r = 12.5$ mm, $r/c = 12.5/0.02 = 625$

$N = 1200/60 = 20$ rev/s

$P = 1250/(25)^2 = 2$ MPa

$$S = (625)^2 \frac{50(10^{-3})(20)}{2(10^6)} = 0.195$$

$h_0/c = 0.525$; $(r/c)f = 4.5$

$Q_s/Q = 0.57$

$h_0 = 0.525(0.02) = 0.0105$ mm Ans.

$f = 4.5/625 = 0.0072$

$T = fWr = 0.0072(1.25)(12.5) = 0.1125$

$H = 2\pi TN = 2\pi(0.1125)(20) = 14.14$ J/s Ans.

$Q_s = 57\%$ of $Q$ Ans.

---

12-8 OMITTED

---

12-9 $r = 75/2 = 37.5$ mm

$\ell/d = 36/75 \cong \frac{1}{2}$, $N = 720/60 = 12$ rev/s

$r/c = 37.5/0.05 = 750$

$P = 2000/[75(36)] = 0.741$ MPa

SAE 20 $\mu = 18.5$ mPa·s

$$S = (750)^2 \frac{18.5(10^{-3})(12)}{0.741(10^6)} = 0.169$$

$h_0/c = 0.29$; $(r/c)f = 5.1$; $P/p_{max} = 0.315$

$h_0 = 0.29(0.05) = 0.0145$ mm Ans.

$f = 5.1/750 = 0.0068$

$T = fWr = 0.0068(2)(37.5) = 0.51$ N·m

$H = 2\pi TN = 2\pi(0.51)(12) = 38.5$ W Ans.

$p_{max} = 0.741/0.315 = 2.35$ MPa Ans.

---

12-10 $r = 25$ mm; $N = 840/60 = 14$ rev/s

$r/c = 25/0.025 = 1000$

$\ell/d = 25/50 = 0.5$

$P = 2000/[25(50)] = 1.6$ MPa

$\mu = 34$ mPa·s

$$S = (1000)^2 \frac{34(10^{-3})(14)}{1.6(10^6)} = 0.2975$$

$h_0/c = 0.395$; $(r/c)f = 7.8$

$Q_s/Q = 0.74$; $Q/rcN\ell = 4.9$

$h_0 = 0.395(0.025) = 0.0099$ mm Ans.

$f = 7.8/1000 = 0.0078$

$T = fWr = 0.0078(2)(25) = 0.390$ N·m

$H = 2\pi TN = 2\pi(0.390)(14) = 34.3$ W Ans.

$Q = 4.9\ rcN\ell$

$4.9\ (25)(0.025)(14)(25) = 1072$ mm³/s

$Q_s = 0.74(1072) = 793$ mm³/s Ans.

---

12-11 $\ell/d = 1$; $P = 700/(1.25)^2$

$= 448$ psi

$N = 3600/60 = 60$ rev/s

Fig. 12-14:

Minimum f: $S = 0.08$

Maximum W: $S = 0.2$

Fig. 12-11: $\mu = 1.42$ μreyn

$$\frac{\mu N}{P} = \frac{1.42(10^{-6})(60)}{448} = 0.190(10^{-6})$$

Eq. (12-7): $\frac{r}{c} = \left(\frac{S}{\mu N/P}\right)^{\frac{1}{2}}$

For minimum f:

$$\frac{r}{c} = \left[\frac{0.08}{0.190(10^{-6})}\right]^{\frac{1}{2}} = 649$$

$c = 0.625/649 = 0.000\ 963$ in Ans.

For maximum W:

$$\frac{r}{c} = \left[\frac{0.2}{0.190(10^{-6})}\right]^{\frac{1}{2}} = 1026$$

$c = 0.625/1026 = 0.000\ 609$ in Ans.

Clearance range is

$\Delta c = 0.000963 - 0.000\ 609$

$= 0.000\ 354$ in

No. This range is entirely too small.

---

12-12 $N = 8$ rev/s; $W = 3000$ N;

$r = 40$ mm; $c = 0.04$ mm; $L = 80$ mm;

$T_1 = 60°C$;

$P = 3000/[80(80)] = 0.469$ MPa

Try $\mu = 12$ mPa·s $T = 81°C$;

$\Delta T = 2(81 - 60) = 42°C$; $S = 0.2048$;

$(r/c)f = 4.6$; $Q/rcN\ell = 4.1$;

$Q_s/Q = 0.56$

Eq. (12-19):

$$\Delta T = \frac{8.30(0.469)}{1 - 0.5(0.56)} \frac{4.6}{4.1} = 6.07°C$$

The discrepancy is $42 - 6.07 = 35.93°C$

Try $\mu = 20$ mPa·s $T = 68°C$;

$\Delta T = 2(68 - 60) = 16°C$; $S = 0.341$;

$(r/c)f = 7.2$; $Q/rcN\ell = 3.86$;

$Q_s/Q = 0.43$

Eq.(12-19): $\Delta T = 9.25°C$

12-12 (Concluded)

Discrepancy is 16 − 9.25 = 6.75°C

Try $\mu$ = 21 mPa·s  T = 65°C;

$\Delta T$ = 2(65 − 60) = 10°C; S = 0.358;

(r/c)f = 7.5; Q/rcNℓ = 3.83;

$Q_s/Q$ = 0.415

Eq. (12-19): $\Delta T$ = 9.62°C

Discrepancy is 10 − 9.62 = 0.38°C  OK

f = 0.0075; $T_2$ = 70°C; $\Delta T$ = 10°C;

H = 45.2 W; $h_0/c$ = 0.67;

$h_0$ = 0.0268 mm; Q = 3922 mm³/s;

$Q_s$ = 1628 mm³/s  Ans.

---

12-13  N = 160/60 = 2.67 rev/s;

W = 8000 N; r = 25 mm; ℓ/d = 3;

c = 0.04167 mm; L = 150 mm; $T_1$ = 25°C

$\mu$ = 330 mPa·s; S = 0.297; $Q_s/Q$ = 0;

(r/c)f = 5.5; Q/rcNℓ = 3.07; $T_{av}$ = 33°C

$\Delta T$ = 16°C  Ans.  $T_2$ = 41°C

$\varepsilon$ = 0.08  Ans.

$P/p_{max}$ = 0.83; $p_{max}$ = 1.067/0.83

= 1.285 MPa  Ans.

---

12-14 and 12-15  OMITTED

PROBLEM  For satisfactory performance of a journal bearing with thick-film lubrication, it is necessary that the largest "stalactite" or "stalagmite" torn from the bushing or journal be able to tumble through the minimum film thickness zone without jamming and gouging more material from the surface causing further damage. Sime the peak-to-valley distance on a ground surface approximates 0.0002 in, Trumpler* recommends that the minimum film thickness be greater than

$$h_0 \geqq 0.0002 + 0.000\,04D$$

where $h_0$ and D are in inches. Review your analysis of any preceding problem of your choice to see if the bearing is satisfactory by this criterion.

---

* Paul R. Trumpler, Design of Film Bearings, Macmillan, 1966 (Copyright held by the author).

---

PROBLEM  Trumpler, in addition to the minimum filmthickness criterion, recommends that the maximum temperature of the lubricant leaving the wedge be less than 250°F, that is $T_{max} \leq$ 250°F, to avoid driving off light hydrocarbon fractions therefy increasing viscosity, maximum film temperature and aggravating the situation to failure. He also recommended that the pressure P at

CONTINUED

---

12-16  N = 20 rev/s; W = 2500 N; r = 19 mm; P = 2500/(38)² = 1.73 MPa

```
P =   1.731302   L/D =  1    R/C =  500.0001    C =  .038
S =              .2888001
S =              .11552
P =   1.731302   L/D =  1    R/C =  678.5715    C =  .028
S =              .212769
VISCOSITY MU =                  40   IN MILLIPAS-S
AVERAGE TEM =                   61.6022  IN DEG C
FRICTION VARIABLE =             4.8
FLOW VARIABLE =                 4.1
FLOW RATIO =     .55
COEF OF FRICTION =              7.073685E-03
TEMP OUT =       73.2044    IN DEG C
POWER LOSS =     42.22298   IN WATTS
```

TRUMPLER PROBLEM 2 (Continued)
at starting (during boundary lubrication) be less than 300 psi, that is

$$P_{start} \le 300 \text{ psi}$$

Additionally he recommended that a design factor $n_d$ be applied to the load and that this factor be at least two, that is

$$n_d \ge 2$$

Apply Trumpler's criteria to a previous problem of your instructor's choice and decide if the contemplated design is satisfactory.

---

12-17 $N = 1750/60 = 29.17$ rev/s; $W = 250$; $r = 0.625$ in; $L = 1.25$ in; $T_1 = 120°F$

$c_{min} = 0.001 - 0.000\ 25 = 0.000\ 75$ in; $c_{max} = 0.001 + 0.000\ 25 = 0.00125$ in

```
P =   160    L/D =  1    R/C =  833.3333     C =  .00075
S =               .3164822
S =               .2531858
S =               .2810362
VISCOSITY MU =                  2.22    IN MICROREYNS
AVERAGE TEM =                   136.7012   IN DEG F
FRICTION VARIABLE =             6.1
FLOW VARIABLE =                 3.96
FLOW RATIO =      .48
COEF OF FRICTION =              .00732
TEMP OUT =        153.4024     IN DEG F
POWER LOSS =      .0224513   IN BTU PER SEC
P =   160    L/D =  1    R/C =  500      C =  .00125
S =               .1139336
S =               .1184909
VISCOSITY MU =                  2.6    IN MICROREYNS
AVERAGE TEM =                   128.9145   IN DEG F
FRICTION VARIABLE =             3.08
FLOW VARIABLE =                 4.33
FLOW RATIO =      .685
COEF OF FRICTION =              .00616
TEMP OUT =        137.8289     IN DEG F
POWER LOSS =      1.889345E-02   IN BTU PER SEC
```

Range of outlet temperatures is 138°F to 153°F <u>Ans.</u>

---

12-18

```
P =   192    L/D =  1    R/C =  625     C =  .002
S =              .1899211
S =              .170929
VISCOSITY MU =                  4.5    IN MICROREYNS
AVERAGE TEM =                   124.2102   IN DEG F
FRICTION VARIABLE =             4.2
FLOW VARIABLE =                 4.19
FLOW RATIO =     .605
COEF OF FRICTION =              .00672
TEMP OUT =       138.4204     IN DEG F
POWER LOSS =     .1266554   IN BTU PER SEC
```

12-18 (Concluded) $N = 1120/60$

$= 18.67$ rev/s

$W = 1200$ lb; $r = 1.25$ in; $c = 0.002$ in;

$L = 2.5$ in; $T_1 = 110°F$

From computer solution $(r/c)f = 4.2$;

$Q/rcN\ell = 4.19$; $Q_s/Q = 0.605$;

$T_{av} = 124.2°F$

(a) $h_0/c = 0.485$

$h_0 = 0.485(0.002) = 0.000\ 97$ in Ans.

(b) $\varepsilon = 0.515$ Ans.

(c) 0.006 72 Ans.

(d) $H = 0.127$ Btu/s Ans.

(e) $Q = 4.19(1.25)(0.002)(18.67)(2.5)$

$= 0.489\ in^3/s$ Ans.

$Q_s = 0.605(0.489) = 0.296\ in^3/s$ Ans.

(f) $P/p_{max} = 0.44$; $p_{max} = 192/0.44$

$= 436$ psi Ans

$\theta_{pmax} = 18°$ Ans.

(g) $\theta_{po} = 80°$ Ans.

(h) $T_{av} = 124°F$ Ans.

(i) $T_2 = 138°F$ Ans.

---

12-19 and 12-20 OMITTED

---

12-21 $\ell' = \frac{1}{2}(2 - \frac{1}{4}) = 0.875$ in

$\ell'/d = 0.875/1.75 = \frac{1}{2}$;

$r/c = 0.875/0.0015 = 583$;

$N = 3000/60 = 50$ rev/s; $P = 600$ psi;

$W = 4r\ell'P = 4(0.875)(0.875)(600)$

$= 1840$ lb

1st trial: $\mu = 1.0$ μreyn at $T = 182°F$

Eq. (12-7):

$$S = (583)^2 \frac{1.0(10^{-6})(50)}{600} = 0.028$$

From charts: $(r/c)f = 1.56$; $\varepsilon = 0.9$

Eq. (12-24):

$$\Delta T = \frac{0.0246}{1 + 1.5(0.9)^2} \frac{1.56(0.028)(1840)^2}{30(0.875)^4}$$

$= 94°F$

$T_{av} = 120 + (94/2) = 167°F$

Discrepancy $= 182 - 167 = 15°F$

Use of a table helps to prevent chart-reading errors.

| Trial No. | 1 | 2 | 3 |
|---|---|---|---|
| $\mu$ | 1.0 | 1.2 | 1.1 |
| T | 182° | 162° | 171° |
| S | 0.028 | 0.034 | 0.031 |
| (r/c)f | 1.56 | 1.70 | 1.60 |
| $\varepsilon$ | 0.90 | 0.89 | 0.9 |
| $\Delta T$ | 94° | 124° | 103° |
| $T_{av}$ | 167° | 183° | 175° |
| Discr. | 15° | -21° | -4° |

Trial #3 is close enough. So

$h_0/c = 0.11$; $h_0 = 0.11(0.0015)$

$= 0.000\ 165$ in Ans.

$P/p_{max} = 0.205$

$p_{max} = 30 + (600/0.205) = 2960$ psi Ans.

---

12-22 OMITTED

---

12-23 $\ell' = \frac{1}{2}(55 - 5) = 25$ mm; $\ell'/d = \frac{1}{2}$;

$r/c = 25/0.042 = 595$;

$P = W/4r\ell' = 10(10^6)/[4(25)(25)]$

$= 4000$ kPa

1st trial: $\mu = 13$ mPa·s at $T = 79°C$

Eq. (12-7):

$$S = (595)^2 \frac{13(10^{-3})(48)}{4000(10^3)} = 0.055$$

From charts: $(r/c)f = 2.3$; $\varepsilon = 0.85$

Eq. (12-25):

$$\Delta T = \frac{1956(10^6)}{1 + 1.5(0.85)^2} \frac{2.3(0.055)(10^2)}{200(25)^4}$$

$= 152°C$

12-23 (Concluded)

$T_{av} = 55 + (152/2) = 131°C$

Discrepancy $\doteq 131 - 79 = 52°C$

| Trial no. | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\mu$ | 13 | 6 | 8 | 8.8 |
| T | 79° | 106° | 95° | 91° |
| S | 0.055 | 0.025 | 0.034 | 0.037 |
| (r/c)f | 2.3 | 1.24 | 1.70 | 1.78 |
| $\varepsilon$ | 0.85 | 0.89 | 0.89 | 0.885 |
| $\Delta T$ | 152° | 36° | 66° | 76° |
| $T_{av}$ | 131° | 73° | 88° | 93° |
| Discr. | 52° | -33° | -7° | 2° |

$\Delta T = 76°C$ Ans.

Eq. (12-21):

$$Q_s = [1 + 1.5(0.885)^2] \times \frac{\pi(200)(0.042)^3(25)}{3[8.8(10^{-6})](25)} = 3830 \text{ mm}^3/\text{s}$$ Ans.

$h_0/c = 0.115$

$h_0 = 0.115(42) = 4.83$ μm Ans.

PROBLEM Journal-bushing assemblies for precision pillow block bearings are randomly selected from journals

$$D^{+0}_{-d} \quad \text{or} \quad 1.500^{+0.0000}_{-0.0005} \text{ in}$$

and bushings

$$B^{+b}_{-0} \quad \text{or } 1.501^{+0.0005}_{-0.0000} \text{ in},$$

all dimensions diametral. The journal speed is to be 1200 rev/min and to carry a load of 600 lb while using an SAE grade 40 lubricant. The oil sump temperature is 100°F, so oil is admitted to the film at this temperature. The bushing has a 1½-in length.

(a) Find the smallest, median and largest radial clearances.

(b) For the tightest possible bearing estimate the minimum film thickness.

(c) The journal formation process results in a random distribution $D \sim U[1.4995, 1.500]$ and, similarly, the bushing bore distribution is $B \sim U[1.5010, 1.5015]$. The resulting radial clearance distribution is triangular in the interval $0.0005 \le c \le 0.0010$ in. Estimate the fraction of the assemblies having a radial clearance less than 0.0006 in.

PROBLEM In the previous problem convince yourself that the standard deviation for the clearance conforms to that predicted by Table 4-4 for the standard deviation of $\underset{\sim}{x} - \underset{\sim}{y}$.

PROJECT The radius of a journal is $\underset{\sim}{y} \sim U[1.000, 1.001]$ in. The radius of a bushing is $\underset{\sim}{x} \sim U[1.002, 1.004]$ in. The radial clearance is $\underset{\sim}{c} = \underset{\sim}{x} - \underset{\sim}{y}$. The distribution of $\underset{\sim}{c}$ is trapezoidal with extremes at c = 0.001 and 0.004 in. The uniform portion exists between 0.002 and 0.003 in.

(a) Compare the mean and standard deviation of $\underset{\sim}{c}$ by calculation and computer simulation.

(b) During your simulation of 10 000 instances of $\underset{\sim}{c} = \underset{\sim}{x} - \underset{\sim}{y}$, collect data for 12 histogram bars and superpose the trapezoidal distribution for comparison.

(c) Compare the probability that $0.0015 \le c \le 0.0035$ by calculation and simulation.

Calculation: $\tilde{c} = 0.0025$ by symmetry.

$\hat{\sigma}_y = (b - a)/2\sqrt{3} = (1.001 - 1)/(2\sqrt{3}$
$= 0.000\ 289$

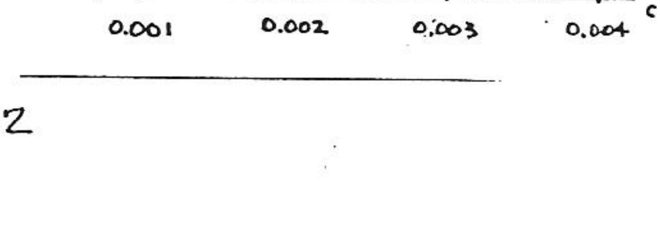
$\hat{\sigma}_x = 0.002/2\sqrt{3}) = 0.000\ 577$

Table 4-4, line 6: $\hat{\sigma}_c = 0.000\ 646$

P = 0.875

Simulation: $\bar{c} = 0.002$, $\hat{\sigma}_c = 0.000\ 645$

P = 0.874. Frequencies: 168, 500, 788, 1045, 1260, 1380, 1255, 1219, 1082, 784, 449, 142.



13-1 $d_P = 17/8 = 2\ 1/8$

$d_G = \frac{N_2}{N_3} d_P = \frac{1120}{544}(2\ 1/8) = 4.375$ in

$N_G = Pd_G = 8(4.375) = 35$ teeth Ans.

$C = (2.125 + 4.375)/2 = 3.25$ in Ans.

---

13-2 $n_G = 1600(15/60) = 400$ rev/min Ans.

$p = \pi m = 3\pi$ mm Ans.

$C = [3(15 + 60)]/2 = 112.5$ mm Ans.

---

13-3 $N_G = 20(2.80) = 56$ teeth Ans.

$d_G = N_G m = 56(4) = 224$ mm Ans.

$d_P = N_P m = 20(4) = 80$ mm Ans.

$C = (224 + 80)/2 = 152$ mm Ans.

---

13-4 $a = 1/P = 1/3 = 0.333$ in Ans.

$b = 1.25/P = 1.25/3 = 0.417$ in Ans.

$c = b - a = 0.0837$ in Ans.

$p = \pi/P = \pi/3 = 1.047$ in Ans.

$t = p/2 = 1.047/2 = 0.523$ in Ans.

Pinion: $d_1 = N_1/P = 21/3 = 7$ in

$d_{1b} = 7 \cos 20° = 6.578$ in Ans.

Gear: $d_2 = N_2/P = 28/3 = 9.33$ in

$d_{2b} = 9.33 \cos 20° = 8.77$ in Ans.

Base pitch: $p_b = p_c \cos \phi$

$= (\pi/3) \cos 20°$

$= 0.984$ in Ans.

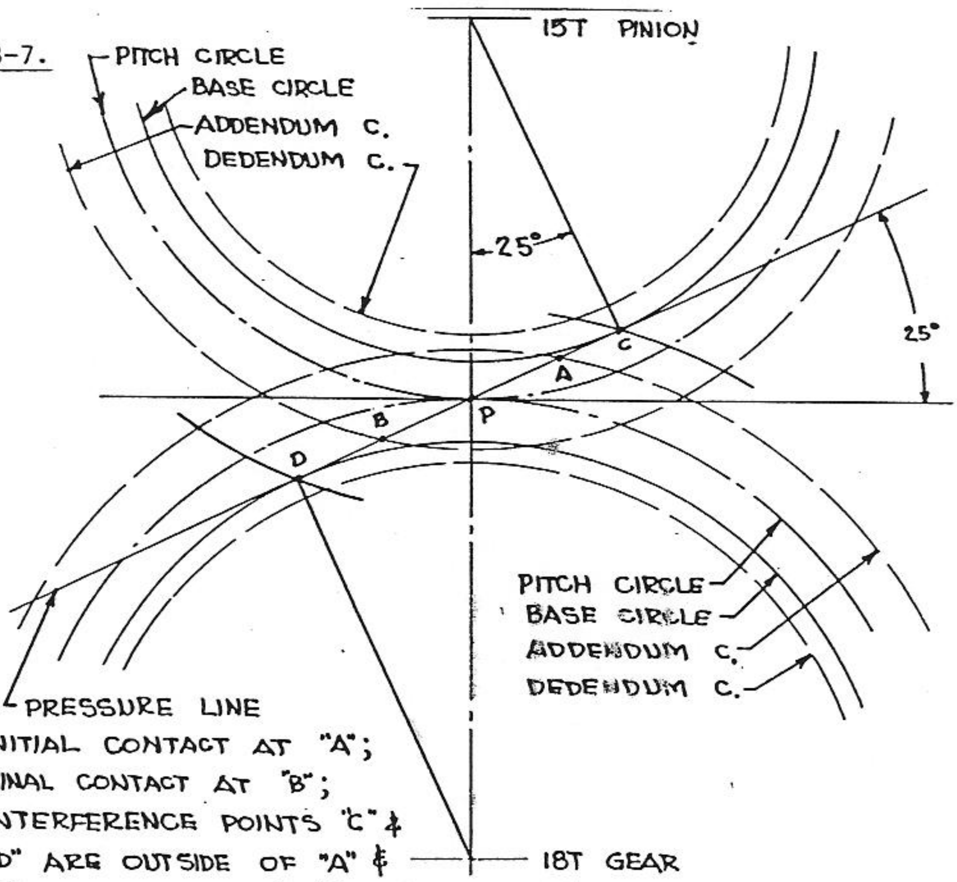
See next page for drawing of gears.

$m_c = L_{ab}/p_b = 1.53/0.984 = 1.55$ Ans.

---

13-5, 13-6 See page

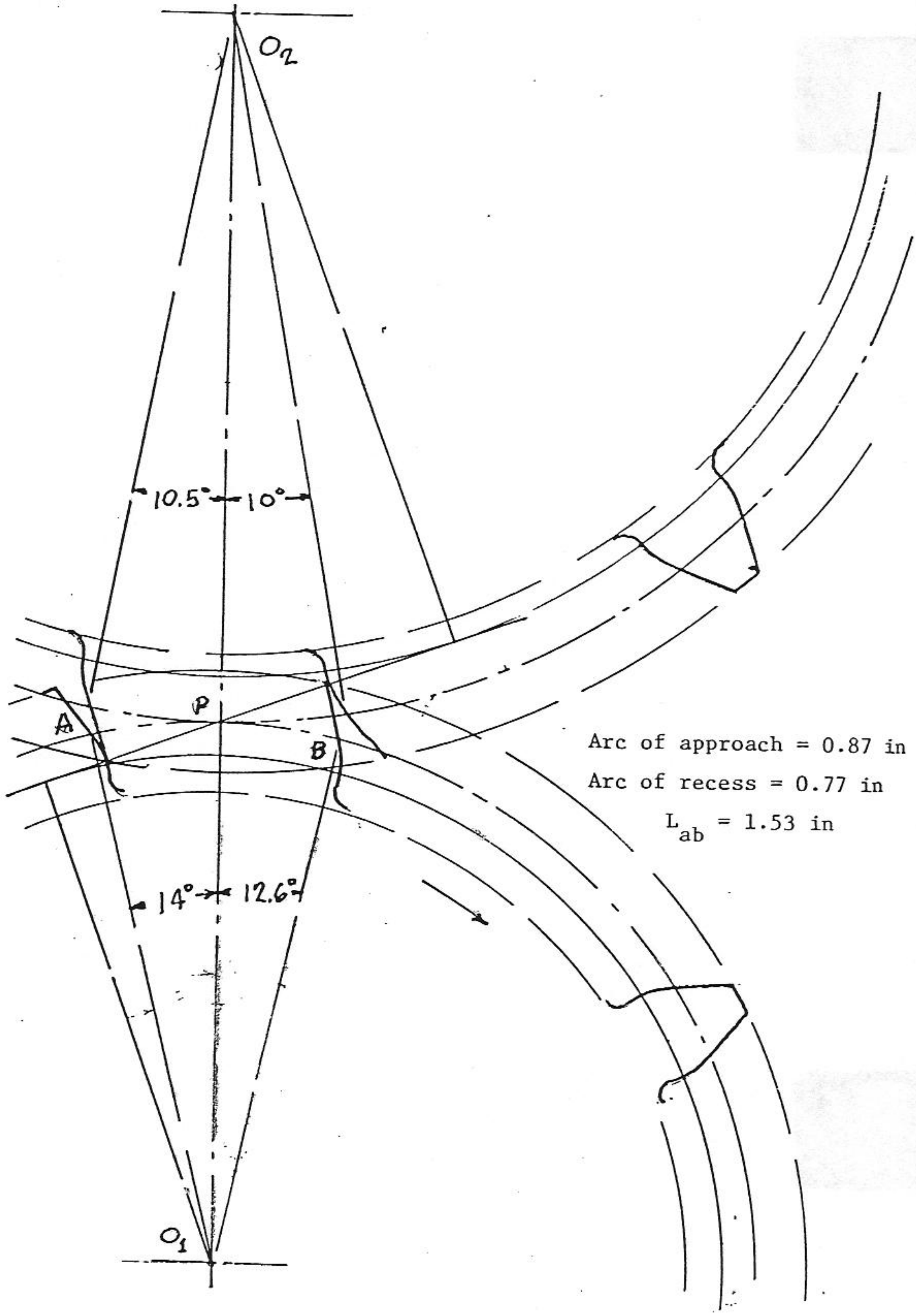
13-7.




INITIAL CONTACT AT "A";
FINAL CONTACT AT "B";
INTERFERENCE POINTS "C" &
"D" ARE OUTSIDE OF "A" &
"B" AND SO THERE IS NO INTERFERENCE.

AB = 1.27", $p_b = \frac{\pi}{3} \cos 25° = 0.95"$, $m_c = \frac{1.27}{0.95} = 1.34$ ANS.

13-4 (Continued)




Arc of approach = 0.87 in

Arc of recess = 0.77 in

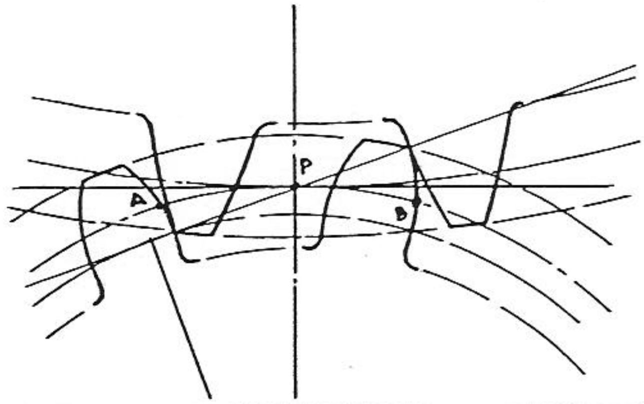
$L_{ab}$ = 1.53 in

13-5.

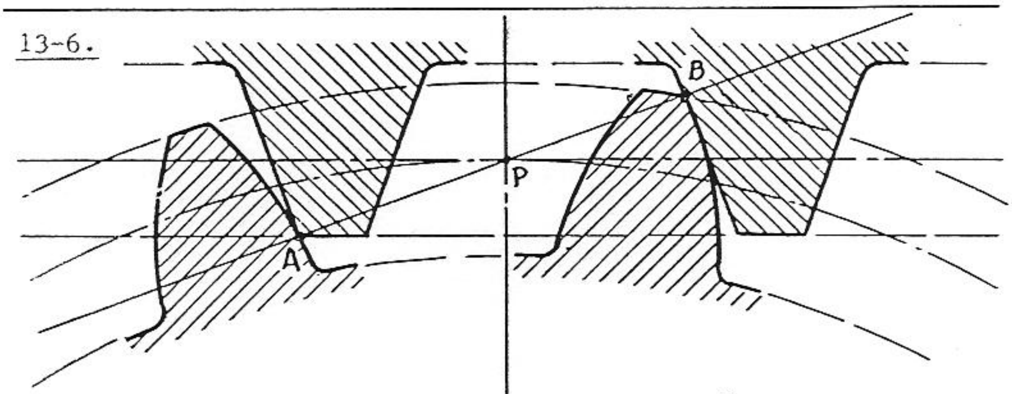
$q_a = 1.07"$, $q_r = 0.99"$,

$q_t = 2.06"$, $p_c = 1.257"$,

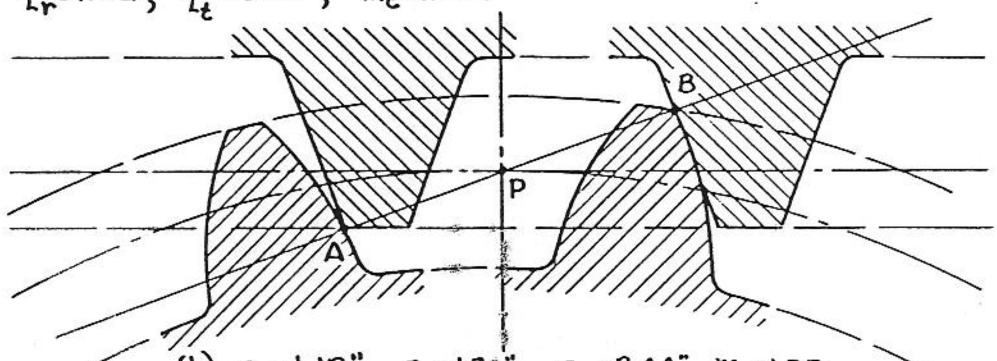
$m_c = 1.64$




13-6.



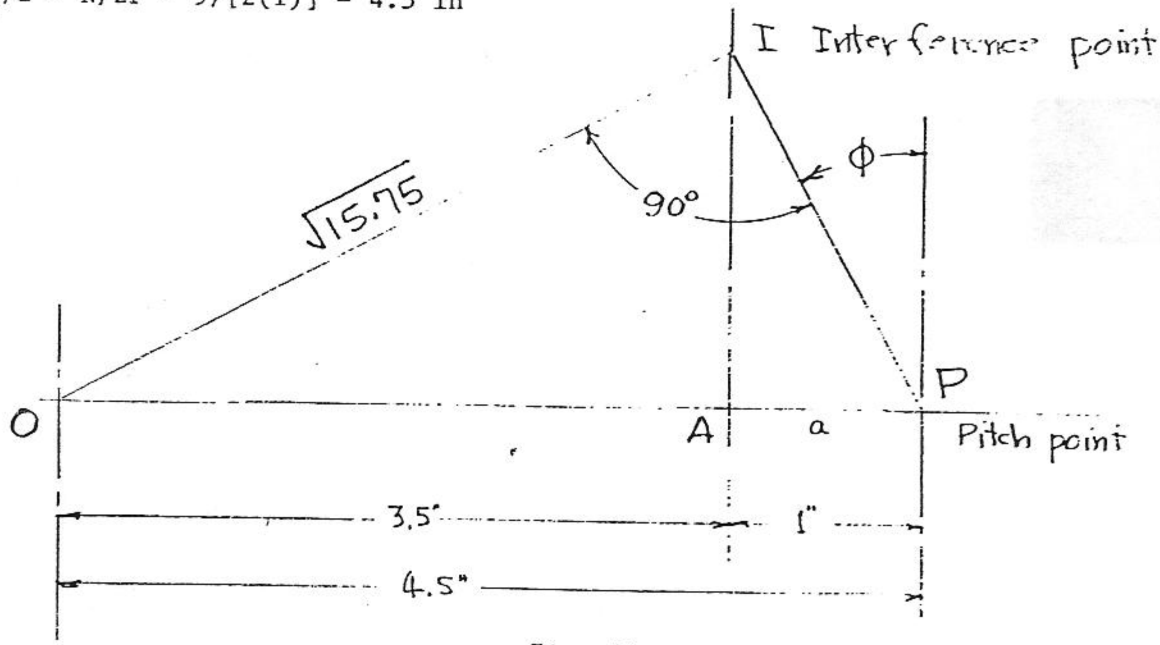

(a) $d = 13"$, $p = \frac{\pi}{2} = 1.57"$, $t = 0.785"$, $a = \frac{1}{2}"$, $b = \frac{5}{8}"$, $q_a = 1.54"$, $q_r = 1.52"$, $q_t = 3.06"$, $m_c = 1.95$.




(b) $q_a = 1.12"$, $q_r = 1.31"$, $q_t = 2.44"$, $m_c = 1.55$

THE PRESSURE ANGLE HAS NOT CHANGED.

13-8 Let P = 1; then a = 1 in. The pitch radius of the pinion is $r_p = d_p/2 = N/2P = 9/[2(1)] = 4.5$ in



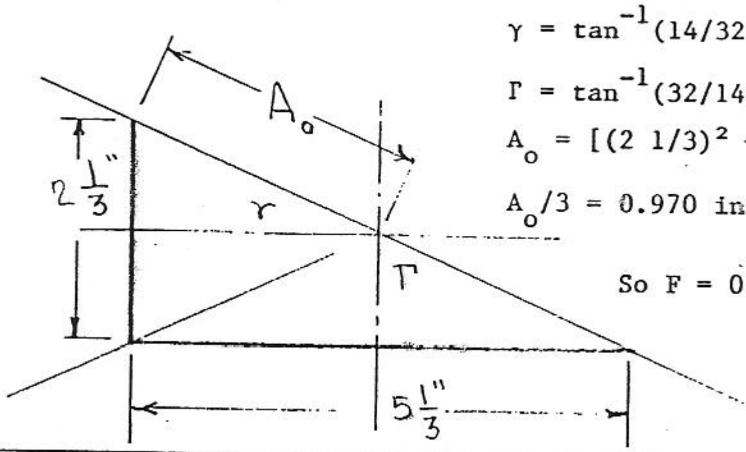

Since $\Delta OIA$ is similar to $\Delta IAP$ we have $\frac{IA}{AP} = \frac{OA}{IA}$ or $\overline{IA}^2 = (\overline{OA})(\overline{AP})$; $\overline{IA} = \sqrt{3.5}$

Then $\overline{OI} = [(3.5)^2 + (\sqrt{3.5})^2]^{\frac{1}{2}} = \sqrt{15.75}$

$$\phi = \cos^{-1}\frac{3.5}{\sqrt{15.75}} \doteq 28.13° \quad \text{Ans.}$$

---

13-9 $d_P = 14/6 = 2\ 1/3$ in; $d_G = 32/6 = 5\ 1/3$ in Ans.




$\gamma = \tan^{-1}(14/32) = 23.63°$ Ans.

$\Gamma = \tan^{-1}(32/14) = 66.37°$ Ans.

$A_o = [(2\ 1/3)^2 + (5\ 1/3)^2]^{\frac{1}{2}} = 2.911$ in

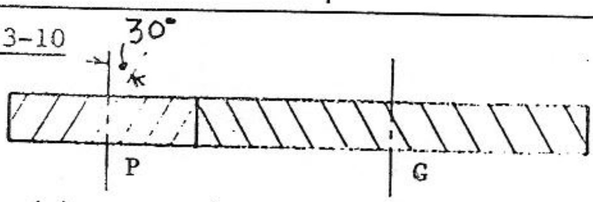
$A_o/3 = 0.970$ in; $10/P = 10/6$ Ans.

$= 1.67$

So F = 0.970 in Ans.

---

13-10




(a) $p_n = \pi/5 = 0.628$ in Ans.

$p_t = p_n/\cos\psi = 0.628/\cos 30°$

$= 0.725$ in Ans.

$p_x = p_t/\tan\psi = 0.725/\tan 30°$

$= 1.256$ in Ans.

(b) $p_{nb} = p_n \cos\phi_n = 0.628 \cos 20°$

$= 0.590$ in Ans.

(c) $P_t = P_n \cos\psi = 5 \cos 30°$

$= 4.33$ teeth/in

13-10 (Concluded)

$\phi_t = \tan^{-1}(\tan \phi_n/\cos \psi)$

$= \tan^{-1}(\tan 20°/\cos 30°) = 22.8°$ Ans.

(d) Table 13-5: $a = 1/5 = 0.20$ in Ans.

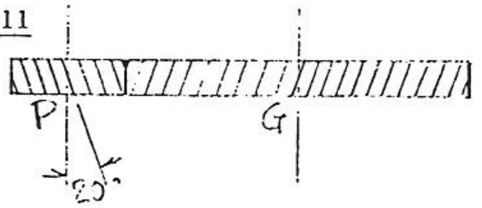
$b = 1.25/5 = 0.25$ in Ans.

$d_P = 17/(5 \cos 30°) = 3.926$ in Ans.

$d_G = 34/(5 \cos 30°) = 7.852$ in Ans.

---

13-11




$\phi_n = 14.5°$; $P_n = 10$ teeth/in

(a) $p_n = \pi/10 = 0.3142$ in Ans.

$p_t = p_n/\cos \psi = 0.3142/\cos 20°$

$= 0.3343$ in Ans.

$p_x = p_t/\tan \psi = 0.3343/\tan 20°$

$= 0.9185$ in Ans.

(b) $P_t = P_n \cos \psi = 10 \cos 20°$

$= 9.397$ teeth/in

$\phi_t = \tan^{-1}(\tan \phi_n/\cos \psi)$

$= \tan^{-1}(\tan 14.5°/\cos 20°)$

$= 15.39°$ Ans.

(c) $a = 1/10 = 0.10$ in Ans.

$b = 1.25/10 = 0.125$ in Ans.

$d_P = 19/(10 \cos 20°) = 2.022$ in Ans.

$d_G = 57/(10 \cos 20°) = 6.066$ in Ans.

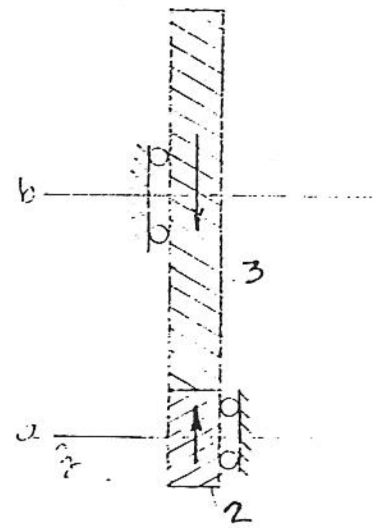
---

13-12 OMITTED

---

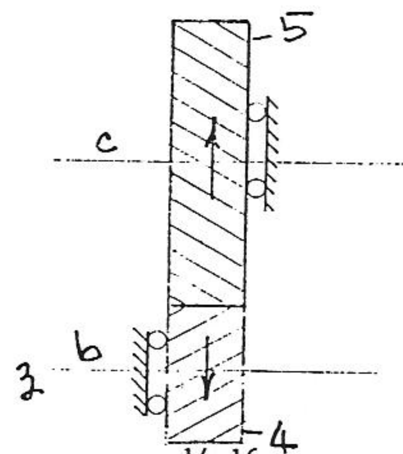
13-13 (a) Axial force of 2 on shaft a is in negative z direction.

Axial force of 3 on shaft b is in positive z direction.




Axial force of gear 4 on shaft b is in the positive z direction.

Axial force of gear 5 on shaft c is in negative z direction.




(b) $n_c = n_5 = \frac{14}{54}\frac{16}{36}(900)$

$= +103.7$ rev/min ccw Ans.

(c) $d_{P2} = 14/(10 \cos 30°) = 1.6166$ in

$d_{G3} = 54/(10 \cos 30°) = 6.2354$ in

$C_{ab} = \frac{1.6166 + 6.2354}{2} = 3.926$ in Ans.

$d_{P4} = 16/(6 \cos 25°) = 2.942$ in

$d_{G5} = 36/(6 \cos 25°) = 6.620$ in

$C_{bc} = 4.781$ in Ans.

---

13-14 $e = \frac{20}{40}\frac{8}{17}\frac{20}{60} = \frac{4}{51}$

$n_d = \frac{4}{51}(600) = 47.1$ rev/min cw Ans.

13-15 and 13-16 OMITTED

13-17 (a) The planet gears act as keys and the wheel speeds are the same as that of the ring gear. Thus

$n_A = n_3 = 1200(17/54) = 377.8$ rev/min Ans.

(b) $n_F = n_5 = 0$; $n_L = n_6$; $e = -1$;

$-1 = \frac{n_6 - 377.8}{0 - 377.8}$; $377.8 = n_6 - 377.8$

or $n_6 = 755.6$ rev/min Ans.

(c) The wheel on the icy surface spins freely leaving no traction for the other wheel. Car is stalled.

13-18 (a) The motive power is divided equally between 4 wheels instead of 2.

(b) Locking the center differential causes 50 percent of the power to be applied to the rear wheels and 50 percent to the front wheels. If one of the rear wheels, say, rests on slippery ice, the other rear wheel has no traction. But the front wheels still provide traction. So you have 2-wheel drive. If the rear differential is locked you have 3-wheel drive because the rear-wheel power is now 50-50 distributed.

13-19 and 13-20 OMITTED

13-21 $e = 9/51$

$n_A + \frac{9}{51}(320 - n_A) = 0$

$n_A = -68.57$ rev/min Ans.

13-22 $e = -8/15$

$(n_F - 5)(-8/15) = -5$

$n_F = +14.375$ turns Ans.

13-23 Let $n_F = n_2 = 0$; then $n_L = n_5$; $e = +0.9999$; then by Eq. (13-24)

$n_L/n_A = 0.0001$ Ans.

(b) Radius $= (101 + 50)(1/10) + 1/10$

$= 15.2$ in

Allowing, say, ½ in clearance gives

ID $= 2(15.2 + 0.5) = 31.4$ in Ans.

13-24, 13-25, 13-26 OMITTED

13-27 (a) $\omega = 2\pi n/60$

$H = T\omega = 2\pi Tn/60$ T in N·m, H in W

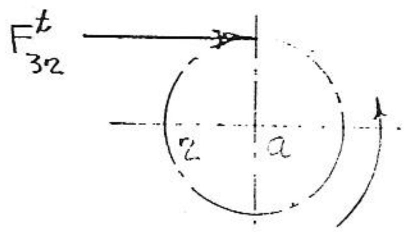
So $T = 60H(10^3)/2\pi n$

$= 9550H/n$ where H is in kW and n in rev/min

$T_a = \frac{9550(75)}{1800} = 398$ N·m

$r_2 = \frac{mN_2}{2} = \frac{5(17)}{2} = 42.5$ mm

So $F^t_{32} = \frac{T_a}{r_2} = \frac{398}{42.5} = 9.36$ kN

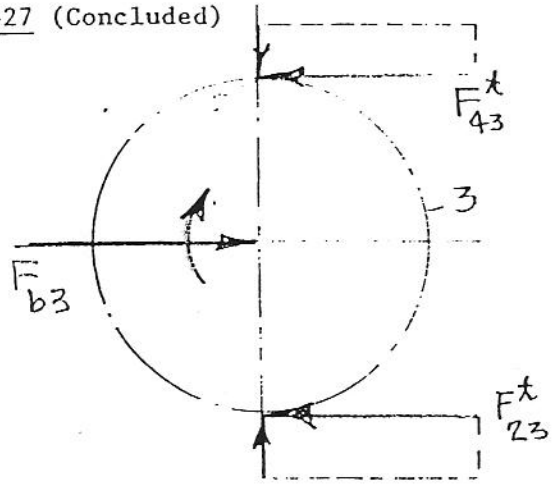



$F_{3b} = -F_{b3} = 2(9.36) = 18.73$ kN

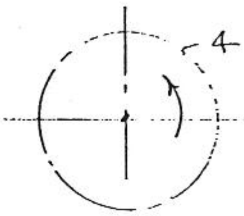
in positive x direction Ans.

(b) $r_4 = \frac{mN_4}{2} = \frac{5(51)}{2} = 127.5$ mm

13-27 (Concluded)

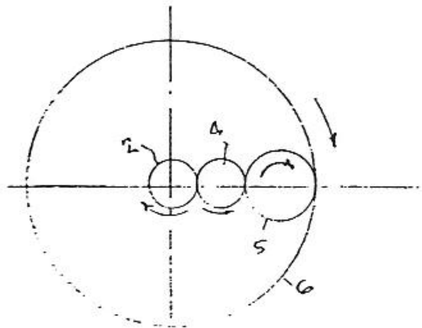



$T_{4c} = +9.36(127.5)$
$= 1193$ N·m ccw




Ans.

13-28




$d_2 = 4$ in; $d_4 = 4$ in; $d_5 = 6$ in;
$d_6 = 24$ in; $e = +\frac{1}{6}$

$n_F = n_2 = 1000$ rev/min

$n_L = n_6 = 0$

Eq. (13-24): $\frac{1}{6} = \frac{0 - n_A}{1000 - n_A}$

$n_A = -200$ rev/min

Input torque: $T_2 = \frac{63\ 000H}{n}$

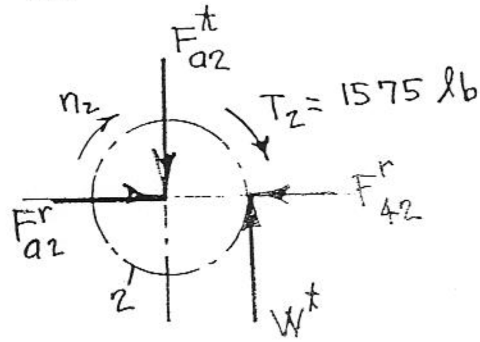
$= \frac{63\ 000(25)}{1000}$

$= 1575$ lb·in

The power out is the same as the power in and so

$T_{arm} = \frac{63\ 000(25)}{200} = 7875$ lb·in Ans.

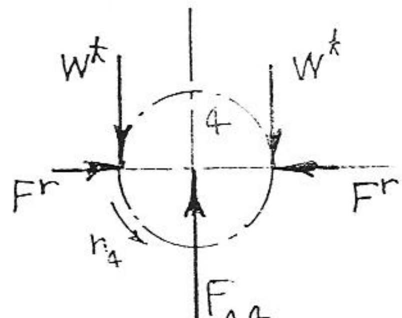
Gear 2




$W^t = 1575/2 = 787.5$ lb

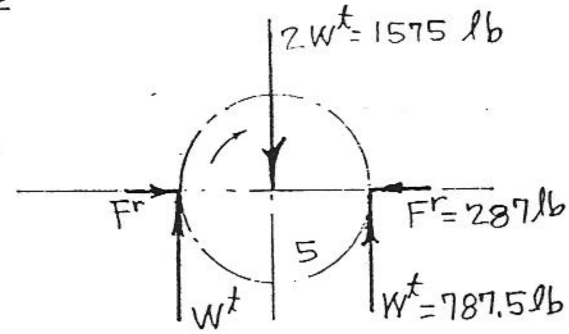
$F^r_{32} = 787.5 \tan 20° = 287$ lb

Gear 4

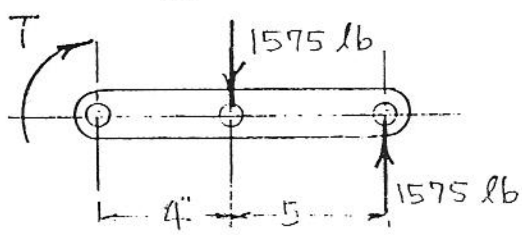



$F_{A4} = 2W^t = 1575$ lb

Gear 5




Arm




$T_{out} = 1575(9) - 1575(4) = 7875$ lb·in
Ans.

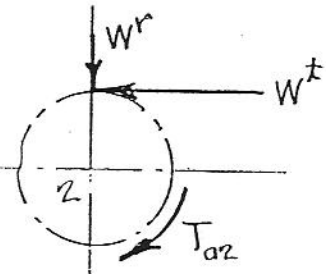
$\underline{13\text{-}29}$ $d_2 = 9$ in; $d_3 = 16$ in; $d_4 = 9$ in;

$d_5 = 24$ in

$$T_{a2} = \frac{63\ 000(200)}{1800} = 7000 \text{ lb}\cdot\text{in}$$

$W^t = 7000/4.5 = 1556$ lb

$W^r = 1556 \tan 20° = 566$ lb

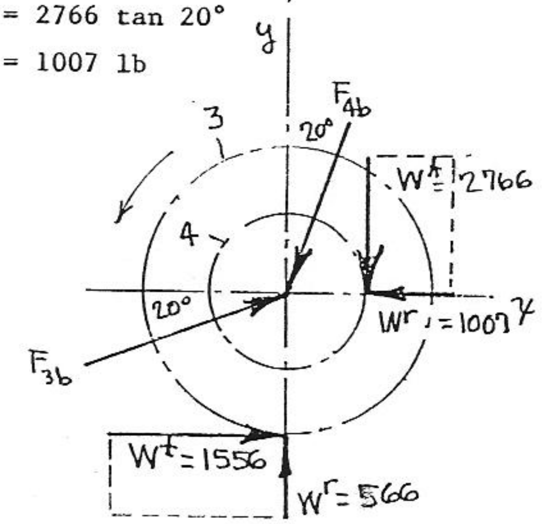



$1556(8) = 4.5W^t$

$W^t = 2766$ lb

$W^r = 2766 \tan 20°$

$= 1007$ lb




Gear 3 exerts $F_{3b} = [(1556)^2 + (566)^2]^{\frac{1}{2}}$

$= 1656$ lb at 20°

So $\underset{\sim}{F}_{3b} = 1656/\underline{20°}$ lb $\underline{\text{Ans.}}$

Gear 4 exerts $F_{4b} = [(2766)^2 + (1007)^2]^{\frac{1}{2}}$

$= 2944$ lb $\underline{\text{Ans.}}$

So $\underset{\sim}{F}_{4b} = 2944/\underline{200°}$ lb $\underline{\text{Ans.}}$

---

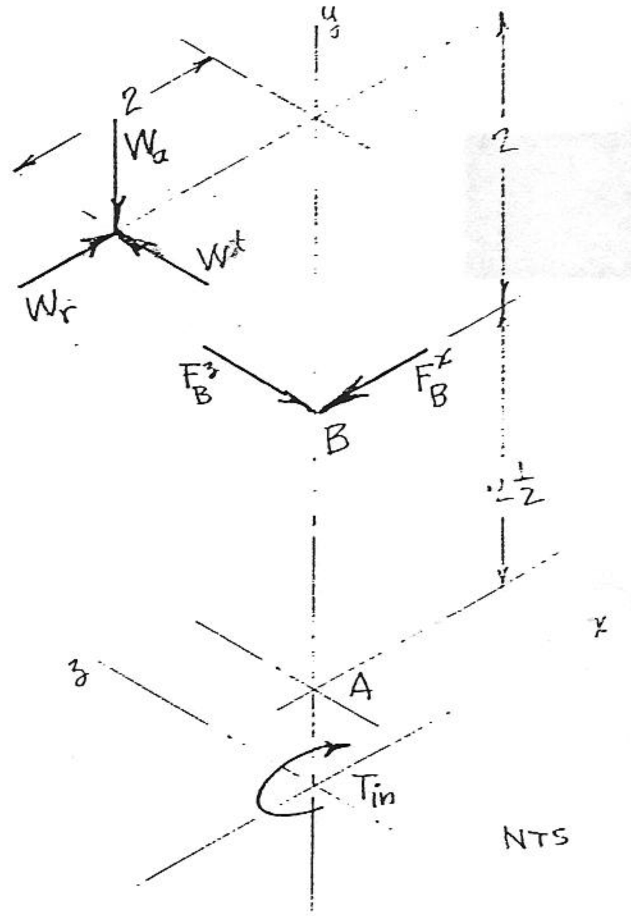
$\underline{13\text{-}30 \text{ and } 13\text{-}31}$ OMITTED

---

$\underline{13\text{-}32}$

$$T_{in} = \frac{63\ 000H}{n} = \frac{63\ 000(2.5)}{240}$$

$= 656$ lb·in




$W_t = T/r = 656/2 = 328$ lb

$\gamma = \tan^{-1}(2/4) = 26.565°$

$\Gamma = \tan^{-1}(4/2) = 63.435°$

$W_r = 328 \tan 20° \cos 26.565° = 106.8$ lb

$W_a = 328 \tan 20° \sin 26.565° = 53.4$ lb

$\underset{\sim}{W} = 106.8\underset{\sim}{i} - 53.4\underset{\sim}{j} + 328\underset{\sim}{k}$ lb

$\underset{\sim}{R}_{AG} = -2\underset{\sim}{i} + 4.5\underset{\sim}{j}$ ; $\underset{\sim}{R}_{AB} = 2.5\underset{\sim}{j}$

$\Sigma \underset{\sim}{M}_A = \underset{\sim}{R}_{AG} \times \underset{\sim}{W} + \underset{\sim}{R}_{AB} \times \underset{\sim}{F}_B + \underset{\sim}{T} = \underset{\sim}{0}$

Solving gives

$\underset{\sim}{R}_{AB} \times \underset{\sim}{F}_B = 2.5F_B^z\underset{\sim}{i} - 2.5F_B^x\underset{\sim}{k}$

$\underset{\sim}{R}_{AG} \times \underset{\sim}{W} = 1476\underset{\sim}{i} + 656\underset{\sim}{j} - 373.8\underset{\sim}{k}$

So $(1476\underset{\sim}{i} + 656\underset{\sim}{j} - 373.8\underset{\sim}{k})$

$+ (2.5F_B^z\underset{\sim}{i} - 2.5F_B^x\underset{\sim}{k}) + T\underset{\sim}{j} = \underset{\sim}{0}$

$F_B^z = -1476/2.5 = -590.4$ lb

13-32 (Concluded)

$T = -656$ lb·in

$F_B^x \doteq -373.8/2.5 = -149.5$ lb

So $F_B = [(-590.4)^2 + (-149.5)^2]^{\frac{1}{2}}$

$= 609$ lb  Ans.

$\underset{\sim}{F}_A = -(\underset{\sim}{F}_B + \underset{\sim}{W})$

$= -(-149.5\underset{\sim}{i} - 590.4\underset{\sim}{k} + 106.8\underset{\sim}{i} - 53.4\underset{\sim}{j} + 328\underset{\sim}{k})$

$= -42.72\underset{\sim}{i} + 53.4\underset{\sim}{j} + 262.4\underset{\sim}{k}$

$F_A$(radial) = 266 lb  Ans.

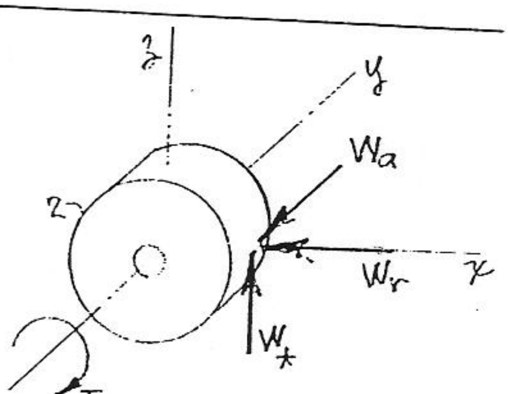
$F_A$(thrust) = 53.4 lb  Ans.

---

13-33 OMITTED

---

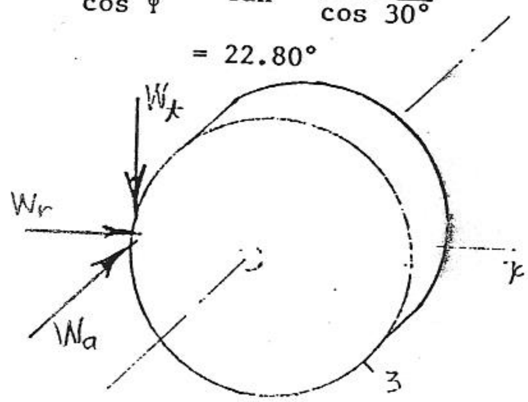
13-34




$P_t = P_n \cos\Psi = 4 \cos 30° = 3.464$ T/in

$\phi_t = \tan^{-1}\dfrac{\tan\phi_n}{\cos\Psi} = \tan^{-1}\dfrac{\tan 20°}{\cos 30°}$

$= 22.80°$




$d_P = 18/3.464 = 5.196$ in

Pinion

$W_r = W_t \tan\phi_t = 800 \tan 22.80° = 336$ lb

$W_a = W_t \tan\Psi = 800 \tan 30° = 462$ lb

So $\underset{\sim}{W} = -336\underset{\sim}{i} - 462\underset{\sim}{j} + 800\underset{\sim}{k}$ lb  Ans.

$W = 983$ lb  Ans.

Gear $\underset{\sim}{W} = 336\underset{\sim}{i} + 462\underset{\sim}{j} - 800\underset{\sim}{k}$ lb  Ans.

$W = 983$ lb  Ans.

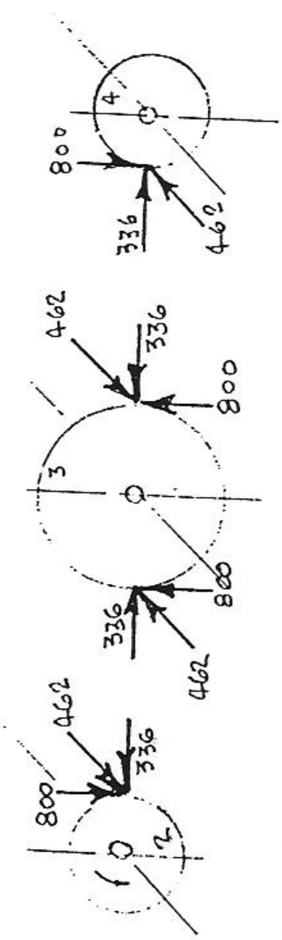
$d_G = 32/3.464 = 9.238$ in

$T = W_t r = 800(9.238) = 7390$ lb·in

T is clockwise about y.

---

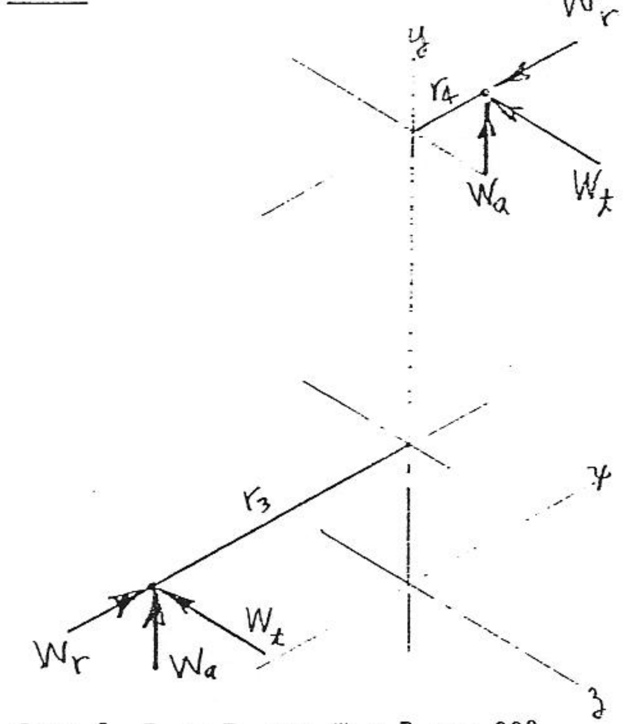
13-35




Note that the idler shaft reaction contains a couple tending to turn the shaft end over end.

13-35

13-36




Gear 3 $\quad P_t = P_n \cos\Psi = 7\cos 30°$

$= 6.062$ teeth/in

$\tan\phi_t = 0.4203;\ \phi_t = 22.8°$

$d_3 = 54/6.062 = 8.908$ in

$W_t = 500$ lb

$W_a = 500 \tan 30° = 288.7$ lb

$W_r = 500 \tan 22.8° = 210.2$ lb

$\underset{\sim}{W}_3 = 210.2\underset{\sim}{i} + 288.7\underset{\sim}{j} - 500\underset{\sim}{k}$ lb $\quad$ Ans.

Gear 4 $\quad d_4 = 14/6.062 = 2.309$ in

$W_t = 500\,\dfrac{8.908}{2.309} = 1929$ lb

$W_a = 1929 \tan 30° = 1113$ lb

$W_r = 1929 \tan 22.8 = 811$ lb

$\underset{\sim}{W}_4 = -811\underset{\sim}{i} + 1113\underset{\sim}{j} - 1929\underset{\sim}{k}$ lb $\quad$ Ans.

---

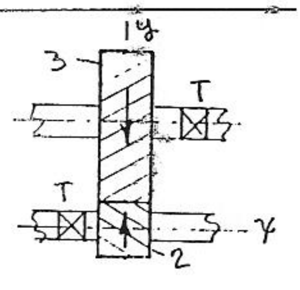
13-37

$P_t = 6 \cos 30°$

$= 5.196$ teeth/in

$d_3 = 42/5.196$

$= 8.083$ in




$\phi_t = 22.8°;\ d_2 = 16/5.196 = 3.079$ in

$T = \dfrac{63\,000(25)}{1720} = 915.7$ lb·in

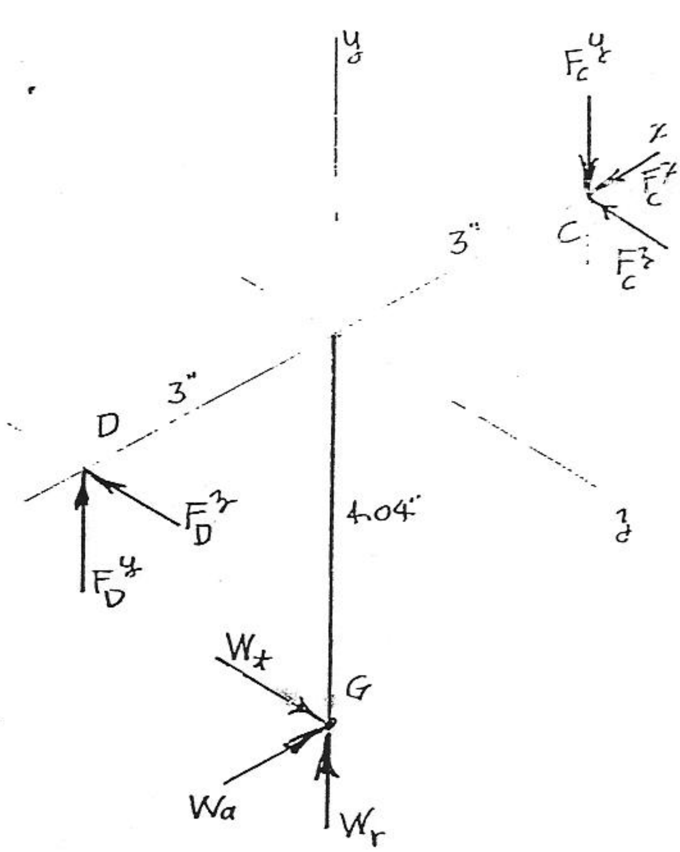
$W_t = T/r = \dfrac{915.7}{3.079/2} = 595$ lb

$W_a = 595 \tan 30° = 344$ lb

$W_r = 595 \tan 22.8° = 250$ lb

$\underset{\sim}{W} = 344\underset{\sim}{i} + 250\underset{\sim}{j} + 595\underset{\sim}{k}$ lb

$\underset{\sim}{R}_{DC} = 6\underset{\sim}{i};\ \underset{\sim}{R}_{DG} = 3\underset{\sim}{i} - 4.04\underset{\sim}{j}$




$\Sigma \underset{\sim}{M}_D = \underset{\sim}{R}_{DC} \times \underset{\sim}{F}_C + \underset{\sim}{R}_{DG} \times \underset{\sim}{W} + \underset{\sim}{T} = 0 \qquad (1)$

$\underset{\sim}{R}_{DG} \times \underset{\sim}{W} = -2304\underset{\sim}{i} - 1785\underset{\sim}{j} + 2140\underset{\sim}{k}$

$\underset{\sim}{R}_{DC} \times \underset{\sim}{F}_C = -6F_D^z\underset{\sim}{j} + 6F_D^y\underset{\sim}{k}$

Substituting and solving (1) gives

$T = +2304$ lb·in

$F_C^z = -297.5$ lb

$F_C^y = -356.7$ lb

13-37 (Concluded)

$$\Sigma \mathbf{F} = \mathbf{F}_D + \mathbf{F}_C + \mathbf{W} = 0 \qquad (2)$$

Substituting and solving gives

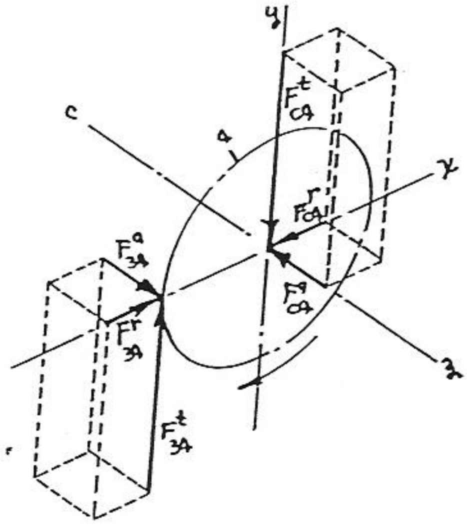
$F_C^x = -344$ lb

$F_D^y = 106.7$ lb

$F_D^z = -297.5$ lb

So $\mathbf{F}_C = -344\mathbf{i} - 356.7\mathbf{j} - 297.5\mathbf{k}$ lb Ans.

$\mathbf{F}_D = 106.7\mathbf{j} - 297.5\mathbf{k}$ lb Ans.




---

13-38 $P_t = 8 \cos 15° = 7.727$ teeth/in

$d_2 = 16/7.727 = 2.07$ in

$d_3 = 36/7.727 = 4.66$ in

$d_4 = 28/7.727 = 3.62$ in

$$V = \frac{\pi d n}{12} = \frac{\pi(2.07)(1720)}{12} = 932 \text{ ft/min}$$

$$W_t = \frac{33\,000(7.5)}{932} = 266 \text{ lb}$$

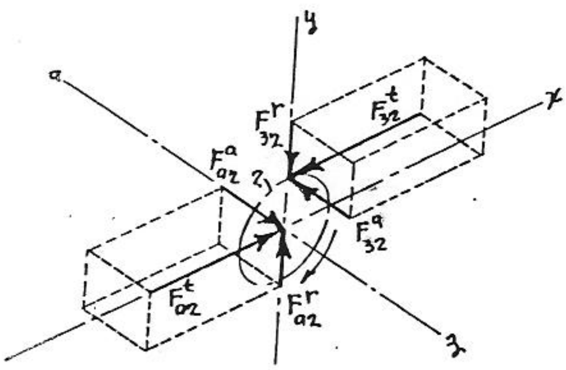
$W_r = 266 \tan 20° = 96.8$ lb

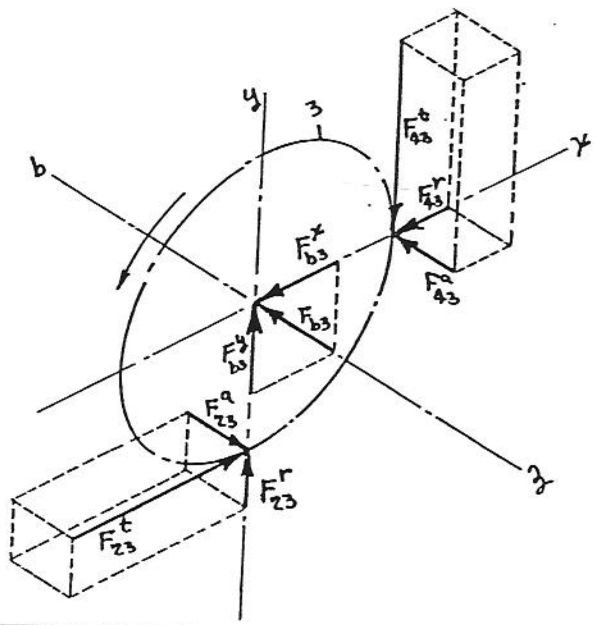
$W_a = 266 \tan 15° = 71.3$ lb

$\mathbf{F}_{2a} = -266\mathbf{i} - 96.7\mathbf{j} - 71.3\mathbf{k}$ lb Ans.

$\mathbf{F}_{3b} = (266 - 96.8)\mathbf{i} - (266 - 96.8)\mathbf{j}$
$= 169\mathbf{i} - 169\mathbf{j}$ lb Ans.

$\mathbf{F}_{4c} = 96.8\mathbf{i} + 266\mathbf{j} + 71.3\mathbf{k}$ lb Ans.

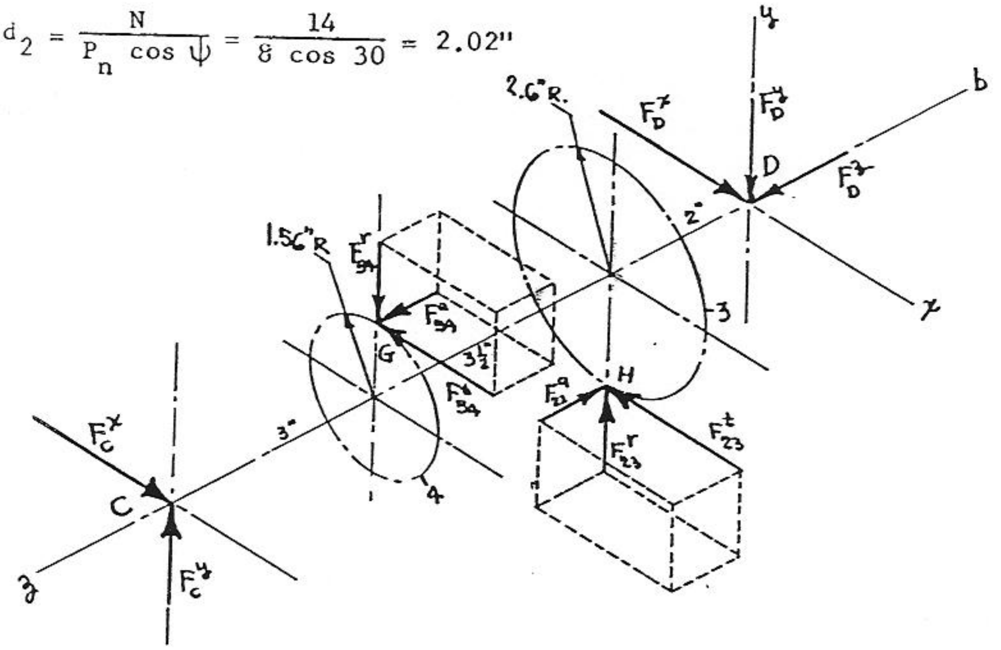



---

13-39

$d_2 = \frac{N}{P_n \cos\psi} = \frac{14}{8\cos 30} = 2.02''$




$d_3 = \frac{36}{8\cos 30} = 5.20$ in; $d_4 = \frac{15}{5\cos 15} = 3.11$ in; $d_5 = \frac{45}{5\cos 15}$
$= 9.32$ in;

For gears 2 and 3: $\phi_t = \tan^{-1}(\tan\phi_n/\cos\psi) = \tan^{-1}(\tan 20/\cos 30)$
$= 22.8$ deg;

For gears 4 and 5: $\phi_t = \tan^{-1}(\tan 20/\cos 15) = 20.6$ deg;

$F_{23}^t = T/r = 1200/(2.02/2) = 1188$ lb; $F_{54}^t = 1188\,\frac{5.20}{3.11} = 1986$ lb;

$F_{23}^r = F_{23}^t \tan\phi_t = 1188 \tan 22.8 = 500$ lb;

$F_{54}^r = 1986 \tan 20.6 = 748$ lb; $F_{23}^a = F_{23}^t \tan\psi = 1188 \tan 30$
$= 686$ lb;

$F_{54}^a = 1986 \tan 15 = 532$ lb

Next, designate the points of action on gears 4 and 3, respectively, as points G and H, as shown. Position vectors are $\vec{R}_{CG} = 1.56j - 3k$, $\vec{R}_{CH} = -2.6j - 6.5k$, $\vec{R}_{CD} = -8.5k$. Force vectors are $\vec{F}_{54} = -1986i - 748j + 532k$, $\vec{F}_{23} = -1188i + 500j - 686k$, $\vec{F}_C = F_C^x i + F_C^y j$, $\vec{F}_D = F_D^x i + F_D^y j + F_D^z k$.

13-39 (Concluded)

Now, a summation of moments about bearing C gives

$$\Sigma\vec{M}_C = \vec{R}_{CG} \times \vec{F}_{54} + \vec{R}_{CH} \times \vec{F}_{23} + \vec{R}_{CD} \times \vec{F}_D = 0$$

The terms for this equation are found to be

$$\vec{R}_{CG} \times \vec{F}_{54} = -1414i + 5958j + 3100k,$$

$$\vec{R}_{CH} \times \vec{F}_{23} = 5035i + 7720j - 3100k, \quad \vec{R}_{CD} \times \vec{F}_D = 8.5F_D^{\,y}i - 8.5F_D^{\,x}j$$

When these terms are placed back into the moment equation, the k terms, representing the shaft torque, cancel. The i and j terms give

$$F_D^{\,y} = -\frac{3621}{8.5} = -426 \text{ lb} \quad \underline{\text{Ans.}} \quad F_D^{\,x} = \frac{(13\ 678)}{8.5} = 1610 \text{ lb} \quad \underline{\text{Ans}}.$$

Next, we sum the forces to zero.

$\Sigma\vec{F} = \vec{F}_C + \vec{F}_{54} + \vec{F}_{23} + \vec{F}_D = 0$; substituting, gives

$$(F_C^{\,x}i + F_C^{\,y}j) + (-1986i - 748j + 532k) + (-1188i + 500j - 686k)$$

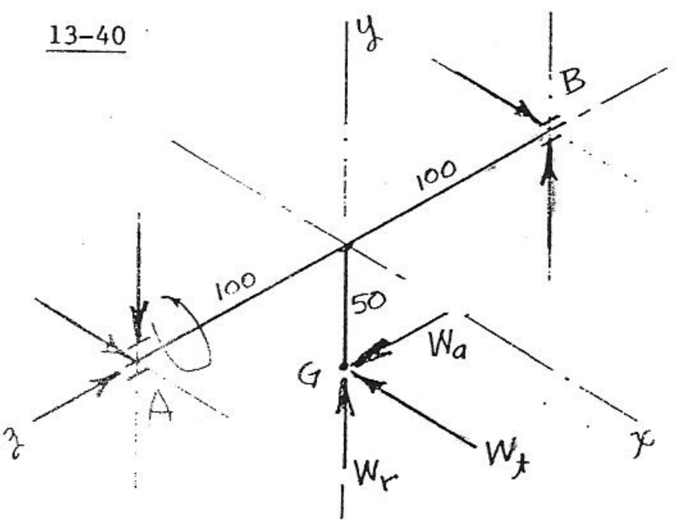
$$+ (1610i - 426j + F_D^{\,z}k) = 0; \text{ solving gives}$$

$F_C^{\,x} = 1986 + 1188 - 1610 = 1564$ lb; $F_C^{\,y} = 748 - 500 + 426 = 674$ lb;

$F_D^{\,z} = -532 + 686 = 154$ lb <u>Ans</u>.

---

13-40




Worm shaft diagram

$$V_W = \frac{\pi d_W n_W}{60} = \frac{\pi(0.100)(600)}{60} = \pi \text{ m/s}$$

$$W_{Wt} = \frac{H}{V_W} = \frac{2000}{\pi} = 637 \text{ N}$$

$$L = p_x N_W = 25(1) = 25 \text{ mm}$$

$$\lambda = \tan^{-1}\frac{L}{\pi d_W} = \tan^{-1}\frac{25}{\pi(100)}$$

$$= 4.550° \text{ lead angle}$$

$$W = \frac{W_{Wt}}{\cos\phi_n \sin\lambda + \mu\cos\lambda}$$

$$V_S = \frac{V_W}{\cos\lambda} = \frac{\pi}{\cos 4.550°} = 3.15 \text{ m/s}$$

in ft/s $V_S = 3.28(3.15) = 10.3$ ft/s

13-40 (Continued)

Use $\mu = 0.043$ from curve A of Fig. 13-42.

$$W = \frac{637}{\cos 14.5° \sin 4.55° + 0.043 \cos 4.55°}$$

$= 5323$ N

$W^y = W \sin \phi_n = 5323 \sin 14.5°$

$= 1333$ N

$W^z = W(\cos \phi_n \cos \lambda - \mu \sin \lambda)$

$= 5323(\cos 14.5° \cos 4.55°$

$- 0.043 \sin 4.55°) = 5119$ N

The force acting against the worm is

$\underset{\sim}{W} = -637\underset{\sim}{i} + 1333\underset{\sim}{j} + 5119\underset{\sim}{k}$ N

A is the thrust bearing.

$\underset{\sim}{R}_{AG} = -0.05\underset{\sim}{j} - 0.10\underset{\sim}{k}$; $\underset{\sim}{R}_{AB} = -0.20\underset{\sim}{k}$

$\Sigma \underset{\sim}{M}_A = \underset{\sim}{R}_{AG} \times \underset{\sim}{W} + \underset{\sim}{R}_{AB} \times \underset{\sim}{F}_B + \underset{\sim}{T} = 0$

$\underset{\sim}{R}_{AG} \times \underset{\sim}{W} = -122.6\underset{\sim}{i} + 63.7\underset{\sim}{j} - 31.85\underset{\sim}{k}$

$\underset{\sim}{R}_{AB} \times \underset{\sim}{F}_B = 0.2F_B^y\underset{\sim}{i} - 0.2F_B^x\underset{\sim}{j}$

Substituting and solving gives

$T = 31.85$ N·m  Ans.

$F_B^x = 318.5$ N; $F_B^y = 613$ N

So $\underset{\sim}{F}_B = 318.5\underset{\sim}{i} + 613\underset{\sim}{j}$ N  Ans.

or $F_B = 691$ N radial  Ans.

$\Sigma \underset{\sim}{F} = \underset{\sim}{F}_A + \underset{\sim}{W} + \underset{\sim}{R}_B = 0$

$\underset{\sim}{F}_A = -(\underset{\sim}{W} + \underset{\sim}{F}_B)$

$= -(-637\underset{\sim}{i} + 1333\underset{\sim}{j} + 5119\underset{\sim}{k} + 318.5\underset{\sim}{i}$

$+ 613\underset{\sim}{j})$

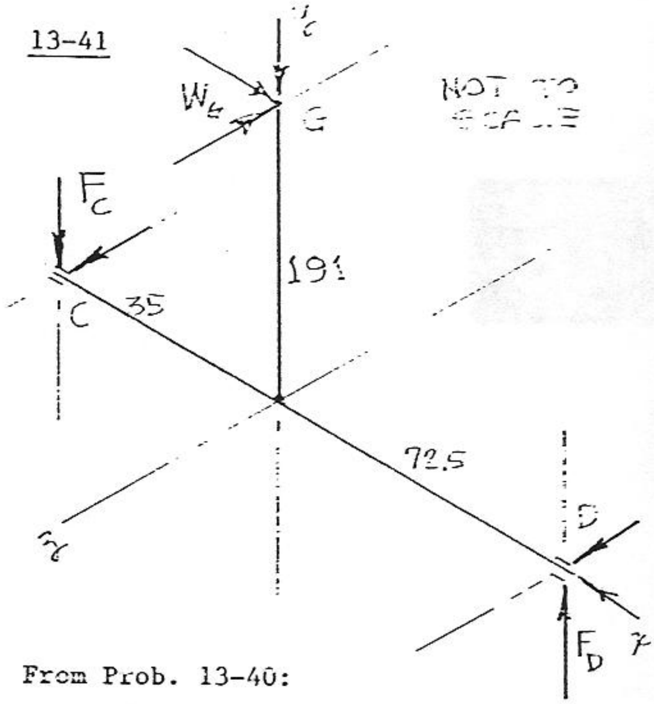
$= 318.5\underset{\sim}{i} - 1946\underset{\sim}{j} - 5119\underset{\sim}{k}$

Radial $\underset{\sim}{F}_A^r = 318.5\underset{\sim}{i} - 1946\underset{\sim}{j}$ N;

$F_A^r = 1972$ N  Ans.

Thrust $\underset{\sim}{F}_A^T = -5119\underset{\sim}{k}$ N  Ans.

13-41




From Prob. 13-40:

$\underset{\sim}{W}_G = 637\underset{\sim}{i} - 1537\underset{\sim}{j} - 5910\underset{\sim}{k}$ N

$p_t = p_x$, so $d_G = \dfrac{N_G p_x}{\pi} = \dfrac{48(25)}{\pi} = 382$ mm

Bearing D to take thrust load.

$\Sigma \underset{\sim}{M}_D = \underset{\sim}{R}_{DG} \times \underset{\sim}{W} + \underset{\sim}{R}_{DC} \times \underset{\sim}{F}_C + \underset{\sim}{T} = 0$

$\underset{\sim}{R}_{DG} = -0.0725\underset{\sim}{i} + 0.191\underset{\sim}{j}$; $\underset{\sim}{R}_{DC} = -0.1075\underset{\sim}{i}$

The position vectors are in meters.

$\underset{\sim}{R}_{DC} \times \underset{\sim}{F}_C = 0.1075F_C^z\underset{\sim}{j} - 0.1075F_C^y\underset{\sim}{k}$

Putting it together and solving gives

$T = 1129$ N·m  Ans.

$\underset{\sim}{F}_C = -95.8\underset{\sim}{j} + 3985\underset{\sim}{k}$ N; $F_C = 3986$ N

$\Sigma \underset{\sim}{F} = \underset{\sim}{F}_C + \underset{\sim}{W} + \underset{\sim}{F}_D = 0$

$F_D = -(F_C + W)$

$= -637\underset{\sim}{i} + 1633\underset{\sim}{j} + 1925\underset{\sim}{k}$ N

$\underset{\sim}{F}_D = 1633\underset{\sim}{j} + 1925\underset{\sim}{k}$ N radial

or $F_D = 2524$ N  Ans. (total radial)

$F_D = -637\underset{\sim}{i}$ N thrust  Ans.

14-1

GIVEN: H= 15 HP SPEED = 1200 REV/MIN N = 22 TEETH
Y = .331 P = 6 TEETH/INCH F = 2 INCHES

RESULTS: D = 3.666667 INCHES V = 1151.916 FT/MIN WT = 429.7187 LB
KV = .5102222 SIGMA = 7.633402 KPSI

---

14-2

GIVEN: H= 1.5 HP SPEED = 700 REV/MIN N = 16 TEETH
Y = .296 P = 12 TEETH/INCH F = .75 INCHES

RESULTS: D = 1.333333 INCHES V = 244.3459 FT/MIN WT = 202.5817 LB
KV = .8308259 SIGMA = 13.18009 KPSI

---

14-3

GIVEN: H = .5 KW SPEED = 1800 REV/MIN N = 18 TEETH
Y = .309 M = 1.25 MM F = 12 MM

RESULTS: D = 22.5 V = 2.120574 M/S WT = .2357853 KN
KV = .7420407 SIGMA = 68.55501 MPA

---

14-4

GIVEN: H = 5 KW SPEED = 200 REV/MIN N = 15 TEETH
Y = .29 M = 5 MM F = 60 MM

RESULTS: D = 75 V = .7853975 M/S WT = 6.366203 KN
KV = .8859329 SIGMA = 82.59627 MPA

---

14-5

GIVEN: H = .15 KW SPEED = 360 REV/MIN N = 16 TEETH
Y = .296 M = 1 MM F = 12.5 MM

RESULTS: D = 16 V = .3015927 M/S WT = .4973597 KN
KV = .9528878 SIGMA = 141.0675 MPA

---

14-6

GIVEN: H = .25 KW SPEED = 400 REV/MIN N = 17 TEETH
Y = .303 M = 1.5 MM F = 18 MM

RESULTS: D = 25.5 V = .5340703 M/S WT = .4681032 KN
KV = .9194958 SIGMA = 62.22795 MPA

---

14-7

GIVEN: H = 1.5 KW SPEED = 900 REV/MIN N = 18 TEETH
Y = .309 M = 2 MM F = 25 MM

RESULTS: D = 36 V = 1.696459 M/S WT = .8841949 KN
KV = .7824065 SIGMA = 73.14542 MPA

---

14-8

GIVEN: H = 3.5 KW SPEED = 1200 REV/MIN N = 19 TEETH
Y = .314 M = 2.5 MM F = 32 MM

RESULTS: D = 47.5 V = 2.984511 M/S WT = 1.172722 KN
KV = .6714726 SIGMA = 69.52595 MPA

---

14-9

GIVEN: H = 10 KW SPEED = 1000 REV/MIN N = 20 TEETH
Y = .322 M = 4 MM F = 50 MM

RESULTS: D = 80 V = 4.188787 M/S WT = 2.387326 KN
KV = .5928785 SIGMA = 62.52593 MPA

---

14-10

GIVEN: H = 25 KW SPEED = 800 REV/MIN N = 21 TEETH
Y = .328 M = 6 MM F = 75 MM

RESULTS: D = 126 V = 5.277871 M/S WT = 4.736758 KN
KV = .5361285 SIGMA = 59.85853 MPA

---

14-11

GIVEN: H= 6 HP SPEED = 50 REV/MIN N = 24 TEETH
Y = .337 P = 5 TEETH/INCH F = 2.5 INCHES

RESULTS: D = 4.8 INCHES V = 62.8318 FT/MIN WT = 3151.271 LB
KV = .9502453 SIGMA = 19.68113 KPSI

---

14-12

GIVEN: H= 15 HP SPEED = 600 REV/MIN N = 16 TEETH
Y = .296 P = 5 TEETH/INCH F = 2.5 INCHES

RESULTS: D = 3.2 INCHES V = 502.6544 FT/MIN WT = 984.7721 LB
KV = .7047819 SIGMA = 9.441028 KPSI

---

14-13 GIVEN: H= 5 HP SPEED = 1000 REV/MIN N = 17 TEETH
Y = .303 P = 8 TEETH/INCH F = 1.5 INCHES

RESULTS: D = 2.125 INCHES V = 556.3233 FT/MIN WT = 296.5902 LB
KV = .6832456 SIGMA = 7.640751 KPSI

---

14-14 GIVEN: H= 2.5 HP SPEED = 600 REV/MIN N = 18 TEETH
Y = .309 P = 10 TEETH/INCH F = 1.25 INCHES

RESULTS: D = 1.8 INCHES V = 282.7431 FT/MIN WT = 291.7843 LB
KV = .8093108 SIGMA = 9.334222 KPSI

---

14-15

GIVEN: H= 1 HP SPEED = 1000 REV/MIN N = 18 TEETH
Y = .309 P = 16 TEETH/INCH F = .75 INCHES

RESULTS: D = 1.125 INCHES V = 294.5241 FT/MIN WT = 112.0452 LB
KV = .8029312 SIGMA = 9.634188 KPSI

---

14-16

GIVEN: H= .5 HP SPEED = 900 REV/MIN N = 20 TEETH
Y = .322 P = 20 TEETH/INCH F = .5 INCHES

RESULTS: D = 1 INCHES V = 235.6193 FT/MIN WT = 70.02823 LB
KV = .8358763 SIGMA = 10.40723 KPSI

---

14-17

GIVEN: H = 15 HP SPEED = 1200 REV/MIN N1 = 22 N2 = 60
P = 6 TEETH/IN F = 2 INCHES PHI = 20 DEG
CP = 2100 SQR(PSI)

RESULTS: D1 = 3.666667 INCHES D2 = 10 INCHES V = 1151.916 FT/MIN
CV = .5102222 WT = 429.7187 LB SIGC = -65.63103 KPSI

---

14-18
GIVEN: H = 1.5 HP SPEED = 700 REV/MIN N1 = 16 N2 = 48
P = 12 TEETH/IN F = .75 INCHES PHI = 20 DEG
CP = 2100 SQR(PSI)

RESULTS: D1 = 1.333333 INCHES D2 = 4 INCHES V = 244.3459 FT/MIN
CV = .8308259 WT = 202.5817 LB SIGC = -94.45623 KPSI

---

14-19
GIVEN: H = 6 HP SPEED = 50 REV/MIN N1 = 24 N2 = 48
P = 5 TEETH/IN F = 2.5 INCHES PHI = 20 DEG
CP = 1960 SQR(PSI)

RESULTS: D1 = 4.8 INCHES D2 = 9.600001 INCHES V = 62.8318 FT/MIN
CV = .9502453 WT = 3151.271 LB SIGC = -99.54811 KPSI

---

14-20

```
GIVEN:   H =  10 KW   SPEED =   1000 REV/MIN   N1 =  20 TEETH
         N2 = 32 TEETH   M =  4 MM   F =  50 MM   PHI =  20 DEG
         CP =  163 SQR(MPA):LPRINT
RESULTS:  D1 =  80 MM  D2 =  128 MM   V =  4.188787 M/S
          CV =  .5928785   WT =  2.387326 KN
         SIGC = -520.0609 MPA
```

14-21

```
GIVEN:   H =  3.5 KW   SPEED =  1200 REV/MIN   N1 =  19 TEETH
         N2 = 30 TEETH   M =  2.5 MM   F =  32 MM   PHI =  20 DEG
         CP =  174 SQR(MPA):LPRINT
RESULTS:  D1 =  47.5 MM  D2 =  75 MM   V =  2.984511 M/S
          CV =  .6714726   WT =  1.172722 KN
         SIGC = -594.6276 MPA
```

14-22

```
GIVEN:   H =  25 KW   SPEED =  800 REV/MIN   N1 =  21 TEETH
         N2 = 44 TEETH   M =  6 MM   F =  75 MM   PHI =  20 DEG
         CP =  163 SQR(MPA):LPRINT
RESULTS:  D1 =  126 MM  D2 =  264 MM   V =  5.277871 M/S
          CV =  .5361285   WT =  4.736758 KN
         SIGC = -477.8649 MPA
```

14-23

```
GIVEN: SPEED =  300 REV/MIN   N1 =  16 TEETH   N2 =  48 TEETH
       QV =  7   CA =  1   CS =  1   CM =  1.3   CF =  1
       H =  5 HP   F =  2 IN.   CP =  2100 SQR(PSI)
       PD =  6 TEETH/IN.

RESULTS: D1 =  2.666667 IN.   D2 =  8 IN.   V =  209.4394 FT/MIN
         WT =  787.8176 LB   I =  .120525   SIGC =  90.20823 KPSI
         CV =  .8634542
```

14-24

```
GIVEN: SPEED =  600 REV/MIN   N1 =  18 TEETH   N2 =  64 TEETH
       QV =  8   CA =  1   CS =  1   CM =  1.6   CF =  1
       H =  4 HP   F =  1.5 IN.   CP =  2100 SQR(PSI)
       PD =  8 TEETH/IN.

RESULTS: D1 =  2.25 IN.   D2 =  8 IN.   V =  353.4289 FT/MIN
         WT =  373.4839 LB   I =  .1254244   SIGC =  84.98319 KPSI
         CV =  .8620029
```

```
14-25
GIVEN: SPEED =  100 REV/MIN  N1 =  20 TEETH  N2 =  36 TEETH
       QV =  6    CA =  1    CS =  1    CM =  1.6    CF =  1
       H =  .12 KW   F =  18 MM   CP =  174 SQR(MPA)   M =  1.5 MM

RESULTS: D1 =  30 MM   D2 =  54 MM   V =  .1570795 M/S
         WT =  .7639443 KN   I =  .1033071   SIGC =  845.1711 MPA
         CV =  .9286817
```

---

```
14-26
GIVEN: SPEED =  1200 REV/MIN  N1 =  21 TEETH  N2 =  26 TEETH
       QV =  7    CA =  1    CS =  1    CM =  1.6    CF =  1
       H =  3 KW   F =  36 MM   CP =  163 SQR(MPA)   M =  3 MM

RESULTS: D1 =  63 MM   D2 =  78 MM   V =  3.958404 M/S
         WT =  .7578813 KN   I =  8.889787E-02   SIGC =  455.8568 MPA
         CV =  .7689626
```

---

```
14-27
GIVEN: SPEED =  1800 REV/MIN  N1 =  17 TEETH  N2 =  52 TEETH
       QV =  6    CA =  1    CS =  1    CM =  1.3    CF =  1
       H =  4 HP   F =  1.25 IN.   CP =  2100 SQR(PSI)
       PD =  10 TEETH/IN.

RESULTS: D1 =  1.7 IN.   D2 =  5.2 IN.   V =  801.1055 FT/MIN
         WT =  164.7723 LB   I =  .1211072   SIGC =  71.0978 KPSI
         CV =  .7261475
```

Based on gear: Table 14-4: $S_C = 65$ kpsi; Fig. 14-8: $C_L = 1.14$. Also 175 Bhn gear, 240 Bhn pinion, and $m_G = 52/17$.

Eq. (14-31): $A = \dfrac{8.98}{1000}\left(\dfrac{240}{175}\right) - \dfrac{8.29}{1000} = 0.004\ 02$

Eq. (14-30): $C_H = 1.0 + 0.004\ 02\left(\dfrac{52}{17} - 1.0\right) = 1.008$

Table 14-7: $C_R = 0.85$; Sec. 14-15: $C_T = 1$

Eq. (14-18): $\sigma_{c,all} = \dfrac{65(1.14)(1.008)}{1(0.85)} = 87.87$ kpsi

Therefore, $n = \sigma_{c,all}/\sigma_c = 87.87/71.1 = 1.24$ Ans.

---

```
14-28
GIVEN: SPEED =  900 REV/MIN  N1 =  15 TEETH  N2 =  30 TEETH
       QV =  6    CA =  1    CS =  1    CM =  1.4    CF =  1
       H =  15 HP   F =  2.5 IN.   CP =  2300 SQR(PSI)
       PD =  5 TEETH/IN.

RESULTS: D1 =  3 IN.   D2 =  6 IN.   V =  706.8578 FT/MIN
         WT =  700.2824 LB   I =  .1071333   SIGC =  93.51778 KPSI
         CV =  .7380446
```

Continued

14-28 (Concluded)

Based on gear Table 14-4: $S_C = 105$ kpsi; $C_L = 1$; $H_G = 240$; $H_P = 300$

Eq. (14-31): $A = \frac{8.98}{1000}\left(\frac{300}{240}\right) - \frac{8.29}{1000} = 0.002\ 93$

Eq. (14-30): $C_H = 1 + 0.002\ 93(\frac{30}{15} - 1) = 1.0029$

Table 14-7 and Sec. 14-15: $C_T = 1$, $C_R = 1$

Eq. (14-18): $\sigma_{c,all} = \frac{105(1)(1.0029)}{1(1)} = 105.3$ kpsi; $n = 105.3/93.5 = 1.13$ Ans.

---

```
 14-29
GIVEN: SPEED =  500 REV/MIN  N1 =  19 TEETH  N2 =  25 TEETH
       QV =  9   CA =  1   CS =  1   CM =  1.3   CF =  1
       H =  6 KW  F =  50 MM  CP =  191 SQR(MPA)  M =  4 MM

RESULTS: D1 =  76 MM  D2 =  100 MM  V =  1.989674 M/S
         WT =  3.01557 KN  I =  9.130681E-02  SIGC =  681.7056 MPA
          CV =  .8869496
```

Based on gear Table 14-4: $S_C = 830$ MPa; $C_L = 1.0$; $H_P = 360$ Bhn; $H_G = 300$ Bhn

Eq. (14-31): $A = \frac{8.98}{1000}\left(\frac{360}{300}\right) - \frac{8.29}{1000} = 0.0025$

Eq. (14-30): $C_H = 1 + 0.0025(\frac{25}{19} - 1) = 1.0008$; $C_T = 1$; $C_R = 1.25$

Eq. (14-18): $\sigma_{c,all} = \frac{830(1.0)(1.0008)}{1(1.25)} = 664$ MPa; $n = 664/681.7 = 0.978$ Ans.

---

```
 14-30
GIVEN: SPEED =  400 REV/MIN  N1 =  22 TEETH  N2 =  32 TEETH
       QV =  5   CA =  1   CS =  1   CM =  1.6   CF =  1
       H =  1 KW  F =  32 MM  CP =  163 SQR(MPA)  M =  2.5 MM

RESULTS: D1 =  55 MM  D2 =  80 MM  V =  1.151916 M/S
         WT =  .8681186 KN  I =  9.522962E-02  SIGC =  524.7932 MPA
          CV =  .7994901
```

Based on pinion $S_C = 450$ MPa; $C_L = 1.25$; $C_H = 1$; $C_T = 1$; $C_R = 0.85$

$\sigma_{c,all} = \frac{450(1.25)(1)}{1(0.85)} = 661.8$ MPa; $n = 661.8/450 = 1.47$ Ans.

---

```
 14-31(PINION)
GIVEN: SPEED =  1200 REV/MIN  J =  .27
       N1 =  16 TEETH  N2 =  56 TEETH
       KA =  1   KS =  1   KM =  1.3
      H =  .5 HP  PD =  20 TEETH/IN.  F =  .5 INCHES

RESULTS: D1 =  .8 INCHES  V =  251.3272 FT/MIN  KV =  .8234976
         WT =  65.65147 LB  SIGMA =  15.354 KPSI
```

14-31A (GEAR)

```
GIVEN: SPEED =  1200 REV/MIN   J =  .395
       N1 =  16 TEETH   N2 =  56 TEETH
       KA =  1    KS =  1    KM =  1.3
     H =  .5 HP   PD =  20 TEETH/IN.   F =  .5 INCHES

RESULTS: D1 =  .8 INCHES   V =  251.3272 FT/MIN   KV =  .8234976
         WT =  65.65147 LB   SIGMA =  10.49514 KPSI
```

---

14-32 (PINION)

```
GIVEN: SPEED =  200 REV/MIN   J =  .335
       N1 =  20 TEETH   N2 =  72 TEETH
       KA =  1    KS =  1    KM =  1.3
     H =  .75 HP   PD =  12 TEETH/IN.   F =  1 INCHES

RESULTS: D1 =  1.666667 INCHES   V =  87.26639 FT/MIN   KV =  .9065834
         WT =  283.6144 LB   SIGMA =  14.56801 KPSI
```

14-32A (GEAR)

```
GIVEN: SPEED =  200 REV/MIN   J =  .42
       N1 =  20 TEETH   N2 =  72 TEETH
       KA =  1    KS =  1    KM =  1.3
     H =  .75 HP   PD =  12 TEETH/IN.   F =  1 INCHES

RESULTS: D1 =  1.666667 INCHES   V =  87.26639 FT/MIN   KV =  .9065834
         WT =  283.6144 LB   SIGMA =  11.61972 KPSI
```

---

14-33 (PINION)

```
GIVEN: SPEED =  900 REV/MIN   J =  .325
       N1 =  18 TEETH   N2 =  96 TEETH
       KA =  1    KS =  1.1   KM =  2
     H =  25 KW   M =  8 MM   F =  100  MM

RESULTS: D1 =  144 MM   V =  6.785835 M/S   KV =  .7203836
         WT =  3.684146 KN   SIGMA =  43.27353 MPA
```

14-33A (GEAR)

```
GIVEN: SPEED =  900 REV/MIN   J =  .43
       N1 =  18 TEETH   N2 =  96 TEETH
       KA =  1    KS =  1.1   KM =  2
     H =  25 KW   M =  8 MM   F =  100  MM

RESULTS: D1 =  144 MM   V =  6.785835 M/S   KV =  .7203836
         WT =  3.684146 KN   SIGMA =  32.70673 MPA
```

---

14-34 (PINION)

GIVEN: SPEED = 600 REV/MIN J = .331
N1 = 19 TEETH N2 = 84 TEETH
KA = 1 KS = 1 KM = 1.3
H = 4 KW M = 3 MM F = 40 MM

RESULTS: D1 = 57 MM V = 1.790706 M/S KV = .7968762
WT = 2.233755 KN SIGMA = 91.74426 MPA

14-34 (GEAR)

GIVEN: SPEED = 600 REV/MIN J = .435
N1 = 19 TEETH N2 = 84 TEETH
KA = 1 KS = 1 KM = 1.3
H = 4 KW M = 3 MM F = 40 MM

RESULTS: D1 = 57 MM V = 1.790706 M/S KV = .7968762
WT = 2.233755 KN SIGMA = 69.81 MPA

---

14-35 (PINION)

GIVEN: SPEED = 1400 REV/MIN J = .295
N1 = 17 TEETH N2 = 120 TEETH
KA = 1 KS = 1 KM = 1.3
H = 12.5 HP PD = 6 TEETH/IN. F = 2 INCHES

RESULTS: D1 = 2.833333 INCHES V = 1038.47 FT/MIN KV = .7005
WT = 397.219 LB SIGMA = 7.496602 KPSI

14-35 (GEAR)

GIVEN: SPEED = 1400 REV/MIN J = .433
N1 = 17 TEETH N2 = 120 TEETH
KA = 1 KS = 1 KM = 1.3
H = 12.5 HP PD = 6 TEETH/IN. F = 2 INCHES

RESULTS: D1 = 2.833333 INCHES V = 1038.47 FT/MIN KV = .7005
WT = 397.219 LB SIGMA = 5.107384 KPSI

Pinion 240 Bhn, $S_t$ = 31 kpsi. Gear No. 30 C. I., $S_t$ = 8.5 kpsi; n = 8.5/5.1 = 1.67 Ans.

---

14-36

GIVEN: SPEED = 720 REV/MIN J = .27
N1 = 16 TEETH N2 = 28 TEETH
KA = 1 KS = 1 KM = 1.6
H = 17.5 HP PD = 5 TEETH/IN. F = 2.5 INCHES

RESULTS: D1 = 3.2 INCHES V = 603.1853 FT/MIN KV = .7912788
WT = 957.4172 LB SIGMA = 14.34029 KPSI

$S_t$ = 31 kpsi; n = 31/14.3 = 2.52 Ans.

```
14-37 (PINION)
GIVEN: SPEED =  360 REV/MIN  J =  .33
       N1 =  19 TEETH  N2 =  77 TEETH
       KA =  1    KS =  1.1    KM =  1.6
     H =  15 KW   M =  6 MM   F =  75  MM

RESULTS: D1 =  114 MM  V =  2.148848 M/S  KV =  .7820878
         WT =  6.980485 KN  SIGMA =  105.7831 MPA
```

```
14-37 (GEAR)
GIVEN: SPEED =  360 REV/MIN  J =  .415
       N1 =  19 TEETH  N2 =  77 TEETH
       KA =  1    KS =  1.1    KM =  1.6
     H =  15 KW   M =  6 MM   F =  75  MM

RESULTS: D1 =  114 MM  V =  2.148848 M/S  KV =  .7820878
         WT =  6.980485 KN  SIGMA =  84.11668 MPA
```

Gear $S_t$ = 69 MPa; PINION $S_t$ = 250 MPa; $n_{pinion}$ = 250/105.8 = 2.36

$n_{gear}$ = 69/84.1 = 0.82 FAILURE

---

```
14-38
GIVEN: SPEED =  1600 REV/MIN  J =  .315
       N1 =  20 TEETH  N2 =  22 TEETH
       KA =  1    KS =  1.2    KM =  2
     H =  50 KW   M =  10 MM   F =  125  MM

RESULTS: D1 =  200 MM  V =  16.75515 M/S  KV =  .627991
         WT =  2.984158 KN  SIGMA =  28.96403 MPA
```

$S_t$ = 69 MPa; n = 69/29.0 = 2.38    Ans.

---

The program that follows is perfectly straightforward. It will request you to insert the input data and then solves all the necessary equations and prints out the results. It accepts only U. S. customary units. It is written in ZBASIC. We believe in general that programs such as this one have no place in this manual. But we thought this one might expedite your work of creating additional gear problem material. You may also find it a convenient base for the creation of more sophisticated gear programs. The program begins on the next page, and is followed by the solution to Problem 14-39.

```
100 PRINT "SOLUTION OF AGMA CONTACT-STRESS EQUATION FOR FULL-DEPTH"
110 PRINT "HELICAL GEARS"
120 PRINT "H = HORSEPOWER; SPEED = REV/MIN OF PINION"
130 PRINT "NP, NG = TOOTH NUMBERS"
140 PRINT "PSI = HELIX ANGLE, DEG.; F = FACE WIDTH, IN"
150 PRINT "PDN = NORMAL DIAMETRAL PITCH, TEETH/IN"
160 PRINT "PHIN = NORMAL PRESSURE ANGLE, DEG."
170 PRINT "QV = QUALITY NUMBER; CP = ELASTIC COEF., SQR(LB/SQ-IN)
180 PRINT "CA = APPLICATION FACTOR; CS = SIZE FACTOR"
190 PRINT "CM = LOAD DISTRIBUTION FACTOR; CF = SURFACE CONDITION FACTOR"
200 PRINT "ENTER DATA SPECIFIED:
210 INPUT "H = ";H: INPUT "SPEED = ";SPEED
220 INPUT "# OF PINION TEETH = ";NP
230 INPUT "# OF GEAR TEETH = ";NG: INPUT "F = ";F: INPUT "PDN = ";PDN
240 INPUT "PHIN = ";PHIN: INPUT "QV = ";QV
250 INPUT "CP = ";CP: INPUT "CA = ";CA: INPUT "PSI = ";PSI
260 INPUT "CS = ";CS: INPUT "CM = ";CM: INPUT "CF = ";CF
270 PSIR = PSI/57.296: PHINR = PHIN/57.296
280 PDT = PDN*COS (PSIR)
290 DP = NP/PDT: DG = NG/PDT:  RP = DP/2: RG = DG/2
300 V = 3.14159*DP*SPEED/12: WT = 33000!*H/V
310 B = ((12 - QV)^(2/3))/4: A = 50 + 56*(1-B)
320 CV = (A/(A + SQR(V)))^B: MG = NG/NP
330 PT = 3.14159/PDT: PX = PT/TAN(PSIR)
340 MF = F/PX
350 PHITR = ATN(TAN(PHINR)/COS(PSIR))
360 ADD = 1/PDN
370 RBP = RP*COS(PHITR): RBG = RG*COS(PHITR)
380 Z1 = SQR((RP + ADD)^2 - RBP^2)
390 Z2 = SQR((RG + ADD)^2 - RBG^2)

400 Z3 = (RP + RG)*SIN(PHITR)
410 IF Z1>=Z3 THEN Z1 = Z3
420 IF Z2>=Z3 THEN Z2 = Z3
430 Z = Z1 + Z2 -Z3
440 PN = (3.14159/PDN)*COS(PHINR)
450 MN = PN/(.95*Z)
460 I = (COS(PHITR))*(SIN(PHITR))*MG/(2*MN*(MG + 1))
470 E = (WT*CA/CV)*(CS/(DP*F))*(CM*CF/I)
480 SIGC = (CP/1000)*SQR(E)
490 LPRINT "  INPUT DATA:"
500 LPRINT "  HORSEPOWER H = "H" SPEED = "SPEED"REV/MIN"
510 LPRINT"  PINION TEETH N SUB P = "NP" GEAR TEETH N SUB G = "NG
520 LPRINT "  FACE WIDTH F = "F"IN"
530 LPRINT "  NORMAL DIAMETRAL PITCH P SUB N = "PDN
540 LPRINT "  NORMAL PRESSURE ANGLE PHI SUB N = "PHIN"DEG"
550 LPRINT "  HELIX ANGLE  PSI = "PSI"DEG"
560 LPRINT "  ACCURACY NUMBER Q SUB V = "QV
570 LPRINT "  ELASTIC COEFFICIENT C SUB P = "CP"SQR(LB/SQ-IN)
580 LPRINT "  FACTORS: C SUB A = "CA" C SUB F = "CF" C SUB M = "CM
590 LPRINT "  C SUB S = "CS
```

```
600 LPRINT
610 LPRINT "  OUTPUT DATA:"
620 LPRINT "  TRANS. DIA. PITCH, EQ. (13-14), CAP P SUB T = "PDT
630 LPRINT "  PINION DIAMETER = "DP"IN"
640 LPRINT "  GEAR DIAMETER = "DG"IN"
650 LPRINT "  TANGENTIAL LOAD, EQ. (13-26), W SUB T = "WT"LB"
660 LPRINT "  CIRCULAR PITCHES: P SUB T = "PT"IN"
670 LPRINT "  EQ. (13-13), P SUB X = "PX
680 LPRINT "  EQ. (14-24), P SUB CAP N = "PN"IN"
690 LPRINT "  PITCH-LINE VELOCITY V = "V"FT/MIN"
700 LPRINT "  RATIOS: EQ. (14-19), M SUB F = "MF
710 LPRINT "  EQ. (14-22), M SUB G = "MG" EQ.(14-21), M SUB N = "MN
720 PHIT = PHITR*57.296
730 LPRINT "  PHI SUB T = "PHIT"DEG. ADDENDUM = "ADD"IN"
740 LPRINT "  BASE CIRCLE RADII: PINION = "RBP"IN  GEAR = "RBG"IN
750 LPRINT "  EQ(14-27), AGMA DYNAMIC FACTOR C SUB V = "CV
760 LPRINT "  EQ(14-23), AGMA GEOMETRY FACTOR I = "I
770 LPRINT "  EQ.(14-16), AGMA CONTACT STRESS SIGMA SUB C = "SIGC"KPSI"
780 LPRINT:LPRINT
790 END
```

```
 14-39
INPUT DATA:
HORSEPOWER H =  4  SPEED =  1800 REV/MIN
PINION TEETH N SUB P =  20  GEAR TEETH N SUB G =  50
FACE WIDTH F =  1.5 IN
NORMAL DIAMETRAL PITCH P SUB N =  10
NORMAL PRESSURE ANGLE PHI SUB N =  20 DEG
HELIX ANGLE  PSI =  25 DEG
ACCURACY NUMBER Q SUB V =  6
ELASTIC COEFFICIENT C SUB P =  2300 SQR(LB/SQ-IN)
FACTORS: C SUB A =  1  C SUB F =  1  C SUB M =  1.2
C SUB S =  1

OUTPUT DATA:
TRANS. DIA. PITCH, EQ. (13-14), CAP P SUB T =  9.063086
PINION DIAMETER =  2.206754 IN
GEAR DIAMETER =  5.516885 IN
TANGENTIAL LOAD, EQ. (13-26), W SUB T =  126.9344 LB
CIRCULAR PITCHES: P SUB T =  .3466358 IN
EQ. (13-13), P SUB X =  .7433663
EQ. (14-24), P SUB CAP N =  .2952131 IN
PITCH-LINE VELOCITY V =  1039.908 FT/MIN
RATIOS: EQ. (14-19), M SUB F =  2.017848
EQ. (14-22), M SUB G =  2.5  EQ.(14-21), M SUB N =  .667899
PHI SUB T =  21.88022 DEG. ADDENDUM =  .1 IN
BASE CIRCLE RADII: PINION =  1.023896 IN  GEAR =  2.559739 IN
EQ(14-27), AGMA DYNAMIC FACTOR C SUB V =  .70036
EQ(14-23), AGMA GEOMETRY FACTOR I =  .1849196
EQ.(14-16), AGMA CONTACT STRESS SIGMA SUB C =  43.35442 KPSI
```

(Continued)

14-39 (Concluded)

Fig. 14-9: $C_L = 1.14$; $C_T = 1$; Table 14-7: $C_R = 0.85$; Table 14-4: $S_c = 105$ kpsi

Eq. (14-31: $A = 8.98(10^{-3})(240/240) - 8.29(10^{-3}) = 0.000\ 69$

Eq. (14-30): $C_H = 1.0 + 0.000\ 69(2.5 - 1) = 1.001$

Eq. (14-18): $\sigma_{c,all} = \dfrac{105(1.14)(1.001)}{1(0.85)} = 141$ kpsi; $n = \dfrac{\sigma_{c,all}}{\sigma_c} = \dfrac{141}{43.4} = 3.25$ Ans.

---

14-40

```
GIVEN: SPEED =  1800 REV/MIN   J =  .482
       N1 =  20 TEETH   N2 =  50 TEETH
       KA =  1   KS =  1   KM =  1.2
       H =  4 HP   PD =  9.060001 TEETH/IN.   F =  1.5 INCHES

RESULTS: D1 =  2.207506 INCHES   V =  1040.262 FT/MIN   KV =  .7003255
         WT =  126.8912 LB   SIGMA =  2.724599 KPSI
```

Table 14-3: $S_t = 31$ kpsi; $\sigma_{all} = S_t K_L / K_T K_R = 31(1.14)/[1(0.85)] = 41.6$ kpsi

$n = 41.6/2.72 = 15.28$ Ans.

---

14-41

```
GIVEN: SPEED =  720 REV/MIN   J =  .444
       N1 =  20 TEETH   N2 =  30 TEETH
       KA =  1   KS =  1   KM =  1.5
       H =  17.5 HP   PD =  4.33 TEETH/IN.   F =  3 INCHES

RESULTS: D1 =  4.618938 INCHES   V =  870.6484 FT/MIN   KV =  .760804
         WT =  663.2987 LB   SIGMA =  4.251199 KPSI
```

Fig. 14-2: $S_t = -274 + 167(240) - 0.152(240)^2 = 31(10^3)$ psi for pinion

$K_L = K_T = K_R = 1$, so $\sigma_{all} = 31$ kpsi and $n = 31/4.25 = 7.29$ Ans.

---

14-42

```
INPUT DATA:
HORSEPOWER H =  17.5   SPEED =  720 REV/MIN
PINION TEETH N SUB P =  20   GEAR TEETH N SUB G =  30
FACE WIDTH F =  3 IN
NORMAL DIAMETRAL PITCH P SUB N =  5
NORMAL PRESSURE ANGLE PHI SUB N =  20 DEG
HELIX ANGLE  PSI =  30 DEG
ACCURACY NUMBER Q SUB V =  7
ELASTIC COEFFICIENT C SUB P =  2300 SQR(LB/SQ-IN)
FACTORS: C SUB A =  1   C SUB F =  1   C SUB M =  1.5
C SUB S =  1
```

14-42 (Concluded)

```
OUTPUT DATA:
TRANS. DIA. PITCH, EQ. (13-14), CAP P SUB T =  4.330133
PINION DIAMETER =  4.618797 IN
GEAR DIAMETER =  6.928195 IN
TANGENTIAL LOAD, EQ. (13-26), W SUB T =  663.319 LB
CIRCULAR PITCHES: P SUB T =  .7255183 IN
EQ. (13-13), P SUB X =  1.25664
EQ. (14-24), P SUB CAP N =  .5904261 IN
PITCH-LINE VELOCITY V =  870.6219 FT/MIN
RATIOS: EQ. (14-19), M SUB F =  2.387318
EQ. (14-22), M SUB G =  1.5  EQ.(14-21), M SUB N =  .7000765
PHI SUB T =  22.79586 DEG. ADDENDUM =  .2 IN
BASE CIRCLE RADII: PINION =  2.129015 IN  GEAR =  3.193523 IN
EQ(14-27), AGMA DYNAMIC FACTOR C SUB V =  .7608066
EQ(14-23), AGMA GEOMETRY FACTOR I =  .1530624
EQ.(14-16), AGMA CONTACT STRESS SIGMA SUB C =  57.11343 KPSI
```

$S_c$ = 85 kpsi; n = 85/57.1 = 1.49 Ans.

---

```
C PROGRAM FOR 14-43 MEDS SIMULATION FOR Cp
    1 print*,'Enter seeds i1 and i2'
      read*,i1,i2
      print*,'Enter mean and sigma for Poisson ratio'
      read*,xnu,sxnu
      print*,'Enter mean and sigma for Young modulus'
      read*,E,sE
    2 print*,'Enter number of simulations'
      read*,n
      sum=0.
      sum2=0.
      do 100 i=1,n
      call gauss(i1,i2,xnu,sxnu,Gxnu)
      call gauss(i1,i2,E,sE,GE)
      f1=(1.-Gxnu**2)/GE
      f2=f1
      Cp=sqrt(1./3.141593/(f1+f2))
      sum=sum+Cp
      sum2=sum2+Cp*Cp
  100 continue
      Cpmean=sum/float(n)
      sigma=sqrt((sum2-(sum**2/float(n)))/float(n-1))
      print*,'Mean Cp=',Cpmean
      print*,'Sigma Cp=',sigma
      print*,' '
      print*,'Enter 1 to do a new problem'
      print*,'Enter 2 to do a new simulation'
      print*,'Enter 3 to stop'
      read*,index
      go to(1,2,3),index
    3 call exit
      end
```

14-43 (Concluded)

```
      Subroutine Gauss(i1,i2,Rmean,Rsigma,G)
      sum=0.
      do 10 i=1,12
      call randu(i1,i2,R)
      sum=sum+R
   10 continue
      z=sum-6.
      G=Rmean+z*Rsigma
      return
      end
```

```
$ run cpsim
Enter seeds i1 and i2
12345 98765
Enter mean and sigma for Poisson ratio
0.287 0.0072
Enter mean and sigma for Young modulus
29.e6 0.73e6
Enter number of simulations
1000
Mean Cp=   2243.314
Sigma Cp=   29.32569

Enter 1 to do a new problem
Enter 2 to do a new simulation
Enter 3 to stop
2
Enter number of simulations
1000
Mean Cp=   2242.402
Sigma Cp=   28.60913

Enter 1 to do a new problem
Enter 2 to do a new simulation
Enter 3 to stop
2
Enter number of simulations
10000
Mean Cp=   2242.780
Sigma Cp=   28.34265

Enter 1 to do a new problem
Enter 2 to do a new simulation
Enter 3 to stop
```

15-1 Table 15-2: $S_t$ = 19 kpsi for the pinion; $S_t$ = 4.6 kpsi for the gear. Assume the gear rules and that $K_T = K_R = K_L = 1$. Then $\sigma_{all}$ = 4.6 kpsi. Use $K_a = K_s = 1$. Fig. 15-5: J = 0.205. Table 15-1: $K_m$ = 1.25. From computer program $d_P$ = 3.33 in, V = 785 ft/min, and $K_v$ = 0.728.

Eq. (15-1):

$$W_t = \frac{K_v F J \sigma_{all}}{K_a P K_s K_m}$$

$$= \frac{0.728(1.25)(0.205)(4600)}{1(6)(1)(1.25)}$$

$$= 114 \text{ lb}$$

$$H = W_t V/33\,000 = 114(785)/33\,000$$

$$= 2.71 \text{ hp} \quad \underline{\text{Ans.}}$$

---

15-2

Eq. (14-31): $A = \frac{8.98}{1000}\left(\frac{300}{175}\right) - \frac{8.29}{1000}$

$$= 0.0071$$

Eq. (14-30):

$$C_H = 1.0 + 0.0071\left(\frac{60}{20} - 1\right) = 1.014$$

As in Prob. 15-1, $C_L = C_T = C_R = 1$.

Table 15-2: $S_c$ = 65 kpsi

Eq. (14-18): $\sigma_{c,all} = \frac{65(1)(1.014)}{1(1)}$

$$= 65.9 \text{ kpsi}$$

Fig. 15-6: I = 0.082. As in Prob. 15-1, $C_a = C_s = C_f = 1$; $C_m$ = 1.25, $C_v$ = 0.728, F = 1.25 in, d = 3.33 in.

Table 15-3: $C_P = 2450\sqrt{\text{psi}}$; V = 785 ft/min

Eq. (15-2):

$$W_t = \left(\frac{\sigma_c}{C_P}\right)^2 \left(\frac{C_v F d I}{C_a C_s C_m C_f}\right)$$

$$= \left(\frac{65\,900}{2\,450}\right)^2 \left[\frac{0.728(1.25)(3.33)(0.082)}{1(1)(1.25)(1)}\right]$$

$$W_t = 144 \text{ lb}$$

$$H = W_t V/33\,000 = 144(785)/33\,000$$

$$= 3.43 \text{ hp}$$

---

15-3 Table 15-3: $C_P = 232\sqrt{\text{MPa}}$

Table 15-2: $S_c$ = 586 MPa

Eq. (14-18): $C_L = C_H = C_T = C_R = 1$,

so $\sigma_{c,all}$ = 586 MPa

Eq. (15-2): $C_a = C_s = C_f = 1$.

Table 15-1: $C_m$ = 1.40

Eq. (14-29): $B = (12 - 5)^{2/3}/4 = 0.9148$

Eq. (14-28): $A = 50 + 56(1 - 0.9148)$

$$= 54.77$$

Eq. (14-27):

$$C_v = \frac{54.77}{54.77 + [200(8.29)]^{\frac{1}{2}}} = 0.601$$

Fig. 15-6): I = 0.066

Eq. (15-2):

$$W_t = \left(\frac{\sigma_{c,all}}{C_P}\right)^2 \left(\frac{C_v F d I}{C_a C_s C_m C_f}\right)$$

$$= \left(\frac{586}{232}\right)^2 \left[\frac{0.601(25)(88)(0.066)}{1(1)(1.40)(1)}\right]$$

$$= 398 \text{ N } (0.398 \text{ kN})$$

$$H = \frac{\pi d n W_t}{60} = \frac{\pi(88)(1800)(0.398)}{60(10^3)}$$

$$= 3.30 \text{ kW} \quad \underline{\text{Ans.}}$$

where $d_P = 22(4) = 88$ mm,

$$\omega = 1800\,\frac{2\pi}{60} = 188.5 \text{ rad/s}$$

and $V = r\omega = \frac{88}{2}\;\frac{188.5}{10^3} = 8.29$ m/s

---

15-4 Table 15-2: $S_t$ = 96 MPa

Eq. (14-17): $K_L = K_T = K_R = 1$ and so $\sigma_{all}$ = 96 MPa

Fig. 15-5: J = 0.21

Eq. (15-1): $K_a = K_s = 1$; $K_m$ = 1.40

15-4 (Concluded)

From Prob. 15-3: $V = 8.29$ m/s, $K_v = C_v = 0.601$

Eq. (15-1):

$$W_t = \frac{\sigma_{all} K_v FmJ}{K_a K_s K_m} = \frac{96(0.601)(25)(4)(0.21)}{1(1)(1.40)}$$

$$= 865 \text{ N} \quad (0.865 \text{ kN})$$

$$H = \frac{\pi d n W_t}{60} = \frac{\pi(88)(1800)(0.865)}{60(10^3)}$$

$$= 7.17 \text{ kW} \quad \underline{\text{Ans}}.$$

---

15-5 (a) Based on bending. $F = 0.71$ in

Table 15-2: $S_t = 19$ kpsi $= \sigma_{all}$, since $K_t = K_L = K_R = 1$.

Fig. 15-5: $J = 0.238$

Also, $K_a = K_s = 1$; $K_m = 1.40$

With this data and the given data as input, a computer solution gave $d_P = 2$ in, $V = 628$ ft/min, $K_v = 0.708$, $W_t = 273$ lb, and $\sigma = 31.9$ kpsi corresponding to $H = 5.2$ hp. So, based on bending these gears should be rated at

$$H = \frac{S_t}{\sigma} = \frac{19}{31.9}(5.2) = 3.10 \text{ hp}$$

(b) Based on contact stress.

Table 15-3: $C_p = 2800$

Table 15-2: $S_c = 120$ kpsi

Use $C_L = C_H = C_T = C_R = 1$, and $\sigma_{c,all} = 120$ kpsi

$C_a = C_s = C_f = 1$; $C_m = 1.40$

Fig. 15-6: $I = 0.078$

At $H = 5.2$ hp, $W_t = 273$ lb so Eq. (15-2) gives

$$\sigma_c = 2800\left[\frac{273(1)}{0.708}\,\frac{1}{2(0.71)}\,\frac{1.40(1)}{0.078}\right]^{\frac{1}{2}}(10^{-3})$$

$$= 195.5 \text{ kpsi}$$

Since 195.5 > 120 the gear will not carry 5.2 hp based on contact stress. In fact it should be rated at

$$H = 5.2\left(\frac{120}{195.5}\right)^2 = 1.96 \text{ hp} \quad \underline{\text{Ans}}.$$

---

15-6 Table 15-4: $\sigma_B = 7.0$ kpsi, $\sigma_C = 1.50$ kpsi

$$V_P = \frac{\pi d_P n}{12} = \frac{\pi(3)(1800)}{12} = 1414 \text{ ft/min}$$

Eq. (15-8): $V_S = \dfrac{1414}{\cos 4.767°} = 1419$ ft/min

Eq. (15-6): $d_G = 24/4 = 6$ in

Eq. (15-4):

$$T_G = \frac{\sigma_C C_V d_G^2 F_G}{30} = \frac{1.50(10^3)(1)(6)^2(1.5)}{30}$$

$= 2700$ lb·in based on contact stress. For bending, with $\psi_G = \lambda_P$, we have, from Eq. (15-5)

$$T_G = \frac{7(10^3)(6)^2(1.5)\cos 4.767°}{1.5(24)}$$

$$= 10\,464 \text{ lb·in}$$

So the contact stress governs, and

$$H = Tn/63\,000 = 2700\left(\frac{1}{24}\right)(1800)/63\,000$$

$$= 3.21 \text{ hp} \quad \underline{\text{Ans}}.$$

---

15-7 (a)

Table 15-4: $\sigma_B = 58.6$ MPa, $\sigma_C = 12.4$ MPa or $\sigma_B = 58.6(10^{-3})$ GPa

$\sigma_C = 12.4(10^{-3})$ GPa

Eq. (13-17): $d_G = \dfrac{N_G p_t}{\pi} = \dfrac{40(6)}{\pi} = 76.39$ mm

Eq. (13-19): $L = p_x N_W = 6(4) = 24$ mm

Eq. (13-20): $\lambda_P = \tan^{-1}\dfrac{24}{\pi(15)} = 27.0°$

15-7 (Continued)

$$V_P = \frac{\pi d_P n}{60}(10^{-3}) = \frac{\pi(15)(1200)}{60(1000)}$$

$$= 0.942 \text{ m/s}$$

Eq. (15-5): $C_V = 1$

Eq. (15-8):

$$V_S = \frac{0.942}{\cos 27.0°} = 1.057 \text{ m/s}$$

Eq. (15-5):

$$T_G = \frac{58.6(10^{-3})(76.39)^2(12)\cos 27°}{1.5(40)}$$

$$= 60.94 \text{ N·m} \quad \text{Ans.}$$

(b)

$$T_G = \frac{12.4(1)(76.39)^2(12)}{1000(30)}$$

$$= 28.94 \text{ N·m} \quad \text{Ans.}$$

---

15-8 OMITTED

---

15-9 $P = 8$, $d_G = 12/8 = 1.5$ in

Use $\sigma_C = 700$ psi

$$V_P = \frac{\pi dn}{12} = \frac{\pi(1)(600)}{12} = 157 \text{ ft/min}$$

$C_V = 1$, $V_S = 157/\cos 45° = 222$ ft/min

Fig. 15-8: $\Omega = 0.52$

Eq. (15-4):

$$T_G = \frac{0.52(700)(1)(1.5)^3(1/1.5)^{\frac{1}{2}}}{75}$$

$$= 13.37 \text{ lb·in}$$

$H = Tn/63\ 000$

$= [13.37(600(8/12)]/63\ 000 = 0.085$ hp Ans.

---

15-10 $\sigma_B = 7.00$ kpsi, $\sigma_C = 1.50$ kpsi

$d_P = 10/6 = 1.667$ in

$d_G = 30/6 = 5.00$ in

$$V_P = \frac{\pi d_P n}{12} = \frac{\pi(1.667)(120)}{12} = 52.4 \text{ ft/min}$$

$V_S = 52.4/\cos 45° = 74.1$ ft/min

$C_V = 1$

$P_n = P_t/\cos\psi = 6/\cos 45° = 8.48$

$p_n = \pi/8.48 = 0.370$ in

For bending, Eq. (15-5):

$$T_G = \frac{7000(30)(0.370)^3}{20} = 532 \text{ lb·in} \quad \text{Ans.}$$

For surface durability, Eq. (15-4):

$\Omega = 0.52$

$$T_G = \frac{0.52(1500)(1)(5.00)^3(1.667/5.00)^{\frac{1}{2}}}{75}$$

$$= 750 \text{ lb·in} \quad \text{Ans.}$$

---

16-1 (a) $\theta_1 = 0°$, $\theta_2 = 120°$, $\theta_a = 90°$, $\sin\theta_a = 1$, $a = 5$ in

Eq. (16-2): $M_f = \dfrac{0.28p_a(1.5)(6)}{1}\displaystyle\int_0^{120} \sin\theta\,(6 - 5\cos\theta)\,d\theta = 18.0p_a$ lb·in

Eq. (16-3): $M_N = \dfrac{p_a(1.5)(6)(5)}{1}\displaystyle\int_0^{120} \sin^2\theta\,d\theta = 56.8p_a$ lb·in

$c = 2(5\cos 30°) = 8.66$ in

Eq. (16-4): $F = \dfrac{56.8p_a - 18.0p_a}{8.66} = 4.48p_a$; $p_a = F/4.48 = 500/4.48 = 111.6$ psi on the RH shoe with cw rotation. Ans.

(b) RH shoe:

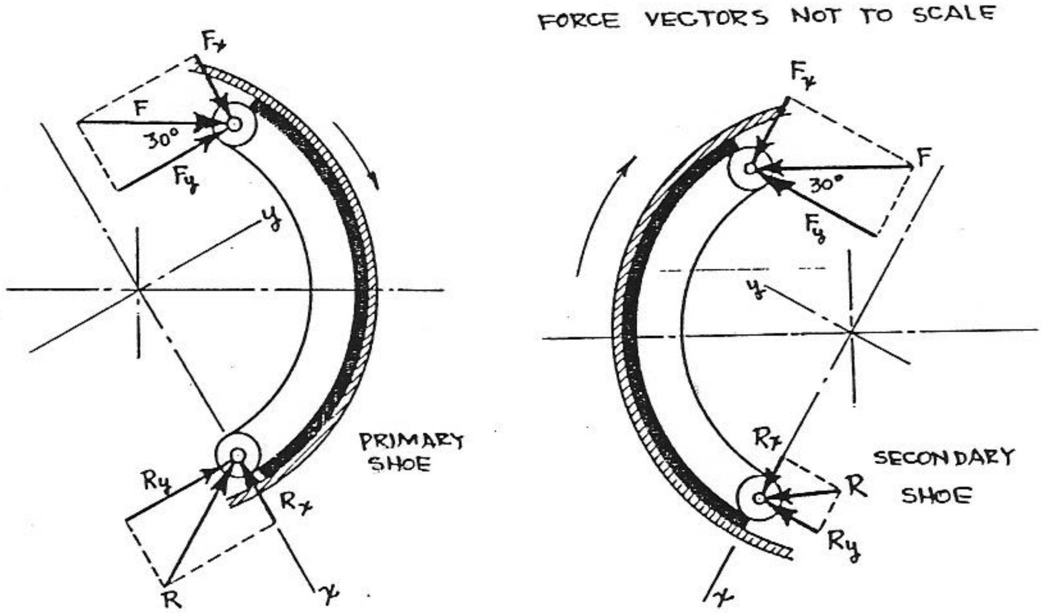
Eq. (16-6): $T = \dfrac{0.28(111.6)(1.5)(6)^2(\cos 0° - \cos 120°)}{1} = 2530$ lb·in

LH shoe: Eq. (16-7): $500 = \dfrac{56.8p_a + 18.0p_a}{8.66}$ ; $p_a = 57.9$ psi

Eq. (16-6): $T = \dfrac{0.28(57.9)(1.5)(6)^2(\cos 0° - \cos 120°)}{1} = 1310$ lb·in

$T_{total} = 2530 + 1310 = 3840$ lb·in Ans.

(c)




RH shoe: $F_x = 500\sin 30° = 250$ lb, $F_y = 500\cos 30° = 433$ lb

EQs. (16-8): $A = \left(\frac{1}{2}\sin^2\theta\right)\Big|_{0°}^{120°} = 0.375$; $B = \left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)\Big|_{0°}^{120°} = 1.264$

Eqs. (16-9): $R_x = \dfrac{111.6(1.5)(6)}{1}\left[0.375 - 0.28(1.264)\right] - 250 = -229$ lb

16-1 (Concluded)

$$R_y = \frac{111.6(1.5)(6)}{1}\ 1.264 + 0.28(0.375) - 433 = 942 \text{ lb}$$

$R = [(-229)^2 + (942)^2]^{\frac{1}{2}} = 969$ lb  Ans.

LH shoe $F_x = 250$ lb, $F_y = 433$ lb

Eqs. (16-10): $R_x = \frac{57.9(1.5)(6)}{1}[0.375 + 0.28(1.264)] - 250 = 130$ lb

$$R_y = \frac{57.9(1.5)(6)}{1}[1.264 - 0.28(0.375)] - 433 = 171 \text{ lb}$$

$R = [(130)^2 + (171)^2]^{\frac{1}{2}} = 215$ lb  Ans.

---

16-2 From Prob. 16-1 we find for the RH shoe

$M_f = 18.0p_a = \frac{18.0fp_a}{0.28} = 64.3fp_a$ lb·in; also $M_N = 56.8\ p_a$ lb·in

Eq. (16-4): $500 = \frac{56.8p_a - 64.3fp_a}{8.66}$, or $p_a = \frac{8.66(500)}{56.8 - 64.3f} = \frac{4330}{56.8 - 64.3\ f}$

and so $\mathbf{p}_a = \frac{4330}{56.8 - 64.3\mathbf{f}}$ ; $\mu_p = \frac{4330}{56.8 - 64.3(0.28)} = 111.6$ psi

To simplify the details, examine $1/p_a$.

$$\frac{1}{p} = \frac{56.8 - 64.3f}{4330} = 0.0131 - 0.014\ 85f = a - bf$$

Table 4-4, lines 3 and 4: $\mu_{(1/p)} = a - b\mu_f = 0.0131 - 0.014\ 85(0.28) = 0.008\ 942$

Table 4-4, line 4: $\hat{\sigma}_{(1/p)} = |b|\ \hat{\sigma}_f = 0.014\ 85(0.03) = 0.000\ 446$

$C_{(1/p)} = \hat{\sigma}_{(1/p)}/\mu_{(1/p)} = 0.000\ 446/0.008\ 942 = 0.0498$

Table 4-4, line 9: $C_p = C_{(1/p)} = 0.0498$; $\hat{\sigma}_p = C_p\mu_p = 0.0498(111.6) = 5.56$ psi

$\mathbf{p}_a = (111.6, 5.56)$ psi; $3\hat{\sigma}_p = 3(5.56) = 16.7$ psi

Therefore $p_{+3\hat{\sigma}} = 111.6 + 16.7 = 128.5$ psi, and $p_{-3\hat{\sigma}} = 111.6 - 16.7 = 95.1$ psi  Ans.

LH shoe: $p_a = \frac{4330}{56.8 + 64.3f}$; $\mu_p = \frac{4330}{56.8 + 64.3(0.28)} = 57.9$ psi

As before, $\frac{1}{p} = \frac{56.8 + 64.3f}{4330} = 0.0131 + 0.014\ 85f = a + bf$

$\mu_{(1/p)} = a + b\mu_f = 0.0131 + 0.014\ 85(0.28) = 0.017\ 25$

$\hat{\sigma}_{(1/p)} = b\hat{\sigma}_f = 0.014\ 85(0.03) = 0.000\ 446$; $C_{(1/p)} = 0.000\ 446/0.017\ 25 = 0.025\ 84$

$C_p = C_{(1/p)} = 0.025\ 84$; $\hat{\sigma}_p = C_p\mu_p = 0.025\ 84(57.9) = 1.494$ psi; $\mathbf{p} = (57.9, 1.494)$

$3\hat{\sigma}_p = 3(1.494) = 4.48$ psi; $p_{+3\hat{\sigma}} = 57.9 + 4.48 = 62.4$ psi, and

$p_{-3\hat{\sigma}} = 57.9 - 4.48 = 53.4$ psi  Ans.

---

COMMENT It is interesting to continue Prob. 16-2 by finding the corresponding braking torques.

$$\underset{\sim}{T} = \frac{f\underset{\sim}{p}_a br^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} = f\underset{\sim}{p}_a\,\frac{1.5(6)^2(1.5)}{1} = 81 f\underset{\sim}{p}_a$$

Now $\underset{\sim}{f}$ and $\underset{\sim}{p}_a$ are correlated. Let us remove the correlation by substituting

$$p_a = \frac{4330}{56.8 - 64.3f};\ T = (81f)\,\frac{4330}{56.8 - 64.3f} = \frac{350\ 730f}{56.8 - 64.3f};\ \mu_T = \frac{350\ 730(0.28)}{56.8 - 64.3(0.28)}$$

$$= 2531\ \text{lb·in}$$

Next, examine the reciprocal of T to simplify the details. For the RH shoe, we have

$$\frac{1}{T} = \frac{56.8 - 64.3f}{350\ 730f} = 0.000\ 162(1/f) - 0.000\ 183$$

$\mu_{(1/T)} = 0.000\ 162(1/0.28) - 0.000\ 183 = 0.000\ 395\ 57$ (from Table 4-4, line 9)

Table 4-4, line 4: $\hat{\sigma}_{(1/T)} = 0.000\ 162\hat{\sigma}_{(1/f)}$

Table 4-4, line 9: $\hat{\sigma}_{(1/f)} = \dfrac{1}{\mu_f}\,\dfrac{\hat{\sigma}_f}{\mu_f} = \dfrac{0.03}{0.28(0.28)} = 0.3826$

$\hat{\sigma}_{(1/T)} = 0.000\ 162(0.3826) = 0.000\ 061\ 99$; $C_{(1/T)} = \dfrac{0.000\ 061\ 99}{0.000\ 395\ 57} = 0.1567$

Table 4-4, line 9: $C_T = C_{(1/T)} = 0.1567$; $\hat{\sigma}_T = C_T\mu_T = 0.1567(2531) = 396.6$ lb·in

So $\underset{\sim}{T}_{RH} = (2531,\ 396.6)$ lb·in; $3\hat{\sigma}_T = 3(396.6) = 1190$ lb·in

Then $(T_{RH})_{+3\hat{\sigma}} = 2531 + 1190 = 3721$ lb·in; $(T_{RH})_{-3\hat{\sigma}} = 2531 - 1190 = 1341$ lb·in

The analysis for the LH shoe is similar. The results are:

$(T_{LH})_{+3\hat{\sigma}} = 1313 + 321 = 1634$ lb·in; $(T_{LH})_{-3\hat{\sigma}} = 1313 - 321 = 992$ lb·in

Total brake torque is $\underset{\sim}{T} = \underset{\sim}{T}_L + \underset{\sim}{T}_R$. Now $T_L$ and $T_R$ are correlated and there seems to be no easy way out. Refer to Charles R. Mischke, MATHEMATICAL MODEL BUILDING, Iowa State University Press, Ames, IA, 1980, p. 381, Eq. (18):

$\mu_T = \mu_{TL} + \mu_{TR} = 1313 + 2531 = 3844$ lb·in since $\rho \cong 1$; next, from Eq. (19),

$\hat{\sigma}_T = (\hat{\sigma}^2_{TL} + \hat{\sigma}^2_{TR} + 2\rho\hat{\sigma}_{TL}\hat{\sigma}_{TR})^{\frac{1}{2}} = [(106.9)^2 + (396)^2 + 2(1)(106.9)(396)]^{\frac{1}{2}} = 503$ lb·in

$\underset{\sim}{T} = (3844,\ 503)$ lb·in; putting it all together finally gives

$(T)_{+3\hat{\sigma}} = 3844 + 3(503) = 5353$ lb·in, and $(T)_{-3\hat{\sigma}} = 3844 - 3(503) = 2335$ lb·in

---

16-3 $\theta_1 = 0$, $\theta_2 = 120°$, $\theta_a = 90°$, $\sin\theta_a = 1$, $a = R = 90$ mm

LH shoe: $C = \displaystyle\int_{\theta_1}^{\theta_2} r\sin\theta\,d\theta = 0.140\int_0^{120}\sin\theta\,d\theta = 0.140(-\cos\theta)\Big|_0^{120} = 0.070$

$$D = -a\int_{\theta_1}^{\theta_2}\sin\theta\cos\theta\,d\theta = -a(\tfrac{1}{2}\sin^2\theta)\Big|_0^{120} = -0.090(0.375) = -0.03375$$

16-3 (Continued)

$$M_f = \frac{fp_a br}{\sin\theta_a}(C + D) = \frac{0.30p_a(0.030)(0.140)}{1}(0.070 - 0.03375) = 4.5675(10^{-5})p_a \text{ N·m}$$

$$\text{Let } E = \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta = \left(\frac{\theta_2}{2} - \frac{1}{4}\sin 2\theta\right)_{0^\circ}^{120^\circ} = 1.2637$$

$$\text{Then } M_N = \frac{p_a bra}{\sin\theta_a}E = \frac{p_a(0.030)(0.140)(0.090)}{1}(1.2637) = 47.77(10^{-5})p_a \text{ N·m}$$

$c = 2(0.90)\cos 30° = 0.1559$ cm; using $F = (M_N - M_f)/c$ for the LH shoe gives

$$1000 = p_a\frac{47.77 - 4.5675}{10^5(0.1559)}\text{ ; or } p_a = 361 \text{ kPa} \quad \underline{\text{Ans}}.$$

$$\text{Also, } T = \frac{fp_a br^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} = \frac{0.30(361)(10^3)(0.03)(0.140)^2[1 - (-0.5)]}{1}$$

$$= 95.5 \text{ N·m} \quad \underline{\text{Ans}}.$$

Similarly, for the RH shoe, we find $p_a = 298$ kPa, $T = 78.8$ N·m, and

$T_{total} = 95.5 + 78.8 = 174.3$ N·m   Ans.

---

16-4. (a) $\theta_1 = 10^\circ$; $\theta_2 = 75^\circ$; $\theta_a = 75^\circ$; some of the terms needed are evaluated as

$$A = \left[r\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta - a\int_{\theta_1}^{\theta_2}\sin\theta\cos\theta\, d\theta\right]$$

$$= r\left[-\cos\theta\right]_{\theta_1}^{\theta_2} - a\left[\frac{1}{2}\sin^2\theta\right]_{\theta_1}^{\theta_2} = 200\left[-\cos\theta\right]_{10}^{75} - 150\left[\frac{1}{2}\sin^2\theta\right]_{10}^{75}$$

$$= 77.5 \text{ mm}$$

$$B = \int_{\theta_1}^{\theta_2}\sin^2\theta\, d\theta = \left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_{10}^{75} = 0.527$$

$$C = \int_{\theta_1}^{\theta_2}\sin\theta\cos\theta\, d\theta = 0.4515$$

Now converting to pascals and metres, we have from Eq. (16-2),

$$M_f = \frac{fp_a br}{\sin\theta_a}A = \frac{0.24[(10)^6](0.075)(0.200)}{\sin 75}(0.0775) = 289 \text{ N·m;}$$

16-4 (Continued)

From Eq. (16-3),

$$M_N = \frac{p_a bra}{\sin \theta_a} B = \frac{[(10)^6](0.075)(0.200)(0.150)}{\sin 75}(0.527) = 1230 \text{ N·m;}$$

Finally, using Eq. (16-4), we have

$$F = \frac{M_N - M_f}{c} = \frac{1230 - 289}{165} = 5.70 \text{ kN} \quad \underline{\text{Ans}}.$$

(b) Use Eq. (16-6) for the primary shoe.

$$T = \frac{fp_a br^2(\cos \theta_1 - \cos \theta_2)}{\sin \theta_a}$$

$$= \frac{0.24[(10)^6](0.075)(0.200)^2(\cos 10 - \cos 75)}{\sin 75} = 541 \text{ N·m}$$

For the secondary shoe, we must first find $p_a$.

$M_N = \frac{1230}{10^6}p_a$; $M_f = \frac{289}{10^6}p_a$; from Eq. (16-7),

$$5.70 = \frac{(1230/10^6)p_a + (289/10^6)p_a}{165}; \text{ solving gives } p_a = 0.619(10)^6 \text{ Pa;}$$

Then

$$T = \frac{0.24[0.619(10)^6](0.075)(0.200)^2(\cos 10 - \cos 75)}{\sin 75} = 335 \text{ N·m;}$$

so the braking capacity is $T_{total} = 2(541) + 2(335) = 1750$ N·m <u>Ans</u>.

(c) Primary shoes:

$$R_x = \frac{p_a br}{\sin \theta_a}(C - fB) - F_x$$

$$= \frac{[10^6](0.075)(0.200)}{\sin 75}[0.4515 - 0.24(0.527)](10)^{-3} - 5.70$$

$$= -0.65 \text{ kN}$$

$$R_y = \frac{p_a br}{\sin \theta_a}(B + fC) - F_y$$

$$= \frac{[10^6](0.075)(0.200)}{\sin 75}[0.527 + 0.24(0.4515)](10)^{-3} - 0 = 9.87 \text{ kN}$$

16-4 (Concluded)

Secondary shoes:

$$R_x = \frac{p_a br}{\sin \theta_a}(C + fB) - F_x$$

$$= \frac{[0.619(10)^6](0.075)(0.200)}{\sin 75}[0.4515 + 0.24(0.527)](10)^{-3} - 5.70$$

$= -\ 0.144$ kN; from Eq. (14-11),

$$R_y = \frac{p_a br}{\sin \theta_a}(B - fC) - F_y$$

$$= \frac{[0.619(10)^6](0.075)(0.200)}{\sin 75}[0.527 - 0.24(0.4515)](10)^{-3} - 0$$

$= 4.02$ kN ; note from figure that $+y$ for secondary shoe is opposite $+\ y$ for primary shoe.

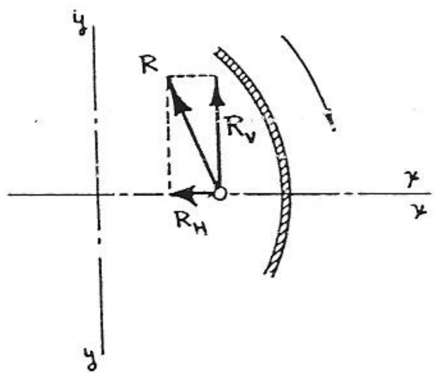
So

$R_H = -\ 0.65 - 0.144 = -\ 0.794$ kN

$R_V = 9.87 - 4.02 = 5.85$ kN

$R = \sqrt{(0.794)^2 + (5.85)^2}$

$= 5.90$ kN Ans.




16-5 $\theta_1 = 45° - \tan^{-1}(150/200) = 8.13°$, $\theta_2 = 98.13°$, $\theta_a = 90°$,

$a = [(150)^2 + (200)^2]^{1/2} = 250$ mm

Eq. (16-8): $A = \frac{1}{2}(\sin^2\theta)\Big|_{8.13}^{98.13} = 0.48$

Let $C = \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = -(\cos\theta)\Big|_{8.13}^{98.13} = 1.1314$

Eq. (16-2): $M_f = \frac{fp_a br}{\sin\theta_a}(rC - aA) = \frac{0.25p_a(0.030)(0.150)}{\sin 90°}[0.15(1.1314) - 0.25(0.48)]$

$= 5.59(10^{-5})p_a$ N·m

Eq. (16-6): $B = (\frac{\theta}{2} - \frac{1}{4}\sin 2\theta)\Big|_{8.13}^{98.13} = 0.925$

Eq. (16-3): $M_N = \frac{p_a bra}{\sin\theta_a}B = \frac{p_a(0.030)(0.150)(0.250)}{1}(0.925) = 1.0406(10^{-3})p_a$ N·m

Using $F = (M_N - M_f)/c$, we obtain $400 = \frac{104.06 - 5.59}{0.5(10^5)}p_a$; or $p_a = 203$ kPa

$$T = \frac{fp_a br^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} = \frac{0.25(203)(10^3)(0.030)(0.150)^2}{1}(1.1314) = 38.76 \text{ N·m}$$

Ans.

16-6 OMITTED

---

16-7. $\theta_2 = 180° - 30° - \tan^{-1}(3/12) = 136°$, $\theta_1 = 20° - \tan^{-1}(3/12) = 6°$, $\theta_a = 90°$,

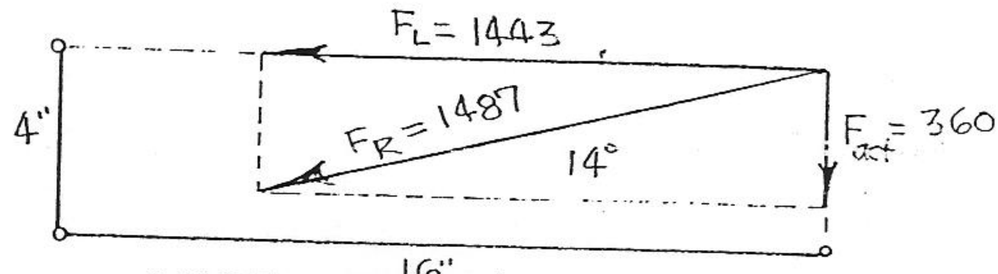
$a = [(3)^2 + (12)^2]^{\frac{1}{2}} = 12.37$ in, $r = 10$ in, $f = 0.30$, $b = 2$ in.

Eq. (16-2): $M_f = \dfrac{0.30(150)(10)}{\sin 90°}\displaystyle\int_{6°}^{136°} \sin\theta(10 - 12.37)\cos\theta)\,d\theta = 12\,800$ lb·in

Eq. (16-3): $M_N = \dfrac{150(2)(12.37)}{\sin 90°}\displaystyle\int_{6°}^{136°} \sin^2\theta\,d\theta = 53\,200$ lb·in

$c_L = 12 + 12 + 4 = 28$ in. Now note that $M_f$ is cw and $M_N$ is ccw. Therefore

$F_L = \dfrac{53\,200 - 12\,800}{28} = 1443$ lb




Eq. (16-6): $T_L = \dfrac{0.30(150)(2)(10)^2(\cos 6° - \cos 136°)}{\sin 90°} = 15\,420$ lb·in

RH shoe: $M_N = 53\,200(\frac{p_a}{150}) = 354.7p_a$, $M_f = 12\,800(\frac{p_a}{150}) = 85.3p_a$

On this shoe, both $M_N$ and $M_f$ are ccw. Also $c_R = (24 - 2\tan 14°)\cos 14° = 22.8$ in

Thus $1487 = \dfrac{354.7 + 85.3}{22.8}\,p_a$; $p_a = 77.1$ psi

Then $T_R = \dfrac{0.30(77.1)(2)(10)^2(\cos 6° - \cos 136°)}{\sin 90°} = 7930$ lb·in

$T_{total} = 15\,420 + 7930 = 23\,400$ lb·in Ans.

---

16-8 $P_1 = \dfrac{p_a bD}{2} = \dfrac{90(4)(14)}{2} = 2520$ lb Ans.

$f\phi = 0.25(2\pi)(270/360) = 1.18$

$P_2 = P_1/e^{f\phi} = 2520/e^{1.18} = 774$ lb Ans.

$T = (P_1 - P_2)(\frac{D}{2}) = (2520 - 774)(7)$

$= 12\,200$ lb·in Ans.

---

16-9 $P_1 = \dfrac{p_a bD}{2}$; $p_a = \dfrac{2P_1}{bD} = \dfrac{2(7.5)}{0.08(0.30)}$

$= 625$ kPa Ans.

$f\phi = 0.28(2\pi)(270/360)) = 1.32$

$P_2 = \dfrac{P_1}{e^{f\phi}} = \dfrac{7.5}{e^{1.32}} = 2.00$ kN

$T = (7.5 - 2.00)(\frac{300}{2}) = 825$ N·m Ans.

---

16-10 OMITTED

---

**16-11** Since $F = \frac{\pi p_a d}{2}(D - d)$,

$$p_a = \frac{2F}{\pi d(D - d)} = \frac{2(5)}{\pi(0.225)(0.3 - 0.225)}$$

$$= 189 \text{ kPa} \quad \underline{\text{Ans.}}$$

$$T = \frac{Ff}{4}(D + d) = \frac{5000(0.25)}{4}(0.3 + 0.225)$$

$$= 164 \text{ N·m} \quad \underline{\text{Ans.}}$$

---

**16-12**
$$p_a = \frac{4F}{\pi(d^2 - d^2)}$$

$$= \frac{4(5)}{\pi[(0.3)^2 - (0.225)^2]}$$

$$= 162 \text{ kPa} \quad \underline{\text{Ans.}}$$

$$T = \frac{Ff}{3}\,\frac{D^3 - d^3}{D^2 - d^2}$$

$$= \frac{5000(0.25)}{3}\left[\frac{(0.3)^3 - (0.225)^3}{(0.3)^2 - (0.225)^2}\right]$$

$$= 165 \text{ N·m} \quad \underline{\text{Ans.}}$$

---

**16-13** First write Eq. (16-25) as

$$\underset{\sim}{T} = \frac{F\underset{\sim}{f}}{4}(D + d) = \frac{5000(0.3 + 0.225)}{4}\underset{\sim}{f}$$

$$= 656.25\underset{\sim}{f}$$

where $\underset{\sim}{f} = (\bar{f}, 0.02)$

Corresponding to $\Phi(z) = 0.001$ we find from Table A-10 $z_\alpha = 3.009$

Also $f = \frac{T}{656.25} = \frac{150}{656.25} = 0.2286$

Since $z = -\frac{f - \bar{f}}{\hat{\sigma}_f}$ we have

$$\bar{f} = z\hat{\sigma}_f + f = 3.009\ (0.02) + 0.2286$$

$$= 0.290 \quad \underline{\text{Ans.}}$$

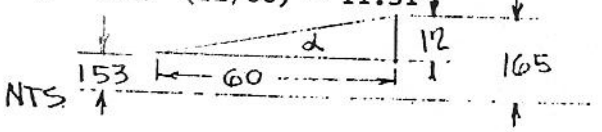
---

**16-14 to 16-17** OMITTED

---

**16-18** (a) By uniform wear

$\alpha = \tan^{-1}(12/60) = 11.31°$




Eq. (16-30):

$$200 = \frac{\pi(0.26)(0.306)p_a}{8 \sin 11.31°}[(0.330)^2$$

$$- (0.306)^2] = 0.002\ 43p_a$$

$$p_a = 82.3 \text{ kPa} \quad \underline{\text{Ans.}}$$

Eq. (16-31):

$$F = \frac{4(200)(\sin 11.31°)}{0.26(0.33 + 0.306)} = 949 \text{ N} \quad \underline{\text{Ans.}}$$

(b) By uniform pressure

Eq. (16-33):

$$200 = \frac{\pi(0.26)p_a}{12(\sin 11.31°)}[(0.330)^3$$

$$- (0.306)^3] = 0.002\ 53p_a$$

$$p_a = 79.1 \text{ kPa} \quad \underline{\text{Ans.}}$$

Eq. (16-34):

$$F = \frac{3(200)(\sin 11.31°)}{0.26}$$

$$\times \frac{[(0.33)^2 - (0.306)^2]}{[(0.33)^3 - (0.306)^3]} = 948 \text{ N} \quad \underline{\text{Ans.}}$$

---

**16-19** OMITTED

---

**16-20** $\omega = 2\pi n/60 = 2\pi(500)/60 = 52.4 \text{r/s}$

$$T = \frac{H}{\omega} = \frac{2(10)^3}{52.4} = 38.2 \text{ N·m}$$

$$F = \frac{T}{r} = \frac{38.2}{12} = 3.18 \text{ kN}$$

The average shear stress in the key is

$$\tau = \frac{3.18(10^3)}{6(40)} = 13.2 \text{ MPa} \quad \underline{\text{Ans.}}$$

The average bearing stress is

$$\sigma = \frac{F}{A} = \frac{-3.18(10^3)}{3(40)} = -26.5 \text{ MPa} \quad \underline{\text{Ans.}}$$

For the stresses in the jaw, it is best to assume that one jaw takes the entire load.

$$r_{av} = \frac{1}{2}\left(\frac{26}{2} + \frac{45}{2}\right) = 17.75 \text{ mm}$$

16-20 (Concluded)

$$F = \frac{T}{r_{av}} = \frac{38.2}{17.75} = 2.15 \text{ kN}$$

Then the bearing and shear stresses are

$$\sigma = \frac{-2.15(10^3)}{10(22.5 - 13)} = -22.6 \text{ MPa} \quad \underline{\text{Ans.}}$$

$$\tau = \frac{2.15(10^3)}{10[0.25\pi(17.75)^2]} = 0.869 \text{ MPa} \quad \underline{\text{Ans.}}$$

---

16-21 $\omega_1 = 2\pi n/60 = 2\pi(1800)/60$

$= 188 \text{ rad/s}$

$\omega_2 = 0$

Eq. (16-36):

$$\frac{I_1 I_2}{I_1 + I_2} = \frac{Tt_1}{\omega_1 - \omega_2} = \frac{320(8.3)}{188 - 0}$$

$= 14.13 \text{ N·m·s}^2$

Eq. (16-37):

$$E = 14.13 \frac{(188)^2}{2}(10^{-3}) = 250 \text{ kJ}$$

Eq. (16-40):

$$\Delta T = \frac{E}{Cm} = \frac{250(10^3)}{500(18)} = 27.8°\text{C} \quad \underline{\text{Ans.}}$$

---

16-22 $\omega_1 = 2\pi(1600)/60 = 168 \text{ rad/s}$

$\omega_2 = 0$

Eq. (16-36):

$$\frac{I_1 I_2}{I_1 + I_2} = \frac{Tt_1}{\omega_1 - \omega_2} = \frac{2800(10)}{168}$$

$= 166.7 \text{ lb·s}^2\text{·in}$

Eq. (16-37):

$$E = 166.7 \frac{(168)^2}{2} = 2.35(10^6) \text{ lb·in}$$

Eq. (16-38):

$H = 2.35(10^6)/9336 = 252 \text{ Btu}$

Eq. (16-39):

$\Delta T = 252/[0.12(40)] = 52.5°\text{F}$ $\underline{\text{Ans.}}$

---

16-23 OMITTED

---

16-24 $n = \dfrac{n_1 + n_2}{2} = \dfrac{260 + 240}{2}$

$= 250 \text{ rev/min}$

$$C_s = \frac{260 - 240}{250} = 0.08 \quad \underline{\text{Ans.}}$$

$\omega = 2\pi(250)/60 = 26.18 \text{ rad/s}$

$$I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{5000(12)}{0.08(26.18)^2}$$

$= 1094 \text{ lb·in·s}^2$

$$I_x = \frac{m}{8}(d_o^2 + d_i^2) = \frac{W}{8g}(d_o^2 + d_i^2)$$

$$W = \frac{8gI}{d_o^2 + d_i^2} = \frac{8(386)(1094)}{[(60)^2 + (56)^2]}$$

$= 502 \text{ lb}$

$w = 0.260 \text{ lb/in}^3$ for cast iron.

$$V = \frac{\pi t}{4}(d_o^2 - d_i^2)$$

$$= \frac{\pi t}{4}[(60)^2 - (56)^2] = 364t \text{ in}^3$$

$$V = \frac{W}{w} = \frac{502}{0.260} = 1931 \text{ in}^3$$

So $t = 1931/364 = 5.3$ in $\underline{\text{Ans.}}$

---

PROJECT A cast-iron flywheel has a speed of 200 rev/min, a rim weight of 4800 lb and a rim section measuring 10 × 10 in.

(a) Find the outside and inside rim diameters, the radius of gyration, and the peripheral velocity.

(b) What are the rim stresses at full speed?

(c) Estimate the energy stored at full speed.

(d) How much energy is released if the speed drops to 180 rev/min?

(e) Determine the angular deceleration corresponding to the speed change in part (d) if the change occurs in one-third revolution.

(f) What torque is required to produce the result of part (e)?

16-25. (a) The useful work done in one revolution of the crankshaft is

$$U = (35)(2000)(8)(0.15) = 84(10)^3 \text{ lb·in}$$

Accounting for friction, the total work done in one revolution is

$$U = \frac{84(10)^3}{1 - 0.16} = 100(10)^3 \text{ lb·in}$$

Since 15% of the crankshaft stroke is 7½ % of a crankshaft revolution, the energy fluctuation is

$$E_2 - E_1 = 84(10)^3 - 100(10)^3(0.075) = 76.5(10)^3 \text{ lb·in} \qquad \underline{\text{Ans.}}$$

(b) For the flywheel, $n = 6(90) = 540$ rpm

So $\omega = \frac{2\pi n}{60} = \frac{2\pi(540)}{60} = 56.5$ rad/s

Since $C_s = 0.1$, we have

$$I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{76.5(10)^3}{0.1(56.5)^2} = 239.6 \text{ lb·in·s}^2 \text{ ; since } I_x = \frac{md^2}{4},$$

$$W = \frac{4gI_x}{d^2} = \frac{4(386)(239.6)}{(48)^2} = 161 \text{ lb} \qquad \underline{\text{Ans.}}$$

---

16-26. (a) Using $n = 36$, $h = 2\pi/36$, gives $U_{total} = 16\ 700$ lb·in

$T_m = (16\ 700)/(2\pi) = 2658$ lb·in  Ans.

$$H = \frac{2\pi Tn}{(33\ 000)(12)} = \frac{2\pi(2658)(240)}{(33\ 000)(12)} = 10.1 \text{ hp} \qquad \underline{\text{Ans.}}$$

(b) The maximum loop begins at 60°, ends at 150°; subtracting $T_m$ gives -86, 2486, 4201, 5230, 5659, 5830, 5916, 5745, 5059, 857, and - 514. $n = 10$, $h = \pi/18$, $U_2 - U_1 = 6370$ lb·in,

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(240)}{60} = 25.1 \text{ rad/s}$$

$$I = \frac{U_2 - U_1}{C_s \omega^2} = \frac{6370}{0.075(25.1)^2} = 134.8 \text{ lb·s}^2\text{·in} \qquad \underline{\text{Ans.}}$$

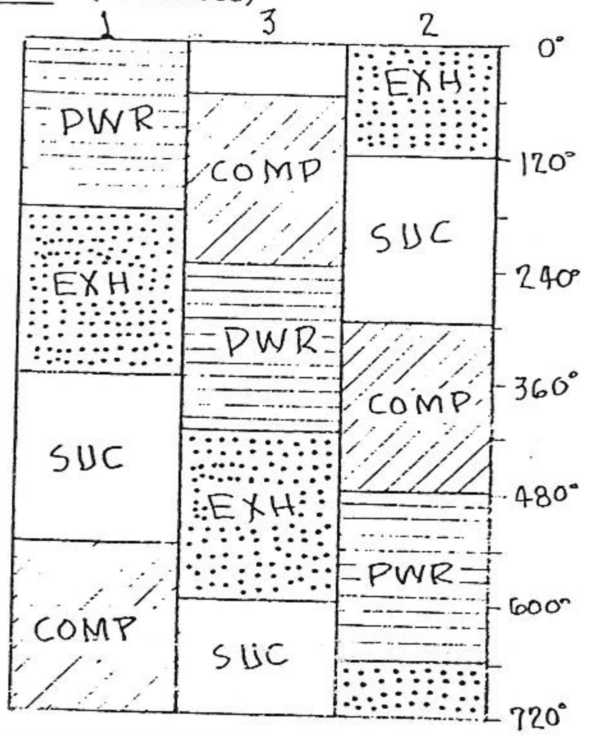
---

16-27.

$$U_{total} = 3490 \text{ lb·in; } T_m = \frac{3490}{(4\pi/3)} = 833 \text{ lb·in} \qquad \underline{\text{Ans.}}$$

The firing order and events for a three-cylinder engine with cranks spaced 120° apart is shown in the figure on the next page.

16-27 (Continued)




| Angle | y | Mult. n | ny |
|---|---|---|---|
| 0° | -833 | 1 | -833 |
| 15° | 1967 | 4 | 7868 |
| 30° | 1257 | 2 | 2514 |
| 45° | 1597 | 4 | 6388 |
| 60° | 1327 | 2 | 2654 |
| 75° | 1007 | 4 | 4028 |
| 90° | 757 | 2 | 1514 |
| 105° | 377 | 4 | 1508 |
| 120° | 233 | 2 | 466 |
| 135° | -30 | 4 | -120 |
| 150° | -301 | 2 | -602 |
| 165° | -649 | 4 | -2596 |
| 180° | -833 | 1 | -833 |
| | | Total | 21 956 |

We shall use the power stroke for the greatest energy fluctuation. The integration interval is, therefore, 0 to $\pi$ rad. Here we perform a tabular integration using Simpson's rule, though it can easily be done by computer. The y values shown in the table below were obtained from Table 16-3 by subtracting 833 lb·in from each term in the power stroke.

Applying Simpson's rule to the total from the table gives

$$U_2 - U_1 = \frac{21\ 956}{3}\frac{\pi}{12} = 1916 \text{ lb·in}$$

$$\text{Since } \omega = \frac{2\pi(2400)}{60} = 251 \text{ rad/s}$$

$$I = \frac{U_2 - U_1}{C_s\omega^2} = \frac{1916}{0.03(251)^2} = 1.014 \text{ lb·s}^2\text{·in}$$

Ans.

16-28 Eq. (16-24): $T = \frac{\pi f p_a d}{8}(D^2 - d^2) = \frac{\pi f p_a}{8}(dD^2 - d^3)$

$\frac{dT}{dd} = \frac{\pi f p_a}{8}(D^2 - 3d^2) = 0; \quad (d/D)^* = 1/\sqrt{3} = 0.577$

Check the stationary point for maximum or minimum

$\frac{d^2T}{dd^2} = \frac{\pi f p_a}{8}(-6d)$; since $\frac{d^2T}{dd^2} < 0$, $(d/D)^*$ locates a stationary point maximum.

---

16-29

$dF = \left(p_a \frac{d}{2r}\right)\frac{2\pi r\, dr}{\sin\alpha}\sin\alpha + \left(\frac{f p_a d}{2r}\right)\frac{2\pi r\, dr}{\sin\alpha}\cos\alpha; \quad dF = \pi p_a d(1 + f\cot\alpha)\, dr$

$F = \pi p_a d(1 + f\cot\alpha)\int_{d/2}^{D/2} dr = \frac{\pi p_a d(D - d)(1 + f\cot\alpha)}{2}$

When F is released: $1 - f\cot\alpha_{cr} = 0$, $f\cot\alpha_{cr} = 1$, $\tan\alpha_{cr} = f$, $\alpha_{cr} = \tan^{-1} f$

Critical angles for various coefficients of friction are:

| f | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|---|
| $\alpha_{cr}$, deg. | 0 | 5.7 | 11.3 | 16.7 | 21.8 | 26.6 |

---

PROJECT Write a computer simulation program to check your results of Prob. 16-2 if you have solved it, or to solve the problem if you have not already done so. Treat the coefficient of friction $\underset{\sim}{f} \sim N(0.28, 0.30)$ and find the mean and standard deviation of $p_a$ on the left shoe, $p_a$ on the right shoe, $T_L$, of the left shoe, $T_R$ of the right shoe, and the total torque T. Be careful to build in the correlation between $p_a$ on the left and right shoes.

A FORTRAN program listing is shown below as well as the results of several simulations.

```
C PROGRAM TO SIMULATE BRAKE PROBLEM 16-2
C DEFINE ARITHMETIC FUNCTION TO CALCULATE STANDARD DEVIATION
      S(SX,SX2,N)=SQRT((SX2-(SX**2/FLOAT(N)))/FLOAT(N-1))
    1 PRINT*,'BRAKE SIMULATION'
      PRINT*,'ENTER SEEDS IX,IY'
      READ*,IX,IY
      PRINT*,'ENTER COEFFICIENT OF FRICTION MEAN AND SIGMA'
      READ*,FMEAN,FSIGMA
    2 PRINT*,'ENTER NUMBER OF SIMULATIONS'
      READ*,N
```

```
C INITIALIZE COUNTERS
      SUMF=0.
      SUMPAL=0.
      SUMPAR=0.
      SUMTL=0.
      SUMTR=0.
      SUMT=0.
      SUMF2=0.
      SUMPAL2=0.
      SUMPAR2=0.
      SUMTL2=0.
      SUMTR2=0.
      SUMT2=0.
C CONDUCT SIMULATION
      DO 100 I=1,N
C OBTAIN A NORMALLY DISTRIBUTED COEFFICIENT OF FRICTION F
      CALL GAUSS(IX,IY,FMEAN,FSIGMA,F)
C USE THIS SINGLE INSTANCE OF F ALL THE WAY THRU TO OBTAIN THE
C NECESSARY CORRELATION
      SUMF=SUMF+F
      SUMF2=SUMF2+F**2
      PAL=4330./(56.8+64.3*F)
      SUMPAL=SUMPAL+PAL
      SUMPAL2=SUMPAL2+PAL**2
      PAR=4330./(56.8-64.3*F)
      SUMPAR=SUMPAR+PAR
      SUMPAR2=SUMPAR2+PAR**2
      TL=81.*F*PAL
      SUMTL=SUMTL+TL
      SUMTL2=SUMTL2+TL**2
      TR=81.*F*PAR
      SUMTR=SUMTR+TR
      SUMTR2=SUMTR2+TR**2
      T=TL+TR
      SUMT=SUMT+T
      SUMT2=SUMT2+T**2
  100 CONTINUE
C FIND THE NECESSARY STATISTICAL PARAMETERS
      FBAR=SUMF/FLOAT(N)
      FSIGMA=S(SUMF,SUMF2,N)
      PRINT*,'FBAR=',FBAR,' FSIGMA=',FSIGMA
      PALBAR=SUMPAL/FLOAT(N)
      PALSIG=S(SUMPAL,SUMPAL2,N)
      PRINT*,'PALBAR=',PALBAR,' PALSIG=',PALSIG
      PARBAR=SUMPAR/FLOAT(N)
      PARSIG=S(SUMPAR,SUMPAR2,N)
      PRINT*,'PARBAR=',PARBAR,' PARSIG=',PARSIG
      TLBAR=SUMTL/FLOAT(N)
      TLSIG=S(SUMTL,SUMTL2,N)
      PRINT*,'TLBAR=',TLBAR,' TLSIG=',TLSIG
      TRBAR=SUMTR/FLOAT(N)
      TRSIG=S(SUMTR,SUMTR2,N)
      PRINT*,'TRBAR=',TRBAR,' TRSIG=',TRSIG
```

```
      TBAR=SUMT/FLOAT(N)
      TSIG=S(SUMT,SUMT2,N)
      PRINT*,'TBAR=',TBAR,' TSIG=',TSIG
      PRINT*,'ENTER 1 FOR ANOTHER PROBLEM'
      PRINT*,'        2 FOR ANOTHER SIMULATION'
      READ*,INDEX
      GO TO(1,2),INDEX
      END
```

```
$ RUN BRAKE
BRAKE SIMULATION
ENTER SEEDS IX,IY
123456789
987654321
ENTER COEFFICIENT OF FRICTION MEAN AND SIGMA
.28
.03
ENTER NUMBER OF SIMULATIONS
10000
FBAR=  0.2799201      FSIGMA=  3.0265549E-02
PALBAR=   57.92789      PALSIG=   1.506860
PARBAR=   111.8773      PARSIG=   5.651194
TLBAR=   1309.728     TLSIG=   108.0792
TRBAR=   2550.472     TRSIG=   404.3007
TBAR=   3860.191     TSIG=   511.9568
ENTER 1 FOR ANOTHER PROBLEM
      2 FOR ANOTHER SIMULATION


2
ENTER NUMBER OF SIMULATIONS
10000
FBAR=  0.2800978      FSIGMA=  3.0435598E-02
PALBAR=   57.91946      PALSIG=   1.518562
PARBAR=   111.9137      PARSIG=   5.685986
TLBAR=   1310.331     TLSIG=   108.6712
TRBAR=   2553.066     TRSIG=   406.8062
TBAR=   3863.396     TSIG=   514.9630
ENTER 1 FOR ANOTHER PROBLEM
      2 FOR ANOTHER SIMULATION
2
ENTER NUMBER OF SIMULATIONS
10000
FBAR=  0.2799848      FSIGMA=  3.0278556E-02
PALBAR=   57.92463      PALSIG=   1.509976
PARBAR=   111.8904      PARSIG=   5.657561
TLBAR=   1309.961     TLSIG=   107.9688
TRBAR=   2551.388     TRSIG=   405.2267
TBAR=   3861.336     TSIG=   512.8618
ENTER 1 FOR ANOTHER PROBLEM
      2 FOR ANOTHER SIMULATION
```

17-1. (a) $V = \frac{\pi dn}{12} = \frac{\pi(6)(1750)}{12} = 2749$ fpm

$$F_1 - F_2 = \frac{(33\ 000)H}{V} = \frac{(33\ 000)(15)}{2749} = 180 \text{ lb}$$

The unit belt weight is $w = 12(\frac{9}{32})(6)(0.035) = 0.709$ lb/ft

$$F_c = \frac{wV^2}{g} = \frac{0.709)(2749/60)^2}{32.2} = 46.2 \text{ lb}$$

$$\theta_s = \pi - 2\sin^{-1}\left(\frac{D-d}{2C}\right) = 180° - 2\sin^{-1}\left[\frac{18-6}{2(8)(12)}\right]$$

$$= 172.833° \ (3.0165 \text{ rad})$$

So $f\theta = 0.30(3.0165) = 0.905$

$$\frac{F_1 - 46.2}{F_2 - 46.2} = e^{[0.3(3.0165)]};\ F_1 = 348.5 \text{ lb},\ F_2 = 168.5 \text{ lb} \quad \underline{\text{Ans.}}$$

$F_i = 258.5$ lb

(b) $e^{f\theta} = e^{[0.2(3.0165)]} = 1.828$

From Eq. (17-6), $F_1 + F_2 = 2F_i = 2(258.5) = 517$ lb

Therefore $F_2 = 517 - F_1$; we now have

$$\frac{F_1 - 46.2}{517 - F_1 - 46.2} = 1.828; \text{ solving gives } F_1 = 321 \text{ lb} \quad \underline{\text{Ans.}}$$

Then $F_2 = 517 - 321 = 196$ lb $\quad$ Ans.

$$H = \frac{(F_1 - F_2)V}{33\ 000} = \frac{(321-196)(2749)}{33\ 000} = 10.4 \text{ hp}$$

So the belt will slip.

(c) $\theta_L = \pi + 2\sin^{-1}\left(\frac{D-d}{2C}\right) = \pi + 2\sin^{-1}\left(\frac{18-6}{2(8)(12)}\right) = 3.27$ rad

$$L = [4C^2 - (D-d)^2]^{1/2} + \frac{1}{2}(D\theta_L + d\theta_s)$$

$$= [(4)(96)^2 - (18-6)^2]^{1/2} + \frac{1}{2}[18(3.27) + 6(3.0165)]$$

$$= 191.62 + 38.44957 = 230.1 \text{ in} \quad \underline{\text{Ans.}}$$

---

17-2. (a)

$$\theta = \pi + 2\sin^{-1}\left(\frac{D+d}{2C}\right) = \pi + 2\sin^{-1}\left(\frac{1.2}{12}\right)$$

$$= 3.342 \text{ rad } (191.5°) \quad \underline{\text{Ans.}}$$

$$L = [4C^2 - (D+d)^2]^{1/2} + \frac{\theta}{2}(D+d) = [4(6)^2 - (1.2)^2]^{1/2} + \frac{3.342}{2}(1.2)$$

$$= 13.9 \text{ m} \quad \underline{\text{Ans.}}$$

17-2 (Concluded)

(b) Since $U = (F_1 - F_2)V$, we have

$F_1 - F_2 = \frac{U}{V} = \frac{60}{25} = 2.4$ kN (1)

$F_c = mv^2 = 2(25)^2(10)^{-3} = 1.25$ kN; $f\theta = 0.38(3.342) = 1.27$;

next, using $\frac{F_1 - F_c}{F_2 - F_c} = e^{f\theta}$, we have

$F_1 - 1.25 = e^{1.27}(F_2 - 1.25) = 3.56F_2 - 4.45$

Then $F_1 - 3.56F_2 = -3.20$ (2)

Solving (1) and (2) simultaneously gives

$F_2 = 2.19$ kN Ans. $F_1 = 4.59$ kN Ans.

---

17-3 $V = \pi dn = \pi(4)(380) = 4775$ ft/min

We shall select a polyamide belt in the F series.

Table 17-5: $C_p = 1$, $C_V = 1$

Eq. (17-12):

$$F_a = \frac{16\ 500(1.1)(60)}{1(1)(4775)} = 228 \text{ lb}$$

If we select an F-2 belt, then

$w = 228/60 = 3.8$ in

So a belt 4-in wide is chosen.

---

17-4 Table 17-2: The allowable tension is 100 lb/in. So $F_a = 10(100) = 1000$ lb

Table 17-5: $C_p = 0.94$

Eq. (17-12):

$$H = \frac{C_p C_V F_a V}{16\ 500 K_S}$$

$$= \frac{0.94(1)(1000)(3600)}{16\ 500(1.3)} = 157.8 \text{ hp} \quad \text{Ans.}$$

Eq. (17-10): $F_1 + F_2 = 2F_i$

$= 2(1000) = 2000$ lb

Eq. (17-8): $F_c = mv^2$

Here $v = 3600/60 = 60$ ft/s

Table 17-2: $\mu = 0.8$, $w = 0.042$ lb/in³, $t = 0.13$ in

$W = 0.042(0.13)(10)(12) = 0.6552$ lb/ft

So $F_c = \frac{Wv^2}{g} = \frac{0.6552(60)^2}{32.2} = 73.3$ lb

Eq. (17-1):

$$\theta_d = \pi - 2\sin^{-1}\frac{(36 - 16)}{2(15)(12)}$$

$= 173.6°$ or 3.03 rad

Eq. (17-9): $\frac{F_1 - 73.3}{F_2 - 73.3} = e^{0.8(3.03)}$

$= 11.29$

or $F_1 - 73.3 = 11.29F_2 - 11.29(73.3)$

$= 11.29F_2 - 828$

We now have

$F_1 - 11.29F_2 = -755$ and

$F_1 + F_2 = 2000$

Solving simulataneously gives

$F_1 = 1776$ lb Ans.

$F_2 = 224$ lb Ans.

---

17-5 and 17-6 OMITTED

17-7 $V = \pi dn/12 = \pi(6.2)(3100)/12$
$= 5030$ ft/min

$H = 0.60(5) = 3.0$ hp capacity needed

Try for a B-section belt.

Table 17-10 : $K_2 = 1.00$ for 92-in pitch length.

Fig. 17-7: $K_1 = 1.00$

Table 17-9: $H_r = 4.00$ hp

Table 17-11: Choose $K_S = 1.30$

Since

$$H = \frac{NK_1K_2H_r}{K_S} \quad \text{we have}$$

$$N = \frac{K_S H}{K_1 K_2 H_r} = \frac{1.30(3)}{1(1)(4.00)} = 0.98$$

So use 1 belt.

Table 17-7: inside circ. = 90 in

Table 17-8: Pitch length = 90 + 1.8
= 91.8 in

So use a B90 belt. Ans.

---

17-8 From Eq. (17-13):

$L_p = 2(12)(12) + 1.57(2)(26) + 0$
$= 369.64$ in pitch length

Table 17- 7 : Select a 390-in inside circum. belt (C, D, or E).

Table 17-11: Select $K_S = 1.4$ for light shock and non-uniform torque.

Fig. 17-7: $K_1 = 1$

Table 17-10: $K_2 = 1.20$ for C belts, 1.10 for D belts, and 1.05 for E belts.

$V = \pi(26)(400)/12 = 2723$ ft/min

Table 17-8: For 26-in E belt

$$H_r = 22.8 + \frac{723}{1000}(30.3 - 22.8)$$

$= 28.2$ rated hp per belt

Required power is $H = 60(1.4) = 84$ hp

So # belts = $84/[(1)(1.2)(28.2)] = 2.48$

Use 3 E390 belts. Ans.

---

17-9 and 17-10) OMITTED

---

17-11 Table 17-14: $H_r = 6.20$ hp

Table 17-16: $K_1 = 0.70$

Table 17-17: $K_2 = 1.7$; use $K_S = 1.0$

$H'_r = 0.70(1.7)(6.20)/1.0 = 7.38$ hp Ans.

In Eq. (17-18) let $A = \dfrac{N_1 + N_2}{2} - \dfrac{L}{p}$

Then $$\frac{C}{p} = \frac{1}{4}\left\{-A \pm \left[A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2\right]^{\frac{1}{2}}\right\}$$

With $p = 0.75$ in, $L/p = 82$, $N_1 = 13$ and $N_2 = 52$ we find $C = 17.96$ in or, say, 18 in. Ans.

---

17-12 $H = 0.70(7.38) = 5.17$ hp

$$T = \frac{63\ 000(5.17)}{300} = 1086 \text{ lb·in}$$

Eq. (17-14): $$D = \frac{0.75}{\sin(180°/13)} = 3.13 \text{ in}$$

$F = T/r = 1086/(3.13/2) = 694$ lb Ans.

---

17-13 (a) Table 17-14: $H_r = 6.45$ hp

Table 17-16: $K_1 = 1.26$

Table 17-17: $K_2 = 3.3$; use $K_S = 1.0$

$H'_r = 1.26(3.3)(6.45)/1.0 = 26.8$ hp Ans.

(b) $V = Npn/12 = 21(0.5)(1200)/12$
$= 1050$ ft/min

Let $H = (F_1 - F_2)V/33\ 000$

Since $F_2 = 0$, we have

$$F_1 = \frac{33\ 000 H'_r}{V} = \frac{33\ 000(26.8)}{1050} = 842 \text{ lb}$$ Ans.

(c) Table 17-13: $F_t = 3130$ lb for a single-strand chain. Figure 17-9 shows that the number of links is proportional to the number of strands. So the factor of safety is

$$n = \frac{4F_t}{F_1} = \frac{4(3130)}{842} = 14.9$$ Ans.

(d) Eq. (17-18):

17-13 (Concluded)

$$\frac{L}{p} = \frac{2(20)}{0.5} + \frac{21 + 84}{2} + \frac{(84 - 21)^2}{4\pi^2(20/0.5)}$$

= 135 pitches Ans.

(e) Force per strand, $F_i$, is

$F_i = 842/4 = 211$ lb

Table 17-13: $d_R = w = 0.312$ in

Eq. (2-93): Use $\nu = 0.3$ and $E = 30$ Mpsi

$$b = \left\{\frac{4(211)[1 - (0.3)^2]/30(10^6)}{\pi(0.312)[(1/0.312) + (1/\infty)]}\right\}^{1/2}$$

= 0.002 85 in

Eq. (2-94):

$$p_{max} = \frac{2(211)(10^{-3})}{\pi(0.002\ 85)(0.312)} = 151 \text{ kpsi}$$

Fig. 2-35: $\left(\frac{\tau_{zy}}{p_{max}}\right)_{max} = 0.30$

$(\tau_{zy})_{max} = 0.30(151) = 45.3$ kpsi Ans.

---

17-14 and 17-15 OMITTED

---

17-16 Table 17-18: For 2-in 6 × 19 monitor steel, $(S_u)_{nom} = 106$ kpsi

Fig. 17-14: $\left(\frac{p}{S_u}\right)_{10^6 \text{cycles}} = 0.0014$

$$F_u = (S_u)_{nom}(A)_{nom} = 106\frac{\pi(2)^2}{4}$$

= 333 kip

In the equation $F_t = (W + w\ell)(1 + \frac{a}{g})$

$W = 4(2) = 8$ kip

Table 17-18:

$w\ell = 1.60d^2\ell = 1.60(2)^2(480)(10^{-3})$

= 3.072 kip

Thus

$$F_t = (8 + 3.072)(1 + \frac{2}{32.2}) = 11.76 \text{ kip}$$

From Eq. (17-19) the bending load $F_b$ is

$$F_b = \sigma A_m = \frac{Ed_w A_m}{D}$$

The metal area is: $A_m = 0.38d^2$

Outer wire dia. is, from Table 17-18),

$d_w = d/13$

Sheave dia. is D = 36 in

Modulus of elasticity, from Table (17-18) is E = 12 Mpsi

Thus

$$F_b = \frac{12(10)^6(2/13)[0.38(2)^2](10^{-3})}{36}$$

= 77.9 kip

For use in Eq. (17-22), $(p/S_u) = 0.0014$, $S_u = 240$ kpsi (p. 691), and D = 36 in.

So

$$F_f = \frac{0.0014(240)(2)(36)}{2} = 12.1 \text{ kip}$$

Factors of safety

Static without bending:

$$n = \frac{F_u}{F_t} = \frac{333}{11.76} = 28.3$$

Static with bending by EQ. (27-19):

$$n = \frac{F_u - F_b}{F_t} = \frac{333 - 77.9}{11.76} = 21.7$$

Static using Fig. 17-13: D/d = 36/2 = 18, $\phi = 1 - 0.08 = 0.92$

$$n = \frac{\phi F_u}{F_t} = \frac{0.92(333)}{11.76} = 26.1$$

Fatigue at $10^6$ cycles:

$$n = \frac{F_f}{F_t} = \frac{12.1}{11.76} = 1.03$$

---

17-17 Table 17-18: $w\ell = 1.60d^2\ell$

$= 1.60(1)^2(90)$

= 144 lb/rope

$$F_t = (W + w\ell)(1 + \frac{a}{g})$$

$$= (\frac{5000}{N} + 144)(1 + \frac{4}{32.2})$$

$$= \frac{5622}{N} + 162 \text{ lb}$$

V = 2(60) = 120 ft/min

Table 17-19: n = 9.2

Table 17-18: $D_{min} = 26(1) = 26$ in

17-17 (Concluded)

Fig. 17-13: (D/d = 26), $\phi = 1 - 0.07$

$= 0.93$

$$F_u = (S_u)_{nom} A_{nom} = 93\left[\frac{\pi(1)^2}{4}\right] = 73.0 \text{ kip}$$

Using $n = \phi F_u/F_t$, we have

$$F_t = \frac{5622}{N} + 162 = \frac{0.93(73.0)(10^3)}{9.2}$$

Solving gives N = 0.78 ropes

So use 1 rope. Ans.

Rope should be checked weekly for any evidence of fatigue.

---

17-18 (a) Table 17-18:

$(S_u)_{nom} = 106$ kpsi

Fig. 17-14): $(p/S_u)_{10^6} = 0.0014$

Page 691: $S_u = 240$ kpsi

Eq. (17-22):

$$F_f = \frac{0.0014(240)(72d)}{2} = 12.1d \text{ kip}$$

Table 17-18: $w\ell = 1.60d^2\ell$

$= 1.60(d^2)(2)$

$= 3.2d^2$ kip

$$F_t = (W + w\ell)(1 + \frac{a}{g})$$

$$= (8 + 3.2d^2)(1 + \frac{2}{32.2})$$

$$= 8.50 + 3.40d^2 \text{ kip}$$

$n = F_f/F_t = 12.1d/(8.50 + 3.40d^2)$

| d | n |
|---|---|
| 0.5 | 0.65 |
| 1.0 | 1.02 |
| 1.5 | 1.124 |
| 1.625 | 1.125 ← |
| 1.75 | 1.120 |
| 2.0 | 1.095 |

Note that n exhibits a stationary maximum at about 1 5/8-in rope diameter.

(b) $F_f = 12.1d$ kip as before

$$F_t = (\frac{8}{4} + 3.2d^2)(1 + \frac{2}{32.2})$$
$$= 2.12 + 3.40d^2 \text{ kip}$$

$n = 12.1d/(2.12 + 3.40d^2)$

| d | n |
|---|---|
| 0.5 | 2.037 |
| 0.5625 | 2.130 |
| 0.625 | 2.193 |
| 0.75 | 2.250 ← |
| 0.875 | 2.242 |
| 1.0 | 2.192 |

Here n exhibits a stationary point maximum at about 3/4-in rope diameter.

---

17-19 and 17-20 OMITTED

---

17-21 $A_m = 0.38d^2 = 0.38(2)^2 = 1.36 \text{ in}^2$

$E_r = 12$ Mpsi, $w = 1.6d^2 = 1.6(2)^2$
$= 6.4$ lb/ft

$w\ell = 6.4(480) = 3072$ lb

Prob. 3-16: $\delta = \frac{P\ell}{AE} + \frac{(w\ell)\ell}{AE}$

Treat the rest of the system as rigid so stretch is due to cage and wire wt.

$$\delta_1 = \frac{1000(480)(12)}{1.36(12)(10^6)} + \frac{3072(480)(12)}{2(1.36)(12)(10^6)}$$

$$= 0.353 + 0.542 = 0.895 \text{ in}$$

The stretch due to the cart, cage, and wire weight is

$$\delta_2 = \frac{10\ 000(480)(12)}{1.36(12)(10^6)} + 0.542 = 4.071 \text{ in}$$

Stretch due to cart and the load is

$$\Delta\delta = \delta_2 - \delta_1 = 4.072 - 0.895 = 3.176 \text{ in}$$

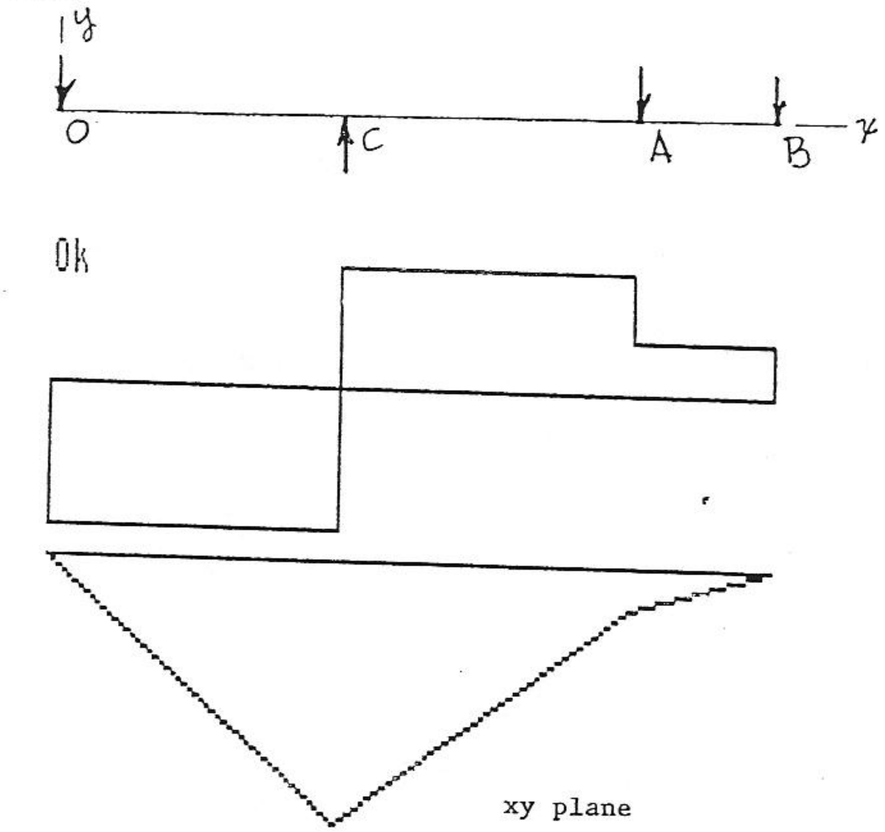
Alternative approach

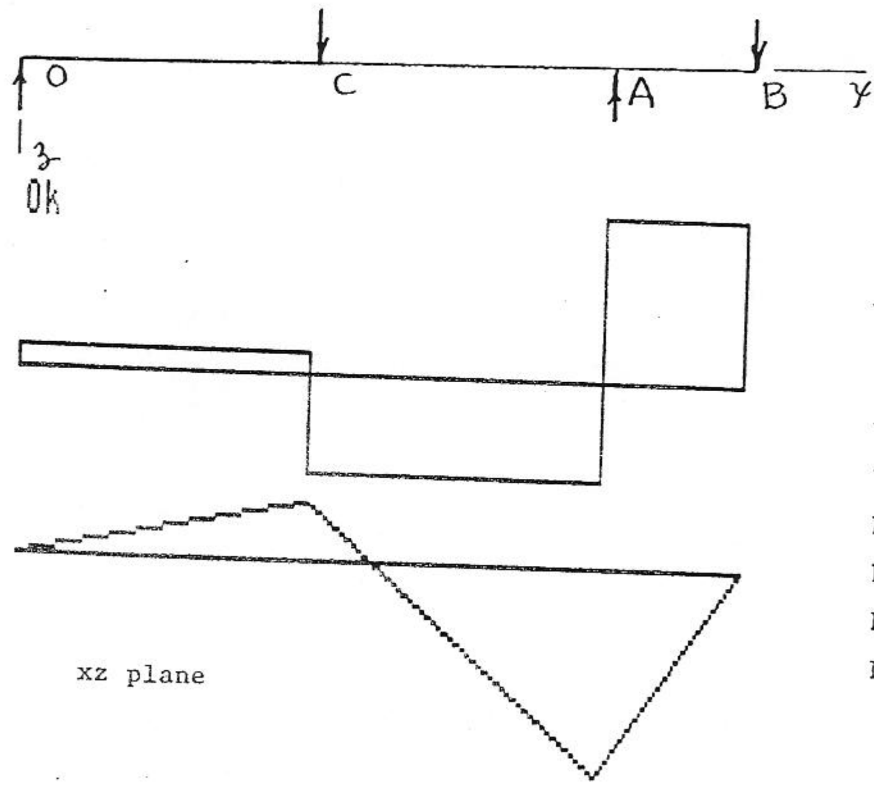
$$\Delta\delta = \frac{\Delta P\ell}{AE} = \frac{(10\ 000 - 1000)(480)(12)}{1.36(12)(10^6)}$$

$$= 3.177 \text{ in}$$

Longer hoists have even greater stretches due to loading, pointing to the need to support the cage during the loading operation.

18-1




Roll forces considered as concentrated. The normal roll force is

$F_y$ (at C) = −30(8) = −240 lb

and the pull is

$F_z$ (at C) = 0.40(240)
= 96 lb

The torque is

T = 96(2) = 192 lb·in

The forces acting on the gear are

$F_z$ (at B) = 192/1.5
= 128 lb

and

$F_y$ (at B) = −128 tan 20°
= −46.6 lb

xy plane

$V_O$ = −131 lb, $R_O$ = −131 lb
$V_C$ = +109 lb
$V_A$ = +46.6 lb, $R_A$ = −62.3 lb
$V_B$ = 0
$M_O$ = 0
$M_C$ = −754 lb·in
$M_A$ = −128 lb·in
$M_B$ = 0

xz plane

$V_O$ = −17.4 lb, $R_O$ =−17.4 lb
$V_C$ = 78.6 lb
$V_A$ = −128 lb, $R_A$ = −206.6 lb
$V_B$ = 0
$M_O$ = 0
$M_C$ = −100 lb·in
$M_A$ = 352 lb·in
$M_B$ = 0

18-1 Concluded)

$M_A = [(-128)^2 + (352)^2]^{\frac{1}{2}} = 375$ lb·in; $M_C = [(-754)^2 + (-100)^2]^{\frac{1}{2}} = 761$ lb·in

Eq. (18-10): $d_A = \left\{\frac{32(3.5)}{\pi(39)(10^3)}\left[(375)^2 + (192)^2\right]^{\frac{1}{2}}\right\}^{1/3} = 0.727$ in  Ans.

$d_C = \left\{\frac{32(3.5)}{\pi(39)(10^3)}\left[(761)^2 + (192)^2\right]^{\frac{1}{2}}\right\}^{1/3} = 0.895$ in  Ans.

EQ. (18-12): $d_A = \left\{\frac{16(3.5)}{\pi(39)(10^3)}\left[4(375)^2 + 3(192)^2\right]^{\frac{1}{2}}\right\}^{1/3} = 0.721$ in  Ans.

$d_C = \left\{\frac{16(3.5)}{\pi(39)(10^3)}\left[4(761)^2 + 3(192)^2\right]^{\frac{1}{2}}\right\}^{1/3} = 0.893$ in  Ans.

---

18-2 Failure may occur at either of the RH shoulders or a the center of the roll.

(a) Assume the 7/8-in section is critical. The moments in the xy and xz planes are M = 46.6(2) = 93.2 lb·in, and M = 128(2) = 256 lb·in. Thus

$M_a = [(256)^2 + (93.2)^2]^{\frac{1}{2}} = 272.4$ lb·in

Eq. (7-4): $S'_e = 0.504(72) = 36.29$ kpsi. Table 7-4: a = 2.70, b = -0.265.

Eq. (7-14): $k_a = 2.70(72)^{-0.265} = 0.869$

Eq. (7-15): $k_b = \left(\frac{0.875}{0.3}\right)^{-0.1133} = 0.886$. Also, $k_c = k_d = 1$

Fig. A-15-9: D/d = 1/0.875 = 1.14, r/d = 0.0625/0.875 = 0.071, $K_t = 1.7$

Fig. 5-16: q = 0.7; $K_f = 1 + 0.7(1.7 - 1) = 1.49$. $k_e = 1/1.49 = 0.671$

Eq.(7-13): $S_e = 0.869(0.886)(1)(1)(0.671)(36.29) = 18.75$ kpsi

Eq. (18-27): $\frac{1}{n} = \frac{32}{\pi(0.875)^3}\left\{\left[\frac{272.4}{18\ 750}\right]^2 + \left[\frac{192}{72\ 000}\right]^2\right\}^{\frac{1}{2}} = 0.225$; n = 4.45  Ans.

(b) Assume the 1-in section is critical at the shoulder. At this section the moments are found to be 210 and 293 lb·in. Thus $M_a = [(210)^2 + (293)^2]^{\frac{1}{2}} = 360$ lb·in

As before, $k_a = 0.869$, $k_b = 0.872$, D/d = 1.25, r/d = 0.0625, $K_t = 1.85$, $K_f = 1.595$, $k_e = 0.627$, and $S_e = 17.24$ kpsi. Thus

$\frac{1}{n} = \frac{32}{\pi(1)^3}\left\{\left[\frac{360}{17\ 240}\right]^2 + \left[\frac{192}{72\ 000}\right]^2\right\}^{\frac{1}{2}} = 0.214$; n = 4.66  Ans.

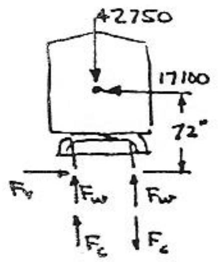
(c) At midroll. Assume half the torque has been used. So $T_m = 192/2 = 96$ lb·in

$k_a = 0.869$, $k_b = 0.851$, $k_c = k_d = k_e = 1$, $S_e = 26.84$ kpsi

$\frac{1}{n} = \frac{32}{\pi(1.25)^3}\left\{\left[\frac{761}{26\ 840}\right]^2 + \left[\frac{96}{72\ 000}\right]^2\right\}^{\frac{1}{2}} = 0.148$; n = 6.76  Ans.

---

18-3




Car, truck frame, and wheels as a free body:

Tipping moment = 17 100(72) = $1.23(10^6)$ lb·in

Force of resisting couple, $F_c = 1.23(10^6)/59.5$

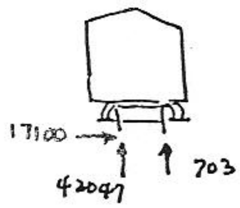
= 20 672 lb

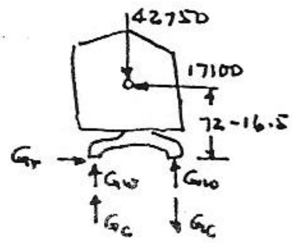
Force supporting weight, $F_w$ = 42 750/2 = 21 375 lb

$F_r$ = 17 100 lb

$R_1 = F_w + F_c$ = 21 375 + 20 672 = 42 047 lb

$R_2 = F_w - F_c$ = 21 375 - 20 672 = 703 lb




Car and truck as a free body:

Tipping moment = 17 100(72 - 16.5) = 949 050 lb·in

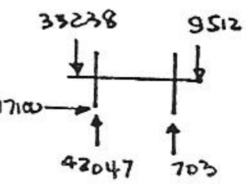
$G_c = \frac{949\ 050}{80}$ = 11 863 lb

Force at journal $G_w$ = 42 750/2 = 21 375 lb

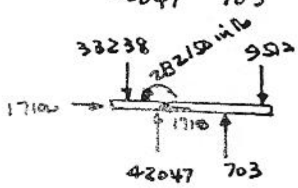
$G_r$ = 17 100 lb

$R_1$ = 21 375 + 11 863 = 33 238 lb

$R_2$ = 21 375 - 11 863 = 9512 lb



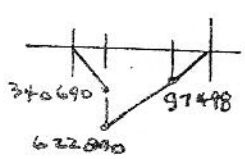

Wheels and axle as a free body




Axle as a free body:

Couple due to flange force = 17 100(33/2)

= 282 150 lb·in




Midspan moment:

M = 33 238(40) + 282 150 - 42 047(29.75)

= 360 772 lb·in

Since curve and wind can be from opposite directions, axle must resist 622 840 lb·in at either wheel seat and resist 360 772 lb·in in the center. Bearing load could be 33 238 lb at other bearing. The tapered axle is a consequence of this. Brake forces are neglected because they are small and induce moment on the ⊥ plane.

18-4 At wheel seat M = 622 840 lb·in

$$\sigma = \frac{32M}{\pi d^3} = \frac{32(622\ 840)}{\pi(7)^3} = 18\ 496 \text{ psi}$$

At midspan

$$\sigma = \frac{32(360\ 772)}{\pi(5.375)^3} = 23\ 665 \text{ psi}$$

The stress at the wheel seat consists of bending stress plus the shrink-fit pressure. The bending stress at the seat is kept less than at mid axle to compensate for the influence of the shrink fit.

---

18-5 and 18-6 OMITTED

---

18-7 Table 7-4: a = 2.7, b = −0.265

Eq. (7-14): $k_a = 2.7(100)^{-0.265} = 0.797$

Eq. (7-15): $k_b = (1/0.3)^{-0.1133} = 0.872$

Eq. (7-22): $k_c = 1$; also $k_d = 1$

Fig. A-15-9: D/d = 1.5, r/d = 0.125, $K_t = 1.58$

Fig. 5-16: q = 0.84

$K_f = 1 + 0.84(1.58 - 1) = 1.487$

$k_e = 1/1.487 = 0.672$

Eq. (7-4): $S'_e = 0.504(100) = 50.4$ kpsi

Eq. (7-13):

$$S_e = 0.797(0.872)(0.672)(50.4) = 23.5 \text{ kpsi}$$

Eq. (18-27):

$$\frac{1}{n} = \frac{32}{\pi(1)^3}\left[\left(\frac{0.800}{23.5}\right)^2 + \left(\frac{0.400}{100}\right)^2\right]^{\frac{1}{2}}$$

from which n = 2.86 Ans.

Based on yielding:

Eq. (18-11):

$$\frac{1}{n} = \frac{32}{\pi(1)^3(70\ 000)}\left[(800)^2 + (400)^2\right]^{\frac{1}{2}}$$

From which n = 7.68

Based on first-cycle yielding:

Eq. (18-25): $r = \frac{(800)^2}{(400)^2}\left(\frac{100}{23.5}\right) = 17.0$

$$\sigma_a = \frac{32M}{\pi d^3} = \frac{32(800)}{\pi(1)^3} = 8149 \text{ psi}$$

Eq. (18-21):

$$n = \frac{S_y}{\sigma_a[1 + (1/17)]} = \frac{70\ 000}{8149[1 + (1/17)]} = 8.11$$

---

18-8 Eq. (7-13) without stress concentration is

$S_e = 0.797(0.872)(50.4) = 35.0$ kpsi

Eq. (18-31):

$$\frac{1}{n} = \frac{32}{\pi(1)^3}\left\{\left[\frac{1.487(800)}{35\ 000}\right]^2 + \left(\frac{400}{70\ 000}\right)^2\right\}^{\frac{1}{2}}$$

= 0.351, and so n = 2.85 Ans.

---

18-9 Eq. (18-35):

$$\frac{1}{n} = \frac{32}{\pi(1)^3}\left[\frac{1.487(800)}{35(10^3)} + \frac{\sqrt{3}(400)}{2(100)(10^3)}\right]$$

= 0.381; n = 2.62 Ans.

---

18-10 Eq. (18-40):

$$\frac{1}{n} = \frac{32}{\pi(1)^3}\left\{\left[\frac{1.487(800)}{35(10^3)}\right]^2 + \frac{3}{4}\left[\frac{400}{70(10^3)}\right]^2\right\}^{\frac{1}{2}}$$

= 0.350; n = 2.86 Ans.

---

18-11 Prob. 18-8: $S_e = 35.0$ kpsi

Fig. A-15-8: D/d = 1.5, r/d = 0.125, $K_{ts} = 1.38$

Fig. 5-17: $H_B = S_u/500 = 100\ 000/500 = 200$

$q = 0.95$; $K_{fs} = 1 + 0.95(1.38 - 1) = 1.36$

Eq. (18-42):

$$\frac{1}{n} = \frac{32}{\pi(1)^3}\left\{\left(\frac{800}{70\ 000}\right)^2 + \left[\frac{1.36(600)}{35\ 000}\right]^2\right\}^{\frac{1}{2}}$$

n = 3.78 Ans.

---

18-12 Eq. (7-4): $S'_e = 0.504(1226)$

$= 618$ MPa

Table 7-4: $a = 4.51$, $b = -0.265$

Eq. (7-14): $k_a = 4.51(1226)^{-0.265}$

$= 0.685$

Eq. (7-15): $k_b = (d/7.62)^{-0.1133}$

Estimate $k_b = 0.80$ for first trial

$k_c = k_d = 1$

Fig. A-15-14: $d_R = d - 2r = 0.75D - \frac{D}{20}$

$= 0.70D$

$D/d = D/0.75D = 1.333$

$r/d = 0.05D/0.75D = 0.067$

$K_t = 1.98$

Fig. 5-16: For first trial use $q = 0.90$

$K_f = 1 + 0.90(1.98 - 1) = 1.88$

Eq. (7-13):

$S_e = 0.685(0.80)(618) = 339$ MPa

Eq. (18-34):

$$d_R = \left\{\frac{32(2.5)}{\pi}\left[\frac{1.88(70)(10^3)}{339} + \frac{\sqrt{3}\ (45)(10^3)}{2(1226)}\right]\right\}^{1/3} = 22.0 \text{ mm}$$

Choose $d = 25$ mm Ans.

Then $D = 25/0.75 = 33.33$ mm

Use $D = 32$ mm Ans.

$r = 32/20 = 1.6$ mm Ans.

$d_R = 25 - 2(1.6) = 21.8$ mm

Eq. (7-15): $k_b = (21.8/7.62)^{-0.1133}$

$= 0.888$

But if d is used, we get

$k_b = (25/7.62)^{-0.1133} = 0.874$

The difference is small

Eq. (5-16): $q = 0.90$, and so $K_f = 1.88$

$S_e = 0.685(0.888)(618) = 376$ MPa

$$\frac{1}{n} = \frac{32}{\pi(21.8)^3}\left[\frac{1.88(70)(10^3)}{376} + \frac{\sqrt{3}(45(10^3)}{2(1226)}\right];\ n = 2.66$$

First cycle yielding:

Eq. (18-36):

$$r = \frac{2(1.88)(70)}{\sqrt{3}(45)} = 3.38$$

$$\sigma_a = \frac{32MK_f}{\pi d^3} = \frac{32(70)(10^3)(1.88)}{\pi(21.8)^3}$$

$= 129$ MPa

Eq. (18-21):

$$n = \frac{S_y}{\sigma_a[1 + (1/r)]}$$

$$= \frac{1130}{129[1 + (1/3.38)]} = 6.76$$

Or, removing $K_f$ gives

$n = 6.76(1.88) = 12.7$

---

18-13 $d = 20$ mm, $D = 20/0.75 = 26.67$

So use $D = 27$ mm Ans.

$r = D/20 = 27/20 = 1.35$ mm Ans.

$d_R = 20 - 2(1.35) = 17.3$ mm

$k_b = (17.3/7.62)^{-0.1133} = 0.911$

$S_e = 0.685(0.911)(618) = 386$ MPa

$D/d_R = 27/17.3 = 1.56$

$r/d_R = 1.35/17.3 = 0.078$

Fig. A-15-9: $K_t = 1.8$

Fig. 5-16: $q = 0.88$

$K_f = 1 + 0.88(1.8 - 1) = 1.70$

Eq. (18-35):

$$\frac{1}{n} = \frac{32}{\pi(17.3)^3}\left[\frac{1.70(70)(10^3)}{386} + \frac{\sqrt{3}(45)(10^3)}{2(1226)}\right]$$

$n = 1.49$ Ans.

Based on first-cycle yielding:

Eq. (18-36): $$r = \frac{2(1.70)(70)(10^3)}{\sqrt{3}(45)(10^3)}$$

$= 3.05$

18-13 (Concluded)

$$\sigma_a = \frac{32(70)(10^3)(1.70)}{\pi(17.3)^3} = 234 \text{ MPa}$$

Eq. (18-21):

$$n = \frac{1130}{234[1 + (1/3.05)]} = 3.64 \quad \underline{\text{Ans.}}$$

---

18-14, 18-15, and 18-16 OMITTED

---

18-17 First change the basis of the stress-concentration factor in Table A-16 to that of a full section. Then

$$\tau = K_{ts}\tau_0 = K'_{ts}\tau'_0$$

$$K_{ts}\frac{32T}{\pi AD^3} = K'_{ts}\frac{32T}{\pi D^3}$$

Therefore $K'_{ts} = K_{ts}/A$

From Table A-16

| a/D | A | $K_{ts}$ | $K'_{ts}$ | |
|---|---|---|---|---|
| 0.05 | 0.95 | 1.77 | 1.85 | |
| 0.075 | 0.93 | 1.71 | 1.84 | |
| 0.10 | 0.92 | 1.68 | 1.83 | ← |
| 0.125 | 0.89 | 1.64 | 1.84 | Minimum |
| 0.15 | 0.87 | 1.62 | 1.86 | stress results |
| 0.175 | 0.85 | 1.60 | 1.88 | with |
| 0.20 | 0.82 | 1.58 | 1.93 | a/D = 0.10 |

A similar result is obtained for bending.

Design rule: Make pins about one-tenth of shaft diameter.

---

18-18

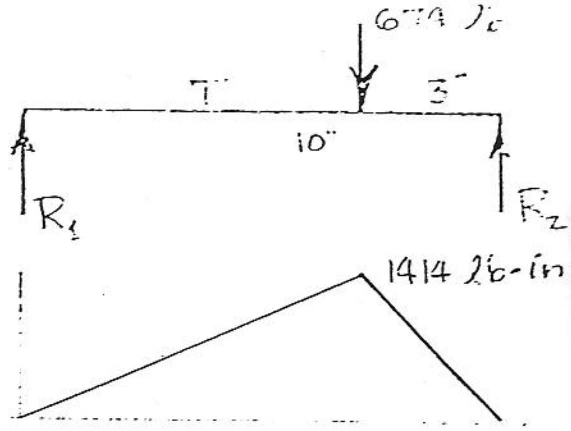
$$T = \frac{63\ 000(4.5)}{112} = 2530 \text{ lb·in}$$

$W_t = 2530/4 = 633$ lb

$W = 633/\cos 20° = 674$ lb

$R_1 = 674(\frac{3}{10}) = 202$ lb

$R_2 = 674(\frac{7}{10}) = 472$ lb




First, discover if problem is stress-controlled, or deflection controlled. If deflection: Limiting slope for deep-groove ball bearings is 0.004 rad. From Prob. 3-19, for RH bearing, with n = 2

$$d = \left|\frac{32Pa(\ell^2 - a^2)}{3\pi E\ell\xi/n}\right|^{\frac{1}{4}}$$

$$= \left|\frac{32(674)(7)[(10)^2 - (7)^2]}{3\pi(30)(10^6)(0.004/2)(10)}\right|^{\frac{1}{4}}$$

$$= 1.08 \text{ in}$$

So an average diameter of 1.20 in will protect the bearing lives.

For strength, use the distortion-energy theory with Eq. (18-34).

$$d = \left[\frac{32n}{\pi}\left(\frac{M}{S_e} + \frac{\sqrt{3}T}{2S_u}\right)\right]^{1/3}$$

Now substitute

$S_e = k_a k_b k_c k_d k_e S'_e \cong S_u/8$, $d = 1.75$ in, and solve for $S_u$.

$$S_u = \frac{32n}{\pi d^3}\left(\frac{M}{1/8} + \frac{\sqrt{3}\ T}{2}\right)$$

$$= \frac{32(2)}{\pi(1.75)^3}\left[\frac{1414}{1/8} + \frac{\sqrt{3}(2530)}{2}\right]$$

$$= 51.3 \text{ kpsi}$$

Choosing AISI 1020 cold-drawn steel in a 2-in size should meet both the deflection and strengths needs.

18-18 (Continued)

Next, choose the bearings and decide on the associated shaft geometry.

Design life: $x_D = \frac{10\ 000(60)(112)}{10^6}$

$= 67.2$ rating lives

Use application factor of 1.2. For the RH bearing, Eq. (11-9):

$$F_R = 1.2(472) \times \left[\frac{67.2}{0.02 + 4.439(1 - 0.99)^{1/1.483}}\right]^{1/3}$$

$= 3820$ lb (17.0 kN)

Use an 02-30-mm deep groove ball bearing.

LH bearing: $F_R = 17.0\ \frac{202}{472} = 7.3$ kN

An 02-15 mm will carry the load but bore must be at least 1 in, dictating an 02-30 mm bearing. So make them both identical.

Table 11-3:

$w = 16$ mm(0.630 in)--make seat 0.750 in

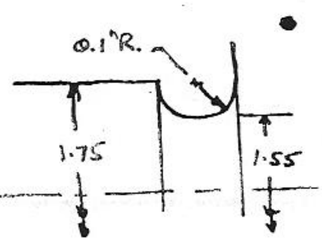
$d_S = 35$ mm(1.38 in)--make 1.40 in

$r_f = 1$ mm(0.039 in)--make 1/32 in

OD = 62 mm(2.441 in)

To minimize machining step quickly from 1.40 in in bearing shoulder to 1.70 in. (A large chamfer will do).

Reentry fillet at shoulder of gear to serve as grinding-wheel relief and provide "square" shoulder for hub.




• $D/d = 1.75/1.55$

$= 1.13$

$r/d = 0.1/0.55$

$= 0.065$

Table A-15-14 : $K_t = 2.0$

Fig. 5-16: $q = 0.75$

$$k_e = \frac{1}{1 + 0.75(2.0 - 1)} = 0.571$$

$k_a = (2.7)(68)^{-0.265} = 0.883$

$k_b = (1.55/0.3)^{-0.1133} = 0.830$

$k_c = k_d = 1$

$S_e' = 0.504(68) = 34.3$ kpsi

$S_e = 0.883(0.83)(1)(1)(0.571)(34.3)$

$= 14.3$ kpsi

$$M = \frac{3 - 0.75}{3}(1414) = 1061 \text{ lb·in}$$

There is not torque at this point.

$$d^3 = \frac{32nM}{\pi S_e}\ ;\ n = \frac{(1.55)^3(\pi)(14.3)(10^3)}{32(1061)}$$

$= 4.93$ ok

Keyways: The coupling uses ¼-in square key. Find length for bearing stress.

$F = T/r = 2532/0.5 = 5064$ lb

$A = L/8$

$$\sigma_b = \frac{5064}{0.125L} = \frac{S_y}{2} = \frac{57\ 000}{2}$$

$$L = \frac{5064(2)}{0.125(57\ 000)} = 1.42 \text{ in; use } 1\tfrac{1}{2} \text{ in}$$

Gear key: $F = T/r = 2532/(1.75/2)$

$= 2894$ lb

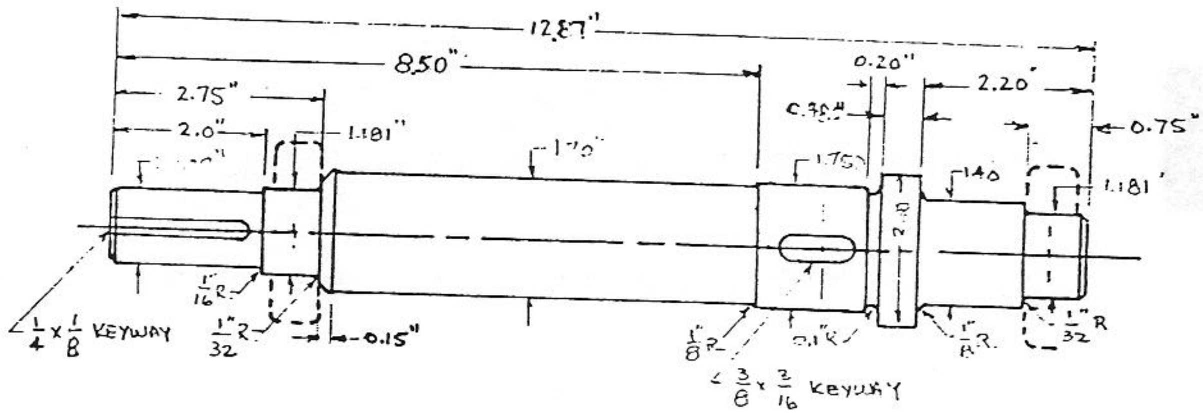
For 3/8-in square key, $A = 3L/16$

$\sigma_b = 2894/(3L/16) = S_y/2 = 57\ 000/2$

$$L = \frac{2(2894)}{\frac{3}{16}(57\ 000)} = 0.54 \text{ in; use 5/8 in.}$$

Fits: A light press fit for gear hub is in order, but until we have the bore tolerance we cannot establish the gear hub seat tolerances. Similarly for the coupling.

18-18 (Concluded)




18-19 Let S = scale factor. Then, in the iconic model $\ell_m = S\ell$.

Eq. (2-29): $\sigma = Mc/I$. If $\sigma_m = \sigma$, then

$$M_m = \frac{\sigma_m I_m}{c_m} = \frac{\sigma S^4 I}{Sc} = S^3 \frac{\sigma I}{c} = S^3 M$$

The load that causes bending is related to reaction and distance.

$$M_m = R_m a_m = \frac{F_m b_m a_m}{\ell_m}; \text{ Solving for } F_m$$

$$F_m = \frac{M_m \ell_m}{a_m b_m} = \frac{S^3 M S\ell}{(Sa)(Sb)} = S^2 F \qquad \underline{\text{Ans.}}$$

For the deflection use Table A-9-6.

$$y_m = \frac{F_m b_m x_m}{6E_m I_m \ell_m}(x_m^2 + b_m^2 - \ell_m^2)$$

$$= \frac{S^2F(Sb)(Sx)}{6ES^4I(S\ell)}(S^2x^2 + S^2b^2 - S^2\ell^2)$$

$= Sy$ Ans. which is expected.

From Prob. 3-19:

$$d = \left|\frac{32Fb(b^2 - \ell^2)}{32E\ell\xi}\right|^{\frac{1}{4}}$$

$$\xi_m = \frac{32F_m b_m(b_m^2 - \ell_m^2)}{32E_m \ell_m d_m^4}$$

$$\xi_m = \frac{32(S^2F)(Sb)(S^2b^2 - S^2\ell^2)}{32E(S\ell)(S^4d^4)}$$

$= \xi$ Ans.

Summary:

Slope: $\xi_m = \xi$

Deflection: $y_m = Sy = y/2$

Moment: $M_m = S^3M = M/8$

Force: $F_m = S^2F = F/4$

For identical material and stress levels.

18-20 OMITTED